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A

COURSE

OF

Experimental Philoſophy.

53264
ac.

BY

J. T. DESAGULIERS, LL. D. F. R. S.

Chaplain to his Grace the DUKE of CHANDOS.

VOL. I.

Adorn'd with Thirty-two COPPER-PLATES.

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God
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TO HIS

ROYAL HIGHNESS

FREDERIC,

Prince of WALES, &c.

SIR,

TO contemplate the Works of GOD, to diſcover Cauſes from their Effects, and make Art and Nature ſubſervient to the Neceſſities of Life by a Skill in joining proper Cauſes to produce the moſt uſeful Effects, is the Buſineſs of a Science, the Grounds and Principles of which I have the Honour to lay at Your ROYAL HIGHNESS's Feet.

I have preſum'd to do this, Philoſophy in all Ages having been thought worthy the Conſideration of Princes; for the greateſt and beſt, who have been handed down to us by Hiſtorians, under the moſt amiable and glorious Characters, have diſtinguiſh'd themſelves by their Attainments in it, and their Encouragement of all Endeavours to advance and improve it.

Amongſt theſe I may juſtly reckon his late MAJESTY, who was, and his preſent MAJESTY, who is, the Patron of a Society, erected for the Advancement of Natural Knowledge, of which one of their Royal Predeceſſors was the Founder.

DEDICATION.

You known Candor (GREAT SIR) will, I hope, excuſe my plain Manner of treating this Subject: Tho' I take the Liberty to aſſure your ROYAL HIGHNESS, that I have ſpared no Pains, that the Work might not be wholly unworthy of Your Patronage.

It preſumes the more upon Your Countenance and Protection, as it is an Account of thoſe Experiments, which I had the Honour to make ſome Years ago at *Hampton-Court*, before his late MAJESTY, by his particular Command; and as their preſent MAJESTIES, who likewiſe honour'd thoſe Experiments with their Preſence, have alſo given me Leave to ſet their Names at the Head of my Subſcribers.

Were I equal to the Deſcription of thoſe excellent Qualities, which make You the preſent Delight, and future Hope of *Britain*; yet I ſhould be guilty of the higheſt Indiſcretion to attempt it in an Addreſs to Yourſelf; ſince none are more offended with Praiſe than thoſe who moſt deſerve it. I ſhall therefore no longer detain Your ROYAL HIGHNESS, than whilſt I beg Leave to ſubſcribe myſelf, with the greateſt Reſpect,

SIR,

Your moſt humble,

moſt obedient,

and moſt dutiful Servant,

J. T. DESAGULIERS.

PREFACE.

ALL the Knowledge we have of Nature depends upon Facts; for without Obſervations and Experiments, our natural Philoſophy would only be a Science of Terms and an unintelligible Jargon. But then we muſt call in Geometry and Arithmetick to our Aſſiſtance, unleſs we are willing to content ourſelves with natural Hiſtory and conjectural Philoſophy. For, as many Cauſes concur in the Production of compound Effects, we are liable to miſtake the predominant Cauſe, unleſs we can meaſure the Quantity of the Effects produc'd, compare them with, and diſtinguiſh them from each other, to find out the adequate Cauſe of each ſingle Effect, and what muſt be the Reſult of their joint Action. When Monſ. Des Cartes's *philoſophical Romance, by the Elegance of its Style and the plauſible Accounts of natural* Phænomena, *had overthrown the* Ariſtotelian *Phyſicks, the World receiv'd but little Advantage by the Change: For, inſtead of a few Pedants, who, moſt of them, being conſcious of their Ignorance, conceal'd it with hard Words and pompous Terms; a new Set of Philoſophers ſtarted up, whoſe lazy Diſpoſition eaſily fell in with a Philoſophy, that required no Mathematicks to underſtand it; and who taking a few Principles for granted, without examining their Reality or Conſiſtence with each other, fancied they could ſolve all Appearances mechanically by Matter and Motion; and, in their ſmattering Way, pretended to demonſtrate ſuch things, as perhaps* Carteſius *himſelf never believ'd; his Philoſophy (if he had*

had ſeen in earneſt) being unable to ſtand the Teſt of the Geometry which he was Maſter of.

It is to Sir Iſaac Newton's *Application of Geometry to Philoſophy, that we owe the routing of this Army of* Goths *and* Vandals *in the philoſophical World; which he has enrich'd with more and greater Diſcoveries, than all the Philoſophers that went before him: And has laid ſuch Foundations for future Acquiſitions; that even after his Death, his Works ſtill promote natural Knowledge. Before Sir* Iſaac, *we had but wild Gueſſes at the Cauſe of the Motion of the Comets and Planets round the Sun; but now he has clearly deduc'd them from the univerſal Laws of Attraction (the Exiſtence of which he has prov'd beyond Contradiction) and has ſhewn, that the ſeeming Irregularities of the Moon, which Aſtronomers were unable to expreſs in Numbers, are but the juſt Conſequences of the Actions of the Sun and Earth upon it, according to their different Poſitions. His Principles clear up all Difficulties of the various* Phænomena *of the Tides; and the true Figure of the Earth is now plainly ſhewn to be a flatted Spheroid higher at the Æquator than the Poles, notwithſtanding many Aſſertations and Conjectures to the contrary. Our* incomparable Philoſopher *has diſcovered and demonſtrated to us the true Nature of Light and Colours, of which the moſt ſagacious and inquiſitive Naturaliſts were entirely ignorant; for while they ſought for the Origin of Colours in the Mixture of Light and Shadow, Sir* Iſaac Newton *found that they were congenial with the Rays of the Sun, and contain'd in Light itſelf; the Surface of colour'd Bodies ſerving only to ſeparate from one another thoſe Rays, that make different Colours; by abſorbing ſome, and reflecting others to our Eyes, ſo as to produce thoſe different Senſations, on which the pleaſing Variety of colour'd Objects depends. His Optics, beſides the Properties of Light, contain a vaſt Fund of Philoſophy; which (tho' he has modeſtly delivered under the Name of* Queries, *as if they were only Conjectures) daily Experiments and Obſervations confirm;*

firm; a notable Inſtance of which may be ſeen in the Rev. Dr. Stephen Hales's *excellent Book of* Vegetable Staticks, *which, by putting ſeveral of Sir* Iſaac's Queries *out of all Doubt, ſhew how well they were founded. I paſs over Sir* Iſaac Newton's *noble Inventions in pure Mathematicks, juſtly admired at home and abroad; becauſe, tho' they have been of great Uſe in the Diſcovery of the Cauſes of natural* Phænomena, *they are foreign to my preſent Subject, which is* Phyſicks; *whoſe Knowledge I am in this* Courſe *endeavouring to convey by Experiments; not only where Things have been diſcover'd that way, but even where they have been deduc'd by a long Train of mathematical Conſequences; having contrived Experiments, which Step by Step bring us to the ſame Concluſions.*

The Thoughts of being oblig'd to underſtand Mathematicks have frighted a great many from the Newtonian Philoſophy: *I have heard ſeveral* Carteſians *ſay, that if the Knowledge of Geometry was neceſſary for their Conviction, they had rather continue in their own Way of Philoſophy than to be at ſo much Trouble; as if a Man could deſerve the Name of a Philoſopher, merely becauſe he reaſons juſtly from Principles when thoſe Principles are either apparently falſe, precarious, or aſſum'd at pleaſure to ſerve the preſent Purpoſe. It is not he, that can ſhew from an* Hypotheſis *how the Celeſtial Motions might be performed; but he, that demonſtrates their real Cauſes, who gives a proper Account of the Syſtem of the World: And it is the ſame of other* Phænomena; *for, unleſs we can demonſtrate what we explain, it is better to own our Ignorance, than to endeavour to paſs our Conjectures upon the World for Solutions. If ever we come to know the Cauſes of the various Operations of* Magnetiſm; *it will ſooner be owing to a Compariſon of the Experiments and Obſervations of* Norman, Pound, *Lord* Paiſley, Graham, Muſchenbroek, Savery, Marcel, *and others (who acknowledge themſelves ignorant of the Cauſes of thoſe ſurprizing Effects) than to twenty* Hypotheſes *of Men, whoſe warm Imaginations ſupply them with what may ſupport*

their Solutions, while daily Obſervations and common Laws of Motion *can eaſily confute them.*

But to return to the Newtonian Philoſophy: *Tho' its Truth is ſupported by Mathematicks, yet its Phyſical Diſcoveries may be communicated without. The great Mr.* Locke *was the firſt who became a* Newtonian Philoſopher *without the help of Geometry; for having aſked Mr.* Huygens, *whether all the mathematical* Propoſitions *in Sir* Iſaac's Principia *were true, and being told he might depend upon their Certainty; he took them for granted, and carefully examined the Reaſonings and* Corollaries *drawn from them, became Maſter of all the Phyſics, and was fully convinc'd of the great Diſcoveries contain'd in that Book: Thus alſo he read the Opticks with Pleaſure, acquainting himſelf with every thing in them that was not merely mathematical *. But ſince Machines have been contriv'd to explain and prove experimentally what Sir* Iſaac Newton *has demonſtrated mathematically, and ſeveral of his own* Experiments *are ſhewn in publick* Courſes; *a great many Perſons get a conſiderable Knowledge of* Natural Philoſophy *by Way of Amuſement; and ſome are ſo well pleas'd with what they learn that way, as to be induc'd to ſtudy Mathematicks, by which they at laſt become eminent Philoſophers. Dr.* John Keill *was the firſt who publickly taught* Natural Philoſophy *by* Experiments *in a mathematical Manner; for he had laid down very ſimple* Propoſitions, *which he prov'd by* Experiments, *and from thoſe he deduc'd others more compound, which he ſtill confirmed by* Experiments; *till he had inſtructed his Auditors in the* Laws of Motion, *the* Principles *of* Hydroſtaticks *and* Opticks, *and ſome of the chief* Propoſitions *of Sir* Iſaac Newton *concerning* Light *and* Colours. *He began theſe* Courſes *in* Oxford, *about the Year* 1704 *or* 1705, *and that Way introduc'd the Love of the* Newtonian Philoſophy. *There were indeed, about the ſame*

* This I was told ſeveral times by Sir *Iſaac Newton* himſelf.

time,

time, Experiments *ſhewn at* London *by the late Mr.* Hauksbee *which were* electrical, hydroſtatical, *and* pneumatical: *But as they were only ſhewn and explain'd as ſo many curious* Phænomena, *and not made Uſe of as* Mediums *to prove a Series of philoſophical Propoſitions in a mathematical Order, they laid no ſuch Foundation for true Philoſophy as Dr.* Keill's Experiments; *tho' perhaps perform'd more dexterouſly and with a finer* Apparatus: *They were* Courſes of Experiments, *and his a* Courſe of Experimental Philoſophy.

When Dr. Keill *left the Univerſity, I began to teach* Experimental Philoſophy, *after the ſame Method that he had done, adding the* Mechanicks *(ſtrictly ſo call'd, that is, the Explanation of mechanical Organs, and the Reaſon of their Effects,) and ſeveral* Optical Propoſitions *in my* Courſes of Experimental Philoſophy; *which ever ſince that time I have endeavour'd to improve, by the Addition of new* Propoſitions *and* Experiments, *and by altering and changing my* Machines, *as I found Things might be made more intelligible to ſuch of my Auditors as were not acquainted with* Mathematicks, *or more ſatisfactory to ſuch as were; eſpecially in what regards the Cauſes of the Motions of the heavenly Bodies, and the* Phænomena *of our* Syſtem. *About the Year* 1713, *I came to ſettle at* London, *where I have with great Pleaſure ſeen the* Newtonian Philoſophy *ſo generally received among Perſons of all Ranks and Profeſſions, and even the* Ladies, *by the Help of Experiments; that tho' ſeveral ingenious Men have ſince that Time with great Succeſs taught (and do ſtill teach)* Experimental Philoſophy *in my (or rather Dr.* Keill's*) manner, I have had as many* Courſes *as I could poſſibly attend; the preſent* Courſe, *which I am now engag'd in, being the* 121*ſt ſince I began at* Hart-Hall *in* Oxford, *in the Year* 1710. *The Satisfaction we enjoy by being any way inſtrumental to the improvement of others, is ſo great, that I can't help boaſting — that of eleven or twelve Perſons, who perform* Experimental Courſes *at this Time in* England, *and other Parts of the World, I have had the*

Honour of having Eight of them for my Scholars; *whoſe further Diſcoveries become an Advantage to myſelf; for what would raiſe Envy in any other Profeſſion, but that of a* Philoſopher, *is receiv'd as a new Acquiſition by all Lovers of* Natural Knowledge, *the Profit being ſhar'd in common, while the Diſcoverer has only the Honour of the Invention.*

For this Reaſon, I never ſcruple making uſe of Machines *and* Inſtruments *contriv'd by others; nor was I ever ſhy of communicating, or even lending, my own; to thoſe, who wanted to imitate them: it is enough to acknowledge the Author of any new Contrivance, which I generally do.*

As the greateſt Part of my Auditors, at whoſe Deſire I have printed this Courſe, *are but little vers'd in* Mathematical Sciences; *the* Lectures *are free from difficult* geometrical Demonſtrations *and* algebraical Calculations; *and the ſame thing is often prov'd by ſeveral Experiments; that where one does not immediately ſtrike with a clear Conviction, another may. I only require Attention and common Senſe, with a very little* Arithmetick, *in my Readers, to qualify them for underſtanding theſe* Lectures; *provided they begin the firſt* Lecture, *and go on regularly, that they may advance from the eaſieſt Truths to thoſe more complex ones, which are deduc'd from them: for otherwiſe many things may ſeem difficult to a Perſon, who ſhould open the Book at random; eſpecially great Part of the laſt* Lecture *of this* Volume, *which yet may be clearly underſtood by all, who have made themſelves Maſters of what goes before. Perhaps the* Mathematicians *may think me tedious and verboſe in my* Lectures; *but ſuch of them as have been uſed to teach, know very well, that one cannot be too plain and explicit with thoſe that are not born with a* Genius *for* Mathematicks *(whatever good Underſtanding they are otherwiſe endow'd with) nay ſometimes one muſt make uſe of ſuch ways of demonſtrating as are not mathematically true, to prepare them for what is a little more abſtract; as I*

have been often forc'd to do to a large Audience, where close Attention is not very common. But I hope the rigid Philosophers *will forgive me, when they find the same Things geometrically demonstrated in the Annotations; in the perusal of which, the* Mathematicians *perhaps will not think their Time wholly lost. However I don't mean to exclude common Readers from the Notes; for those that have carefully read and understood the* Lectures, *will thereby be qualified for comprehending what is in the* Annotations.

To prevent the Publick from being impos'd upon, I must not omit mentioning, that several Years ago, some Persons published a Book of Experimental Philosophy *in my Name, without my Knowledge, which they endeavour'd to pass upon the World for my* Lectures. *It is a thin* Quarto, *and was at that Time call'd,* A System of Natural Philosophy: *And as those, who were capable of such a Thing, may, very probably, if they have any of the Books left, endeavour to sell them by giving them the same Title as my Book, I thought proper to give this Caution. But while I am complaining of others, I might be thought to ascribe to myself what is not my own, if I did not acknowledge, that most of what I have said of the* Bow *and* Spring *in the last* Lecture *of this* Volume, *as also Part of what I have said of the* Fly *and the* Battering Ram, *was copied from some Papers lent me by* William Jones, *Esq.*

A COURSE

A COURSE OF Experimental Philoſophy.

LECTURE I.

THE Spirit of Diſputing, and an earneſt Deſire to obtain the Victory, rather than come at the Truth, has been one of the greateſt Impediments to the Improvement of natural Knowledge. And nothing occaſions more wrangling, than when different Perſons annex different Ideas to the ſame Words; for Men can never rightly underſtand one another, till they mean the ſame Things by the ſame Expreſſions. No Diſputes ariſe in pure Mathematics, becauſe the Definitions of the Terms are premiſed; and every body that reads a Propoſition has the ſame Idea of every Part of it: So that when any Man is ſo weak as to think to bring one Demonſtration in oppoſition to another, he is to be diſregarded by every one that would reaſon like a

Lect. I. Philoſopher: For we may immediately put an end to all Mathematical Controverſies, by ſhewing, either that our Adverſary has not ſtuck to his Definitions, or has not laid down true Premiſes, or elſe that he has drawn falſe Concluſions from the Principles he has laid down; or at leaſt own, that we do not underſtand ſome part of his Demonſtration, and deſire him to explain it: for unleſs we can ſhew where the Error or Paralogiſm is, we muſt not condemn by the Lump, but acquieſce with him in what he has proved.

IT is true, that in mixed Mathematics, where we reaſon mathematically upon Phyſical Subjects, we cannot give ſuch juſt Definitions as the Geometricians or Logicians do: We muſt be content with Deſcriptions, and they will be of the ſame Uſe as Definitions, provided we are always conſiſtent with ourſelves, and always mean the ſame Things by thoſe Terms, that we have once explained: For to ſay, that others have taken the Words we uſe in a different Senſe, can be no valid Objection; becauſe it may be anſwered, that then they meant not the ſame Things as we do. Therefore in this Courſe, when we make uſe of ſuch Words, as have been variouſly underſtood, we ſhall ſhew the particular Meaning, in which we would have them taken: And when we are obliged to coin new Terms, as it will often happen in the Deſcription of Machines, we ſhall always explain them the firſt Time they are mentioned.

1. BY the Word *Matter*, we underſtand all that has Extenſion and Reſiſtance: And becauſe all Bodies, whether ſolid or fluid, are extended and do reſiſt, therefore we ſay, that all Bodies are made of Matter.

2. THE *Carteſians* would have Matter to conſiſt in Extenſion alone; but Extenſion without Reſiſtance is nothing but mere *Space*. For though they affirm that one cannot have an Idea of Extenſion without Body; it is contrary to Experience: Since if we take a Cube out of a cubic Box, which it exactly filled, we may very eaſily conceive the Length, Breadth, and Depth of the empty Box; and it muſt be a ſecond Idea, that will give us a Notion either of ſome other Body coming in to fill the Box, or of its Sides coming together by the Preſſure of ambient Bodies.

3. THAT

Lect. I.

3. THAT Matter is the same in all Bodies, is evident from its Definition; though the common Acceptation of the Word would give us another Notion; for in our common Discourse we say, *an Instrument of* Wood, Brass, *or* Iron, *or of any other Matter*, as if the Difference of Bodies consisted in their different Kind of Matter: Whereas all Matter is *homogeneous*, or of the same Nature in all Bodies, whether solid or fluid, hard or soft, more or less heavy; whether they belong to the Earth, or any other Part of the Universe: as for Example, the Matter of Cork does not essentially differ from the Matter of Flesh, or that of Gold or Diamonds. But the whole Variety of Bodies *, and the different Changes that happen in them, entirely depend upon the Situation, Distance, Magnitude, Figure, Structure, Powers, and Cohesion of the Parts that compound them.

* Ann. 1.

4. THAT Mercury resists more than Water *, and Water more than Air, is not owing to the one being of a more resisting Matter than the other; but to the greater Number of Particles contained in the same Space in the heavier Body; and often to the stronger Cohesion of Parts: and then a lighter may resist more than a heavier Body, to a Force impress'd to separate its Parts; as Wood will resist more than Water, and a Diamond more than Gold. But where there is little or no Cohesion of Parts, even in the subtilest Fluids, we find a Resistance: For Light condensed by a Burning-Glass, though many thousand times rarer than Air, has a considerable Resistance, as appears from its separating the Parts of Bodies so forcibly, and so soon, when they are placed in the Focus of the Glass: Nay, and when the Rays of Light are as much dispersed as they are here on Earth, coming directly from the Sun, they have a sensible impelling Force, as appears by observing, that the Vapour arising from a Comet (which makes its Tail) is always driven toward that Side of the Comet, which is opposite to the Sun; and that happens whether the Comet be going towards, or coming from the Sun, even at Distances equal to and greater than that of the Earth. And if there be a Medium finer than * Light (as we have Reason to believe from some Phænomena) even that Medium has a Resistance, whereby it refracts, reflects, and bends the Rays of Light near the Surfaces and Sides of Bodies.

* Ann. 2.

* Ann. 3.

Lect. I.

5. THAT Quantity, and consequently Matter, is divisible *in Infinitum* *, has been several Ways demonstrated by Mathematicians; and one cannot conceive a Particle of Matter ever so small, but what is still divisible; for since it is a Body, it must have a Top, a Bottom, and a Middle, unless we suppose the Top and Bottom to be the same, which is absurd: and if so, we may conceive such a small Particle to be divided in the Middle. But then we are not by such a Division to imagine the Parts to be separated from each other, any more than (when we divide a cubic Space of two Inches into eight cubic Spaces of an Inch) those new Cubes are to be supposed removed from each other, or taken out of the two Inch Cube which contains them.

* Ann. 4.

Ann. 5. 6. As to the actual Division of Matter * by separating the Parts from each other, it is not possible beyond a certain Degree; because there are *Atoms*, or extremely small Parts, which are called the constituent or component Parts of natural Bodies, which the Wise and Almighty Author of Nature did at first create as the original Particles of Matter, from which all corporeal Natures were to arise, that are without Pores, solid, firm, and impenetrable, perfectly passive, and moveable: So that the utmost Mechanical Division that we can arrive at, does only separate some of these first Parts from one another, and alter their Contact; for mixed and compounded Bodies are destroyed by such a Separation, and not by breaking the original Particles to pieces. These primary Particles being perfectly solid, must be much more hard and firm than any Bodies that can be made out of them with Pores or hidden Vacuities interspersed; that is, so perfectly hard and firm, that they can never be worn away or diminished: For it is not reasonable to suppose that there should be any Force or Power in the ordinary Course of Nature, that can divide that into several Parts, which God in his first Creation of Things has made *One*. As long therefore as the original Particles remain entire, there may for ever be Bodies made or composed of them, which shall have the same Nature and Texture: But if *these* could be *broken*, *worn away* or *diminished*, then the Nature of corporeal Things, which is dependent on these, might be changed. Earth and Water composed of either such Particles as have been worn or broken, or of their Fragments, could not have, at

at this Day, the same Nature and Texture, as that original Earth and Water, which was composed of these Particles when they were sound and entire. Wherefore that the Nature of Things should last, and their natural Course continue the same; all the Changes made in Bodies must arise only from the various Separations, new Conjunctions and Motions of these original Particles. Lect. I.

7. THESE must be imagined of an unconceivable Smallness*; but by the Union of several of them together, there are made bigger Lumps or Parts of the first Composition (as they are called) which have Interstices between them, because the first Particles do not touch in every Part of their Surface; and those Interstices are called the Pores of the first Composition. Likewise by the uniting of several of these Lumps there are formed *Moleculæ*, or Lumps of the second Composition, which have Pores of the second Composition, larger than the former: And so on to the several Compositions before we come to Bodies of a sensible Magnitude. Hence it follows, that there must be a great deal of Vacuity * interspersed in all Bodies, according as they are made up of fewer or more Compositions; and all Spaces are not equally full of Matter *. This will be plainly shewed by an Experiment. * Ann. 6. * Ann. 7. * Ann. 8.

EXPERIMENT I. *Plate* I. *Fig.* I.

8. UPON B the Brass Plate of the Air-Pump (which we shall hereafter describe) set a tall Cylindric Glass-receiver A B open at both Ends of about five Inches Diameter, and seven or eight Foot high. Let it be made Air-tight by means of a wet Leather upon the Plate B, and another upon the Mouth of the Receiver A under the Covering Plate D, to which Plate underneath is screwed the Machine *P s p s* contrived for letting fall Bodies *in Vacuo* at the same Instant of Time. For when the Wire W (which slips to and fro through the Collar of oiled Leathers *c*, that the Air may no way escape) is drawn up by its Hook *h*, the square horizontal Plate *p* being brought up to a narrow Part of the Brass Springs *s*, *s*, causes them to open so as to let the Plate P (moving on an Hinge) fall into a vertical Position; upon which the Bodies that were laid on it drop at the same Moment.

THEN

Lect. I. THEN upon the square Plate P lay a Down Feather, and a Guinea just by the Side of it (See *Fig.* 2. where the Plate P is in an horizontal Situation, as it rests on the Return of one of the Springs *s*, and the Collar *c* is separated from the covering Plate and Springs, through both which it must be screwed when on the Receiver) and having exhausted the Air from the Receiver; by pulling up the Wire W, let the Guinea and Feather drop, which falling both with the same Velocity (as at C) will come to the Bottom at the same Time: But if the Air had not been exhausted, the Guinea would have been at Bottom before the Feather had fallen a quarter of the Height of the Receiver.

9. IF the Receiver had been perfectly full before the Air was drawn out of it (though then there was much more Vacuum * than Matter) yet it is evident, that there must be a great deal of Vacuity in it after the Air is pumped out; because the Resistance is so far diminished, that the Feather falls at least four Times faster than it did in the common Air. For whatever fine Air was left in the Receiver; whatever Particles of Light, or whatever subtile Effluvia penetrate the Glass, and get into the Receiver; all that Matter is much less in Quantity than the Air taken out, because the Resistance is diminished. For to say that after Exhaustion the Receiver is as full as before, would be as absurd, as to say, that a Gallon of Beer turned into Froth (so as to reach from the Bottom to the Top of the Receiver) fills it as full as eight Gallons of Beer without Froth, which it is capable to contain.

* Ann. 9.

10. GRAVITY may be looked upon as a Property of Matter, which though not essential, is yet universal, and in one Sense inseparable from it; that is, all Parcels of Matter, however modified, (or all Bodies) have a Gravitation or Attraction towards one another; as will be hereafter demonstrated in respect of heavenly as well as terrestrial Bodies: The Tendency of heavy Bodies towards the Center of the Earth, being owing to the same Cause, that makes the Sun and Planets tend towards one another. N. B. *When we use the Words* Gravity, Gravitation, *or* Attraction; *we have a Regard not to the Cause, but to the Effect; namely to that Force, which Bodies have when they are carried towards each other,*

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PL. I.

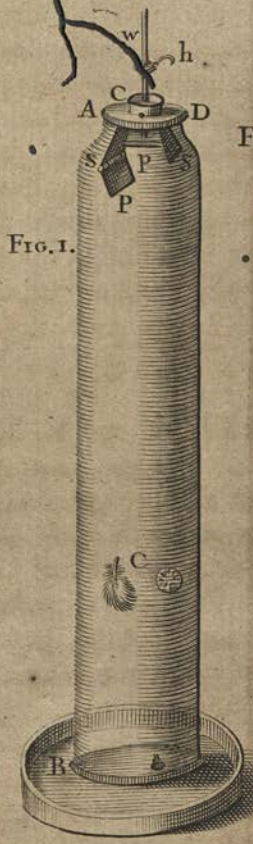

FIG. I.

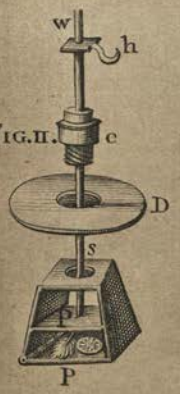

FIG. II.

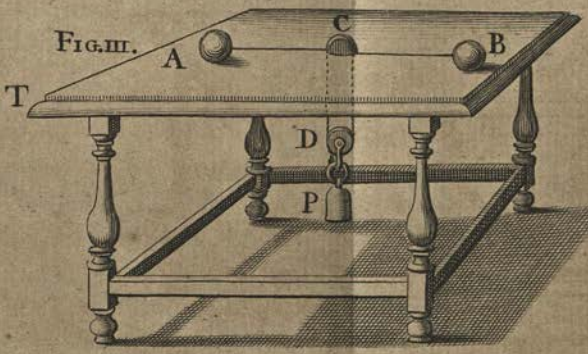

FIG. III.

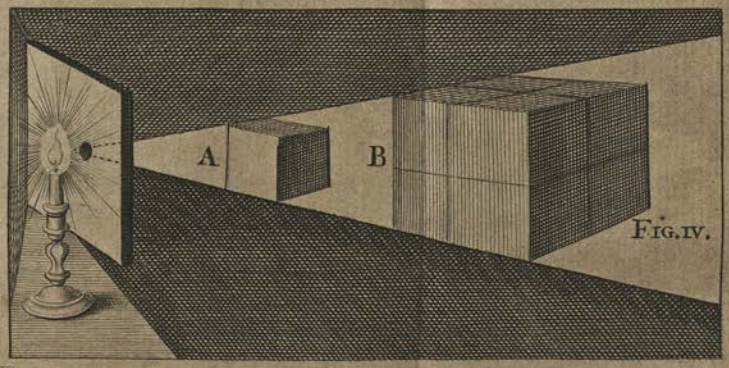

FIG. IV.

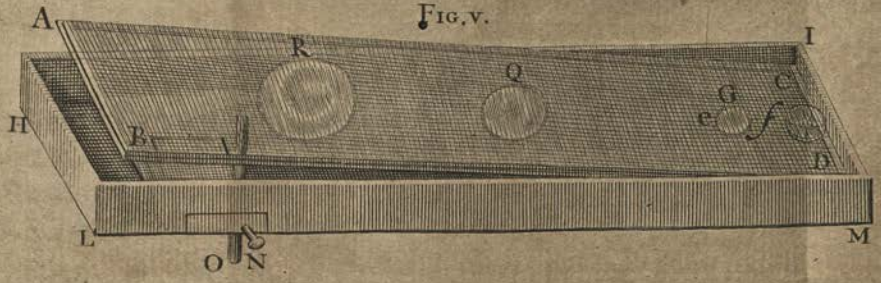

FIG. V.

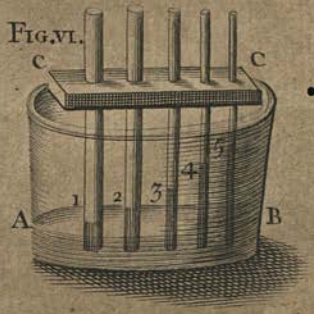

FIG. VI.

other, which (at equal distances) is always proportionable to their Quantity of Matter; whether it be occasioned by the Impulsion of any subtile Fluid, or by any unknown and unmechanical Power concomitant to all Matter. Lect. I.

11. If all the Matter in the Universe was contained in two equal Balls or Spheres, placed at some Distance from each other; these Spheres would move towards one another with equal Velocity, so as to meet in the middle of their first Distance. But if the Spheres be supposed unequal in any Proportion; they would meet in a Point as much nearer the great Ball, as the great Ball would be bigger than the other.

12. The Reason why we do not perceive this mutual Attraction in such Bodies, as we daily handle, is, that our Earth having infinitely more Matter than those Bodies, attracts them so strongly as to make their mutual Tendency towards each other insensible. So it happens in respect of a Load-stone and a Piece of Iron; which when let fall at a little Distance from one another, do not appear to move towards each other; though we find their Effect sensible when brought near together.

This will be illustrated by the following Experiment.

Experiment II. *Plate* 1. *Fig.* 3.

Upon the Table TAB set the two Balls A, B, equal in Bigness and Weight, (for Example, of two Ounces each) at the Distance of a Foot from the Hole C, and two Foot from each other. Now if the Earth was annihilated, or was removed to an infinite Distance, and the Table did not attract, the two Balls would come to each other and meet at C; but Gravity, or the Attraction of the Earth, pressing them against the Table, they remain at rest: but to overcome that Pressure, and make them act as if the Earth was away, let a String of abot 30 Inches long be tied to the Ball A, then brought through the smooth Hole in the Table C, then round the Pulley under the Table, and so up again through the Hole C, and at last fastened to the Ball B. Let the Weight P of four Ounces hang from the Center of the Pulley D: Then if you let go both the Balls at the same Time, they will

Lect. I. will come to each other equally faſt, and meet juſt over the Hole.

13. If inſtead of the Ball A, you ſubſtitute another, which weighs but one Ounce (whether it be leſs than the other, or as big, but hollow, or of light Wood) and this Ball A be let go from the Diſtance of one Foot, and the heavier Ball B from the Diſtance of ſix Inches from the Hole (hanging three Ounces at the Pulley) they will both meet again at the Hole, the lighter Ball going through twice the Space that the other does.

14. In the firſt Caſe, the Quantity of Matter in the two Balls, which is four Ounces, being conſidered as divided into two, cauſes a two Ounce Ball on each Side to move towards the Hole, which conſequently move through an equal Space in the ſame Time.

15. In the ſecond Caſe, the Quantity of Matter which is three Ounces, being divided into two, does on one Side cauſe a Ball of two Ounces to move towards C, and on the other a Ball of one Ounce to move towards the other; which laſt moves twice as faſt, becauſe it has but half the Quantity of Matter, weighing but half.

* Ann. 10. N. B. *Any Weight may hang to the Pulley*, provided it be not too light; becauſe it only ſerves to overcome that Gravity which preſſing the Balls againſt the Table, hinders them from moving towards each other; as they would do, if the Earth did not exiſt, or was removed to an infinite Diſtance.*

16. If the Ball B be ſuppoſed infinitely greater than the Ball A, the Velocity of the Ball A will then become infinitely greater than that of the Ball B, ſo that B will ſtand ſtill, and A move through the whole Diſtance between A and B; the whole Quantity of Matter belonging to both, being now wholly attributed to B alone, and A looked upon as a Point without any ſenſible Quantity of Matter, and conſequently unable to move the Ball B by its Attraction. This is applicable to the Earth and all the Bodies about it, which in reſpect of them is looked upon as immoveable; whilſt they, as ſo many phyſical Points, move in the Lines deſcribed by their Centers of Gravity, as they fall to the Ground,

Ground, without having any Regard to the Quantity of Matter, which they contain, whereby they attract the Earth towards them. Thus also since the Sun contains almost 230000 Times more Matter than the Earth, this last is looked upon as a Point describing an Ellipsis about the Sun, called the *Magnus Orbis*, whilst the Sun, that attracts the Earth, is considered as immoveable in one of the *Foci* of that Ellipsis: Or rather the Moon and Earth together may be considered as reduced to one Point, which is their common Center of Gravity, and which describes the Orbit above mentioned. Lect. I.

17. The Earth in respect of the Bodies about it, and the Sun in respect of the Planets and Comets, and all the Planets in respect to the Satellites and other circumambient Bodies, do exert a greater or lesser Force of Attraction, according as the Bodies are nearer or farther off; for Gravity, * being a Virtue diffused from an attracting Body every Way in right Lines, decreases in the same Manner as all other Virtues propagated from a Center round about. So Light and Heat become weaker, as we recede from the lucid or hot Body. This Decrease of Virtue is in a reciprocal duplicate Proportion of the Distance; that is, at twice the Distance, the Virtue is four times weaker, and at three times the Distance, nine times weaker, *&c.* Thus for Example, if the Earth was three times farther from the Sun, it would be nine times less attracted, nine times less enlightned, and nine times less heated than it is at present: In like manner, if it was four times farther, it would be sixteen times less affected by those Qualities. So on the contrary, if it was three or four times nearer than it is now, it would be nine or sixteen times more affected. * Ann. 11.

This Proportion of Increase or Decrease of Qualities diffused every Way, may be illustrated by the following

Experiment III. *Plate* 1. *Fig.* 4.

18. Take a Candle, so small that its Rays, that are diffused every Way, may proceed as it were from one Point; (for if the Candle be large, its Light must be made to pass through a small Hole in a Pastboard:) and if a Cube * of an Inch, as A, be held up upon a Needle, at the Distance of one Foot from the Candle, * Ann. 12.

Lect. I. dle, its Shadow will cover the Surface of a two Inch Cube B, held at the Diſtance of two Foot from the Candle; which laſt Surface is four Times larger than that of the firſt Cube, as appears by applying the firſt Cube upon the laſt. Hereby it is evident, that if the firſt Cube was twice as far from the Candle it would receive but the 4th Part of the Light; and but the 9th Part, if it was three Times as far; becauſe when held at one Foot from the Candle, its Shadow will cover a three Inch Cube held three Times as far. In the ſame manner will a Sphere of an Inch Diameter at one Foot from the Candle intercept all the Light, which would fall upon a two Inch Sphere at two Foot, or a three Inch Sphere at three Foot from the Candle, their Surfaces being as 1, 4 and 9, proportionable to the Squares of their Diameters.

19. As it is eaſier to raiſe moſt Bodies from the Ground than to break them in pieces; that Force by which their Parts cohere, is ſtronger than their Gravity. That Force, whatever be its Cauſe, we ſhall call the *Attraction of Coheſion*. This Attraction is ſtrongeſt, when the Parts of the Bodies touch one another; but decreaſes much faſter than Gravity, when the Parts that were before in Contact, ceaſe to touch; and when they come to be at any ſenſible Diſtance, this Attraction of Coheſion becomes almoſt inſenſible.

THIS Attraction is always the ſtrongeſt, where the Contact is the greateſt. As for Example, two Boards of Fir or Oak, being glued in the Middle along the Grain, are not ſo eaſily broken aſunder in the glued Place, as any where elſe; becauſe there are more Pores, and conſequently fewer touching Parts, along the Wood any where elſe, than where the Glue is; for when a Joint is ſhot (as the Workmen call it) or the two Pieces of Wood made ſmooth, in order to join them, the Glue which is ſpread on the Pieces fills the Pores, and cauſes the Wood not only to touch where it did before, but even in the Interſtices where it did not touch; becauſe thoſe little Spaces are filled with Glue, that ſupplies the Place of Wood. On the contrary, when the Wood is more ſolid *, or has fewer Pores than the Glue, it does not hold ſo faſt where it is glued, as in the other Parts of the Wood; which may be ſeen in Brazil-wood, Ebony, or Lignum-vitæ, and in Metals. The Parts of Glaſs, which are almoſt round, touching but in a few

* Ann. 13.

Points, are easily separated, and therefore it breaks easily: And Fluids (whose Parts seem to be spherical) have scarce any Cohesion, except so much as serves to make Drops, whose Roundness plainly proves an Attraction of Cohesion: For if this Roundness depended upon the Pressure of the Air, the Drops would not be round *in Vacuo*; and if it depended upon the external Pressure of any other Fluid whatever, two Drops could never unite into one; because the Figure of any Potion of a Fluid * pressed every way, by the same or any other Fluid, cannot be altered by that Pressure; whereas from a mutual Attraction of the little Globules * that compose the Drop, it must become round; and it will continue so, because then there will be the greatest Contact possible between all the Parts. A Drop of Water, or any other Liquor, does indeed become flat where it touches a Table, or any plane Body, that does not repel it; this is owing to the Attraction of the Table, and to the Gravity of the Drop, if the Table be horizontal. Lect. I. * Ann. 14. * Ann. 15.

How such an Encrease of Contact increases this Attraction, is more evident by the following

Experiment IV. *Plate* 1. *Fig.* 5.

20. Having moistened or thinly smeared over with Oil of Oranges * two Glass Planes A B C D, (eighteen Inches long, and three or four Inches broad) lay them upon one another in an horizontal Position in the wooden Frame H I L M, having first put a Drop of the same Oil upon the undermost Plane at G, and laid a thin Plate or Piece of Money upon the said Plane, between D C, to hinder the upper Plane from touching it at that End whilst their other Ends A B are in close Contact. The Drop being large enough to reach the upper Plane, will immediately be flattened, and move on towards the touching Ends, continually encreasing its Diameter, as at Q and R, and likewise moving faster as it spreads. Nay, though the Planes be raised up at their touching Ends by means of the Prop O N, the Drop will continue to advance towards the touching Ends, but not so fast. When the Drop is at G, a small Elevation of the Planes will retard it; if they be raised a little higher, it will stop; if still higher, the Gravity of the Drop will act more strongly than the Attraction of the Planes, and make the Drop move * Ann. 16.

Lect. I. downwards towards C D. When the Drop is at Q, the Plane will require a much greater Elevation to ſtop the Drop, or make it move downwards; and ſtill a greater when it is at R. Now when the Planes are raiſed ſo high as to make the Drop run back; if the upper Plane be preſſed by the Finger a little above the Drop, ſo as to come nearer to the lower Plane, the Drop will not only ſtop, but move upwards; becauſe it ſpreads more by that Preſſure and touches ſo many more parts of the Glaſs than it would have done, that even at this Elevation of the Planes, their Attraction overcomes the Gravity of the Drop. That it moves at firſt towards A B, is owing to the greater Attraction of the Planes at *e*, where they are nearer together than at *f*; and that the Velocity of the Drop encreaſes, is owing to its moving towards A B, where the Attraction muſt encreaſe continually * as the Planes come nearer and nearer.

* Ann. 17.

SEVERAL other Circumſtances of the Attraction of Coheſion are ſhewn by the following Experiments.

EXPERIMENT V. *Plate* 1. *Fig.* 6.

21. HAVING fixed into a piece of Wood or Cork C C, ſeveral little Glaſs Tubes, the Diameters of whoſe Bores are leſs than one another, (the biggeſt being about $\frac{1}{5}$ of an Inch) let the Ends of thoſe Tubes be dipped into any tinged Liquor that will adhere to Glaſs, as red Water; and the Liquor will ſpontaneouſly riſe in all of them, but always higheſt in thoſe whoſe Diameter is leaſt; as appears at 1, 2, 3, 4, 5, when they are dipped in the red Water of the Veſſel A B.

IN very ſmall capillary Tubes, the Liquor will riſe very high, but then the Colour will be imperceptible: But to make the riſing Liquor viſible in that caſe, the Tube may be applied to one's Finger after it is pricked ſo as to make a drop of Blood iſſue out, which will riſe very quick, and be viſible though the Tube be as ſmall as an Hair. See *Plate* 2. *Fig.* 1.

ANY porous Body will have the Effect of capillary Tubes; thus Water will riſe up into a piece of Bread, or into a piece of Loaf Sugar whoſe lower end is dipped into the Liquor; but it will riſe much higher into Loaf Sugar, becauſe its Interſtices are ſmaller than thoſe of the Bread.

Ex-

EXPERIMENT VI. *Plate* 2. *Fig.* 2.

22. TAKE two ſquare Planes of Glaſs A B C D, and having moiſtened them with Water, ſet them upright in a Veſſel of the ſame Water, preſſing the Sides together at A B, but keeping the oppoſite Sides D C aſunder, by putting a thin Plate between them. The Water will riſe between the Planes in the Curve *e f g*; where it is to be obſerved, that the Fluid always goes higheſt where the Planes are neareſt, namely towards A B, juſt as it happens in the ſmall Glaſs Tubes.

THAT none of theſe Phænomena ariſe from the Preſſure of the Air, is evident by making the Experiments *in Vacuo*; which may be eaſily performed by means of the Wire *w* (*Plate* 1. *Fig.* 2.) which can move up and down through its Collar of Leathers, without admitting the external Air into the Receiver, as it riſes or depreſſes the Planes or the Tubes faſtened to it. *The whole Apparatus for this will be hereafter deſcribed.*

EXPERIMENT VII. *Plate* 2. *Fig.* 3.

23. IN a clean Glaſs that is not full, Water and ſeveral Liquors will be higher at A B where they touch the Glaſs than in the middle C; but that Elevation is hardly ſenſible but near the Sides, becauſe the Attraction of Coheſion reaches but a little way. Quickſilver does the reverſe in this caſe; and in ſmall Tubes it does not riſe ſo high as the Surface of the reſt of the Quickſilver in the Veſſel in which the Tubes are held. The Reaſon of theſe Phænomena * is, that Water and thoſe Liquors are attracted by Glaſs more than it is attracted by it ſelf; and that on the contrary, Quickſilver attracts Quickſilver more than Glaſs attracts it *. * Ann. 18. * Ann. 19.

EXPERIMENT VIII. *Plate* 2. *Fig.* 4.

LET A B be a Cylindrick Glaſs Veſſel filled up to the Line A B with Quickſilver, whoſe Surface is loweſt at A and B the Sides of the Glaſs, where it riſes with a Convexity. If an open Tube of a ſmall bore D E be preſſed againſt the inſide of the Glaſs (to render the Experiment viſible) its lower Orifice

Lect. I. Orifice being almost at the Bottom of the Glass, the Mercury will rise in the said Tube no higher than F below the Surface A B of the Mercury in the Vessel. But to avoid all Cavils about something in the Tube that might be supposed to stop the Mercury, let the Experiment be made in this Manner.

LET the Tube being quite full of Mercury, (and kept so by the Finger pressed on the upper Orifice D,) be put into the Vessel, whose Bottom is a little convex, below the Surface of the Mercury as low as E; upon the Removal of the Finger, the Mercury in the Tube will fall below F, and then rise above it, and after some Vibrations settle at F, the Point to which it rose before.

EXPERIMENT IX. *Plate* 2. *Fig.* 5.

24. INTO the Cylindrick Glass Jar A B, of about five Inches Diameter and 1 $\frac{1}{2}$ Inch deep, pour gently about one Pound of Quicksilver, and a circular part of the Bottom as C C will remain uncovered; then if you shake the Jar to make the Quicksilver come together, the whole Bottom will be covered; but if without any Shake you continue to pour on more Mercury you may put in a Pound or two more before the Bottom be quite covered, the Pit C C becoming continually less, but deeper; and if then you bring the Mercury together to cover the Bottom and destroy the Pit, a Finger being pressed against the Bottom through the Mercury, will, when taken away, leave a Pit as before, the Mercury there remaining convex at the Sides of the Pit; as appears in the vertical Section of the Glass and Quicksilver in the lower Figure 5. *Plate* 2.

EXPERIMENT X. *Plate* 2. *Fig.* 6.

25. HAVING put in still more Mercury till there be no Pit left, lay a Piece of Iron Wire C C about two or three Inches long and $\frac{1}{20}$ of an Inch thick upon the Surface of the Mercury, where it will swim, making a Dent on both Sides as at D; which happens because the Mercury attracts it self more than it does the Iron; so in the case of *Fig.* 5. it was less attracted by the Glass than by it self, and therefore made the Pit: but when once the Sides of the mercurial Pit C C were brought to touch, they never parted again of themselves.

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Plate II.

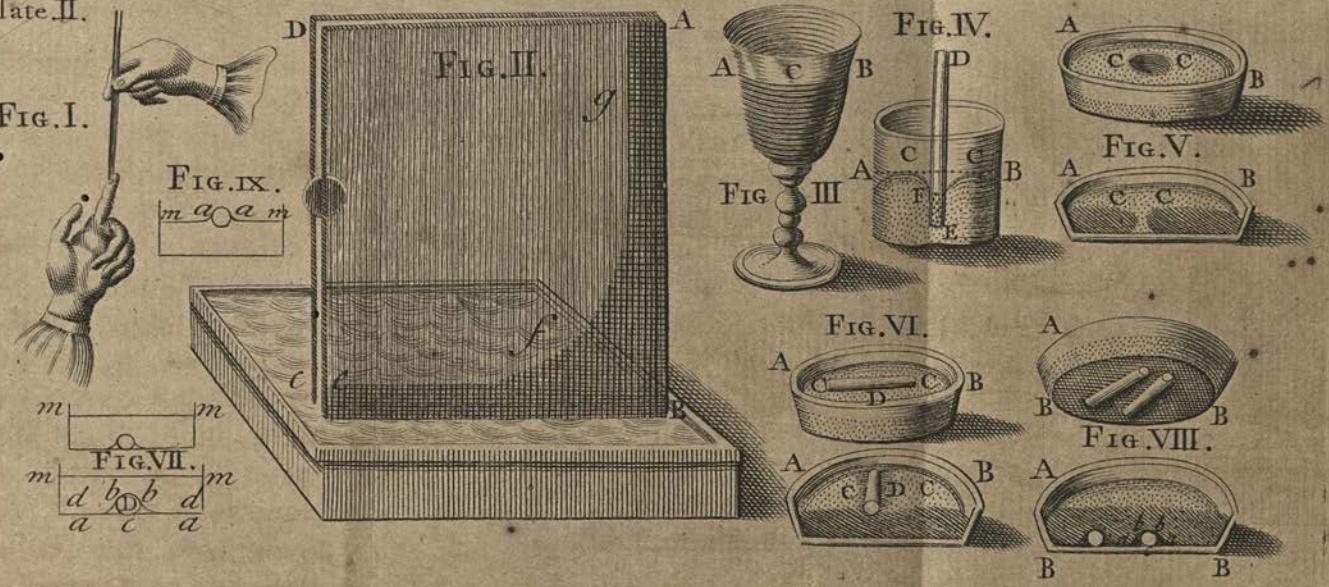
Fig. I.
Fig. II.
Fig. III
Fig. IV.
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Fig. IX.

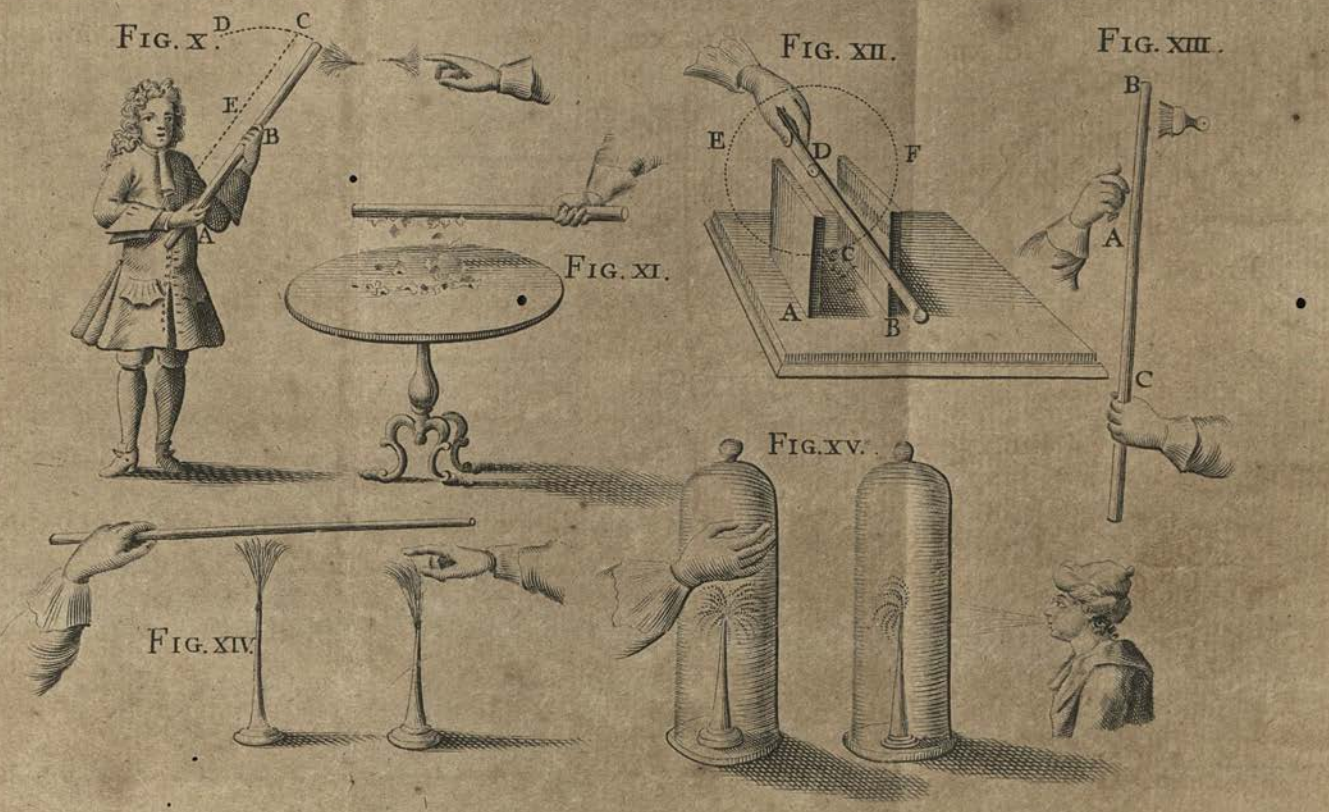


Fig. X.
Fig. XI.
Fig. XII.
Fig. XIII.
Fig. XIV.
Fig. XV.

EXPERIMENT XI. *Plate* 2. *Fig.* 7.

26. IF *m m* represent the Surface of the Mercury, and the Wire which swam at the Top be pressed down to the Bottom, (where D represents its transverse Section,) the Wedges of Mercury *b c a*, that go under the Wire so as very nearly to meet at *c*, will not remain in their Places, but by the rest of the Mercury they will be attracted towards *d*, *d*, so as to leave the Spaces *b c a* without any Mercury; as appears in the Section of the Glass, Wire, and Mercury, *Fig.* 8. And that it is so in fact, appears by looking at the under side of the Glass in the upper *Fig.* 8. where the Wire or Wires (if there be more than one) become visible through the Bottom of the Glass, which could not be unless the Mercury went away from under them; because a Cylinder can touch a Plane but in an invisible Line: And as a farther Proof of this, the Wires, though specifically lighter than Mercury, remain at Bottom, as being pressed only downwards by the Arch of Mercury over them; which could not happen if the Mercury could insinuate it self under.

27. NOW if the Experiment be made with a Silver Wire of the same Bigness as the Iron one, when it is laid at Top, the Mercury will rise up round it as at *a a*, *Fig.* 9. and this Wire will not remain at the Bottom of the Glass, though pushed down to it, but always emerge; neither does it become visible through the Bottom of the Glass, though held down against it with the Finger; nay, even when the Finger is seen on each side of it. This happens because Silver attracts Mercury more than Mercury attracts it self. But to shew that this Attraction is only strong in Contact or extremely near; let the Silver Wire be made a little foul by putting it into the Fire, and then the same thing will happen to it as to the Iron Wire; because the Attraction of Cohesion is insensible, at the Distance of the Thickness of the thin Skin that there covers the Silver.

28. THAT there should be a Contact to bring Bodies to cohere very much, is evident from the soldering of Lead or Brass with a Mixture of Lead and Tin, commonly called soft Solder.

Lect. I. Solder. For if theſe Metals be not well cleaned, they can never be ſoldered perfectly tight; and in Lead, after the Parts that are to be joined have been made foul by rubbing them over with Chalk, and then with Mallows or any Green-Herb, ſo as to make a thin Skin; the Work-man ſcrapes clean the two Edges of the Lead that are to be joined, that the Solder may come cloſe enough to the Metal; and when the Solder is poured on hot out of a Ladle, then ſpread with an Iron, it faſtens ſtrongly where the Metal was cleaned, but does not at all ſtick where the Skin remains that was made by the Chalk and Juice of the Mallows. It has been obſerved, that the Air alone will foul or make a Skin over the Part of the Lead that has been cleaned, and therefore Greaſe or Tallow is commonly rubbed on after ſcraping; for the Parts of any fat or inflammable Subſtances being much finer than thoſe of Air, will allow the Solder to come much cloſer to the Lead than a Plate of Air, or what ſeparates from the Air to ſtick to the Lead.

If two leaden Balls of about an Inch Diameter be pared with a Knife, ſo as to cut off a Segment of about $\frac{1}{4}$ of an Inch Diameter, and they be preſſed together (giving them a little Twiſt) pretty hard, they will ſtick together with a great Force; ſometimes ſo as to hold above 100 *lib.* See the 6th Volume of the *Philoſophical Tranſactions*, abridged by Meſſrs *Reid* and *Gray*, Part 2d, Page 2 and 3.

29. In what Proportion of the Diſtance this Attraction of Coheſion encreaſes or decreaſes is not yet fully known; but from ſome Phænomena, we have reaſon to believe that it decreaſes in a biquadratic Ratio of the encreaſed Diſtance; that is, at twice the Diſtance it acts 16 times more weakly, and at 3 times the Diſtance 81 times more weakly, *&c.* For it becomes inſenſible at the leaſt ſenſible Diſtance.

30. There is in Nature another ſort of Attraction not ſo ſtrong as the Attraction of Coheſion, but ſtronger than Gravity; whoſe Proportion of Decreaſe, in the Removal of Bodies attracting, is nearly as the Cube and a Quarter of the encreaſed Diſtance*: And this is the Magnetical Attraction. As for Example, if a Loadſtone attracts a Piece of Iron with a certain Force at a given Diſtance; at twice the Diſtance, the Attraction will be 10 times weaker: And at three times the Diſtance, the Attraction will be

* Ann. 20.

$33\frac{1}{3}$ times weaker. But as Magnetism is a particular Virtue that affects only Load-stones and Iron and Steel, we shall refer a fuller Account of it to another Place; because we are now only considering general Properties of Bodies. We shall only observe, that the Load-stone repels as well as attracts; for that Pole of the Stone, which attracts one End of a touch'd Needle, will repel the other. Lect. I.

31. THERE are several other Instances in Nature of a repellent Power in Bodies *; and very often the same Bodies that attract one another at certain Distances, and under some Circumstances, do repel one another at different Distances, and under other Circumstances. * Ann. 21.

THIS may be seen upon the Dissolution of Salts in Water, That the Parts of the Salts attract one another, appears from uniting into hard Lumps when the Water is evaporated, so that the Particles come so near to each other, as to be within the Power of the Attraction. That they repel one another at farther Distances, appears from the regular Figures into which they coalesce, when by the Evaporation of part of the Fluid in which they float, they are brought within each other's Sphere of Attraction; these regular Figures depending entirely upon the Equality of their Distances one from another before this Evaporation, and this Equality of Distance being owing to an Equality of repelling Force.

32. A repelling Force is also prov'd by the Production of Air and Vapours; for those Particles which are forc'd out of Bodies by Heat and Fermentation, as soon as they are out of the Sphere of Attraction of the Bodies, do immediately recede from them, and from one another with a great Force, and avoid coming together again; so as sometimes to take up about a Million of Times the Space which they did before in a dense Body.

33. THE Attraction and Repulsion in the same Body at a considerable Distance, is evident in several electrical Experiments.

IF you rub a Piece of Amber with a dry Hand or a woollen Cloth, it will put into motion Threads, Feathers and light Bodies, attracting and repelling them at a distance: And therefore the Name of *Electricity* has been given to that attracting and repelling Force which is excited in any other Body by the same Friction as is given to the Amber. Wax, Rosin, Sulphur, Silks, Paper, Ribbons,

Lect. I. Ribbons, Hair, and Feathers, and ſeveral other Bodies have this Property: But Glaſs has it more than any other.

EXPERIMENT XII. *Plate* 2. *Fig.* 10.

34. TAKE a Glaſs Tube of about 1 ½ Inch Diameter, and rubbing it from one End to the other with a dry Hand pretty briſkly, it will attract a Feather, or any light Body, at a conſiderable Diſtance, from one to eight or ten Feet. After a Feather has been attracted and ſtuck to the Tube for ſome time, it will fly off of it ſelf, and never come to the Tube again (which conſtantly repel the Feather in the Air, whenever it is brought near it) till it has touch'd ſome other Body; as a Finger or a Stick. And if the Finger be held pretty near the Tube, the Feather will alternately fly from the Finger to the Tube, always ſtretching out its Fibres the way that it is going, and that before it comes off from the Finger or the Tube. *In driving the Feather about the Room with the Tube, it muſt now and then be rubb'd afreſh to excite the Electricity, which continually grows weaker, after the Friction is over.*

EXPERIMENT XIII. *Plate* 2. *Fig.* 11.

35. IF ſeveral little Pieces of Leaf-gold, or Leaf-braſs, be laid upon a Stand or ſmall Table, and the rubb'd Tube be held over them at the diſtance of a Foot or two; the Pieces of Leaf-gold will move from the Table to the Tube with great Swiftneſs: And often by the Attraction and Repulſion they will move backwards and forwards without touching either the Tube or the Table. But if two Books, or two Pieces of Wood of the ſame Size be ſet up on end on the Table on each ſide of the Leaf-gold, as at A, B; (*Plate* 2. *Fig.* 12.) ſo that their Diſtance A B be equal to the Height of one of them: Then the Tube being held between their Tops, as at D, will have no Power to move the Leaf-gold, tho' the Diſtance from the Leaf-gold be but ſix Inches, when the Tube juſt before attracted the Leaf-gold at a Foot or two; but if the Pieces of Wood be remov'd, without giving the Tube a new Friction, it will attract and repel the Leaf-gold as before. When the Pieces of Wood are not remov'd, the Tube will not put the Leaf-gold in Motion, till the Diſtance D C from the Tube to the Gold be leſs than half the Diſtance A B of the Pieces of Wood: As if this

this Effect could not be produc'd while the Sphere of Attraction represented by the Circle ECF (whose Center is in the middle of the Section of the Tube at D) is disturb'd by the Pieces A and B. Lect. I.

EXPERIMENT XIV. *Plate* 2. *Fig.* 13.

36. To know when the Tube is sufficiently rubb'd to make Experiments with it, you must move your Fingers Ends nimbly by the Tube, as if you went to strike it in a Direction perpendicular to its Axis, but the Fingers need not come nearer than a quarter of an Inch from the Tube: Then the Effluvia or fine Parts that fly from the Tube will snap against the Finger, or (beating back from it) against the Tube, so as to be heard with a Noise like the crackling of a green Leaf in the Fire; but not so loud.

ACCORDING to the State of the Air, the Tube requires more or less rubbing. In Weather that is hot and moist, the Tube requires a great deal of Friction before it will snap, and attract and repel the most strongly; and then at best its Virtue will diffuse it self but a little way: So that the same Tube, which in dry cold Weather gave Motion to the Fibres of a Feather at the Distance of eight or ten Feet, will scarce act sensibly at the Distance of two Feet when it rains in Winter.

IF the Tube be warm'd at the Fire without rubbing, it will have no Effect: Its Electricity will also be less if you rub the Tube long enough to make it very warm; and then you must let it cool before you use it again.

IT is not amiss to set the Tube or any Glass to be rubb'd before the Fire to dry it before you use it, especially if it be pretty thick, provided it is not much heated.

37. IT is remarkable, that if the Tube be rubb'd in the dark, the Effluvia will appear lucid; and when it is made to snap (as in the 13th *Fig.* of *Plate* 2.) there appears a Light upon the Fingers Ends, as at A; and if a little Brush be held near the Tube, as at B; or drawn along it without touching, just after it is rubb'd, Sparks of Light like Stars will appear upon every Hair of the Brush: but the same part of the Tube will not snap or give Light twice together in the same Place * without a new Friction.

* Ann. 22.

EXPERIMENT XV. *Plate* 2. *Fig.* 14.

38. IF the rubb'd Tube be brought near a Down Feather tied to the upper part of a little Stick ſtanding upon a Foot, the Feather will ſtretch out its Fibres towards the Tube; but upon the Approach of a Finger between the Tube and Feather, they will be repell'd by the Finger; tho' they will be again attracted by it as ſoon as the Tube is remov'd; and then the Fibres will turn back again to the Stick and be attracted by it, when the Finger is taken away.

EXPERIMENT XVI. *Plate* 2. *Fig.* 15.

39. IF you ſet a Glaſs Receiver, about five Inches wide, and twenty Inches high, over the Stick and Feather, having firſt dried the Receiver at the Fire or in the Sun; upon rubbing the Glaſs with one or both Hands from Top to Bottom, the Feather will ſtretch its Fibres every way like the Radii of a Sphere, when the Hand is remov'd from the Receiver. But if whilſt you rub the Receiver, or after rubbing, you only move the Hand upwards and downwards, the Fibres of the Feather will (notwithſtanding the Interpoſition of the Glaſs) follow the Motion of the Hand: And if the Tube be rubb'd within a Foot or two of the Receiver, the Feather in the Receiver will likewiſe follow the Motion of the Hand rubbing the Tube. When the Tube has its Electricity excited by Friction, if it be brought near the outſide of the Receiver, the Feather will ſtretch its Fibres towards the Tube; and upon the Removal of the Tube, turn back to the Stick; though ſometimes this laſt Phænomenon will happen at the approach of the Tube, and the Fibres will ſtretch out again when the Tube is taken away: Nay, ſometimes there ſeem to be Fits of Attraction and Repulſion; for whilſt the Tube is held near the outſide of the Receiver, the Fibres of the Feather will be alternately extended and contracted, without any new Friction given to either of them.

40. JUST after the Receiver has been rubb'd, if you blow towards the Feather, (ſee the 15th *Fig.* *Plate* 2.) its Fibres will fly from the Blaſt; and they will alſo fly from an Hand mov'd briſkly

briskly towards the Glass, yet so as not to touch it; but the Experiment will not do twice without rubbing the Receiver a-new. Lect. I.

41. Most of these Experiments, if not all, will succeed even when the Air has been pump'd out of the Receiver: Only there will be this difference, upon rubbing it *in Vacuo*, that the Light excited will be of a Purple Colour, in a much greater Quantity, and all within the Glass: And whereas Bodies would be attracted before, when held near the outside of the Glass, now that Power will cease, and the Virtue will exert it self wholly inwards. The same will happen to the Tube when exhausted, as also to a Glass Globe * whirl'd by means of a Wheel, and so rubb'd by the Hand; as is described more at large by the late Mr. *Hawksbee* in his Book of Experiments, where he has given a large Account of a great many Electrical Experiments that he made. * Ann. 23.

42. I shall say no more on this Subject now, because I shall have Occasion to consider it more fully in another Part of my Course: And the Intent of this Lecture is only to shew,—That those Properties of Bodies, such as Gravity, Attractions, and Repulsions, by which we shall hereafter explain several Phænomena, are not occult Qualities or supposed Virtues, but do really exist, and are by Experiments and Observations made the Objects of our Senses. These Properties produce Effects, according to settled Laws, always acting in the same Manner under the same Circumstances: And, though the Causes of those Causes * are not known; since we do not reason about these hidden Causes, it is plain that we reject occult Qualities, instead of admitting them in our Philosophy, as the *Cartesians* always object to us. * Ann. 24.

ANNOTATIONS upon the Firſt LECTURE.

I. [3——*But the whole Variety*, &c.]

Annotat. Lect. I.

IF we conſider the Bricks of which a Building conſiſts as its ſmalleſt or firſt Parts; we ſhall find that, however ſimilar they are, their different Diſpoſition, in reſpect to each other, will produce very different Parts of the Edifice: An Arch, a Wall, a Chimney, a Pier, a ſquare or a round Pillar, a Globe or a Cube are compos'd of the ſame Sort of Bricks; and ſuch as ſerved for one Part, when pulled aſunder, will as readily ſerve for another. So in the wonderful Edifice of the Univerſe, there is no need of a Difference in the Atoms or firſt Particles, of which the ſeveral Parts are compounded; the ſame Atoms being as proper to make Land as Sea, to make Gold as Clay: And, when we do not conſider the Soul that actuates the Matter, a particular Diſpoſition of the firſt Atoms makes all the Difference betwixt a lifeleſs Lump, and the Body of a curiouſly organiz'd Animal.

One may bring various Examples of Matter trac'd through ſeveral Bodies, whoſe Changes depend upon the different Texture and Poſition of the Parts.

When the Water of Rivers, Seas, and Lakes, is ſo rarified by the Heat of the Sun, as to become ſpecifically lighter than Air (which will happen, when it takes up above 900 times the Space in Vapour that it did in Water) it will riſe up ſo high as to form Clouds of various Colours, which float about at that Height, where the Air is of the ſame ſpecific Gravity as the Clouds.

When the Winds, by carrying off ſome of the Air above, cauſe that which is below to become ſpecifically lighter by its Expanſion; the Clouds, retaining the ſame ſpecific Gravity as they had before, do then deſcend; and, by the Reſiſtance which they meet with in their Deſcent, are chang'd into Rain, which falling down to the Earth, does in a great meaſure run back into the Rivers and Seas; but ſome Part of it runs into the Earth, and is imbibed by the Seed of Plants. If we conſider ſo much of it as gets into the Grains of Wheat that are ſown, the Appearance of it is much

chang'd

chang'd in the green Blade of Corn, then in the Straw, afterwards in the Ear, and in the Grains contained in it. The Wheat, by being ground in the Mill, puts on the Form of Flour; which being made up into Paste and then baked, is again changed into the Crust and Crumb of Bread. The nourishing Parts of the Bread (after it has been eaten by Man, and has passed through the Stomach) do out of the Intestines pass through the Lacteal Vessels into the *Receptaculum Chyli*, and thence up the *Ductus Thoracicus* into the left Subclavian Vein, or (as an ingenious * Anatomist has lately discovered) into the left internal Jugular Vein; where mixing with the Blood, it goes along with it through the Heart and Lungs, where it receives vivifying Particles from the Air; and returning into the Heart, is from thence by the Action of the left Ventricle of the Heart, and the Arteries, carried to the Extremities of the Body. There some Part of this new Blood circulates back again; whilst other Parts are changed into the Substance of Bones, some into Membranes, some into Hairs, some into Nails; and other Parts of it passing through the Glands, are turned into Sweat, and so become Water again, as at first.

Anno at. Lect. I.

* Mr. *St. André.*

If we had considered such Drops of Rain as impregnated Linseed; we might have traced it through the Stalks of the Plant, Flax made of that, Thread spun from the Flax, Linnen made of the Thread, a white Pulp made of the Rags of the worn Linnen beaten up with Water at the Paper-Mills, Paper made of that Pulp thinly spread upon a fine Net-Work of Wire; and lastly the Smoke, which burnt Paper affords, is again easily reduced to Water.

Solids become Fluids, as Metals do by the Action of Fire, or being dissolved in acid *Menstruums*; and Fluids put on the Form of Solids, as Mercury will be made hard by the Fumes of Lead: And two Chymical Liquors will immediately upon their Mixture coalesce into a firm Substance.

It is to their particular Figure that Machines and Instruments owe their Usefulness; Clocks, Mills and other mechanical Engines being only valuable, when their Parts that are made to communicate Motion have their proper Fitness.

The same kind of Glass produces great Variety of Effects, according to the different Figure of its Surface: as is known to all that have seen Optic Glasses.

The Difference of Seasons which makes such Changes on the Surface of our Earth, and even in the Bodies of Animals, is entirely owing to the different Situation of the several Parts of the Earth in respect to the Sun.

Those that would read more upon this Subject, may consult Dr. *John Keil's Introductio ad veram Physicam.* Edit. 3. Lect. 7.

Annotat. Lect. I.

2. [4. *That Mercury resists*, &c.] Mercury weighs $13\frac{2}{3}$ times more than Water, and is found to resist just $13\frac{2}{3}$ times more; and Water which weighs between 800 and 900 times more than Air, is found to resist just so much more. This Sir *Isaac Newton* found by making Experiments upon Pendulums of Wood and Lead in the Air, Pendulums of Lead in Water, and of Iron in Mercury. See his *Principia*, Edition 2d. *Book* 2. *Prop.* 31. and *Prop.* 40. where he demonstrates that the Resistance of Fluids is as their Quantity of Matter.

I made an Experiment before the *Royal Society* about two Years ago with a Ball of Gold of an Inch Diameter, which, being suspended by a String, did first oscillate in Water and then in Mercury: And it appeared that 42 Vibrations in Water destroyed as much of the Motion of the Golden Pendulum as three Vibrations in Quicksilver. And letting the Ball of Gold fall in a Copper Tube four Foot long, and $4\frac{1}{2}$ Inches Diameter, filled with Mercury, (from an Height of three Foot ten Inches) and nicely observing the Time of its Fall, I found by comparing several Experiments, that the Resistance of the Medium (discovered by the Method taught in the said 40th *Prop.* of the *Principia*) agreed so exactly with Sir *Isaac Newton*'s Theory, as not to differ one Tenth of an Inch in the Space that the Ball fell through, which was three Foot ten Inches.

3. [4—*And if there be a Medium finer than Light*, &c.] See Sir *Isaac Newton*'s Optics, second Edition, Book 3d. Queries 18, 19, 20, and 21.

4. [5——*That Quantity*, &c. *is divisible* in infinitum, *&c.*] Of the many Demonstrations brought to prove this Assertion, I shall only mention two that are very plain and obvious. The first taken from Dr. *s'Gravesande*'s Introduction to Sir *Isaac Newton*'s Philosophy; Part 1st, N°. 18.

Plate 3. *Fig.* 1.

Let there be a Line A D perpendicular to B F, and another as G H at a small Distance from A, also perpendicular to the same Line. With the several Centers C, C, C, &c. and Distances C A, C A, &c. describe Circles cutting the Line G H in the Points *e*, *e*, *&c.* The greater the Radius A C is, the less is the Part *e* G. Since the Radius may be augmented *in infinitum*, therefore the Part *e* G may be diminished in the same Manner; and yet it can never be reduced to nothing, because the Circle can never coincide with the right Line B F.

The next is from Dr. *John Keil*'s *Introductio ad veram Phys.* Lect. 3. *De magnitudinum Divisibilitate.*

Let A B (*Plate* 3. *Fig.* 2.) be a Perpendicular between the Parallels C D, E F. From the Point C in one of the Parallels draw the Line C G to a Point G on the other Parallel, on the other side of the Perpendicular A B,

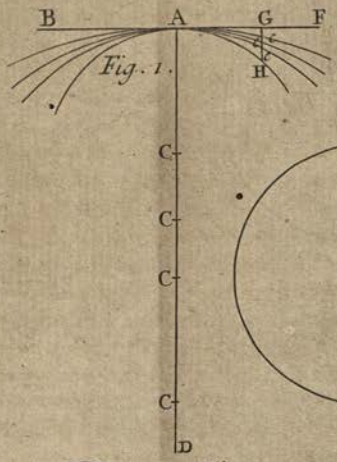
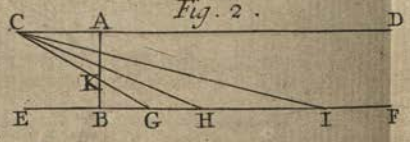


Fig. 1.



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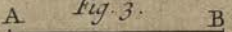

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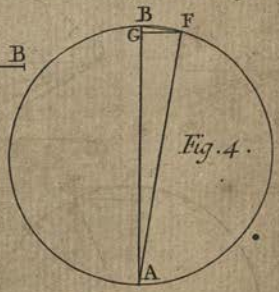

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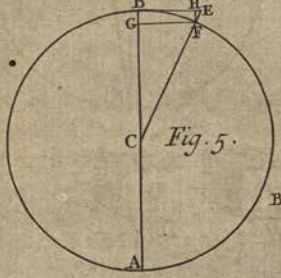

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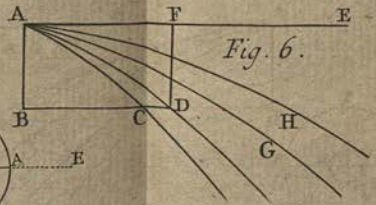

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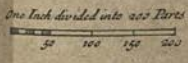

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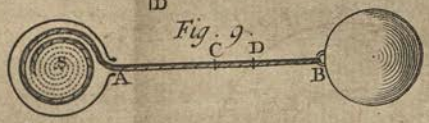

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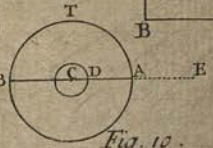

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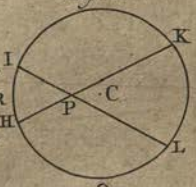

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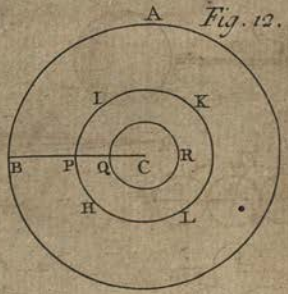

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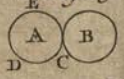

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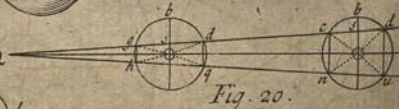

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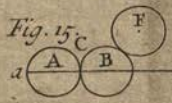

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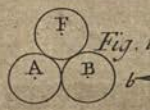

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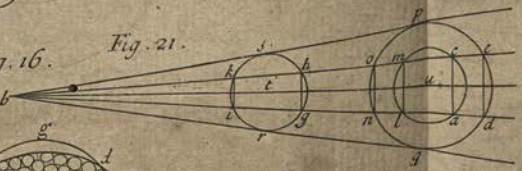

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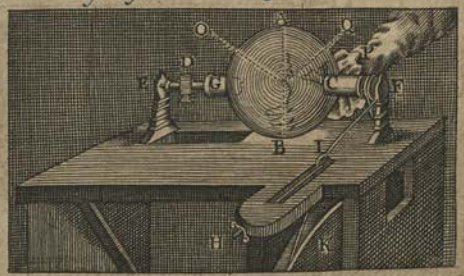

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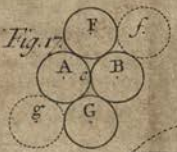

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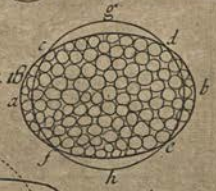

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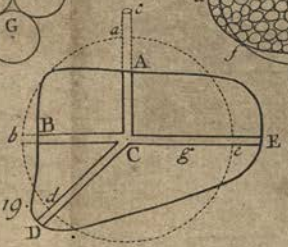

Fig. 19.

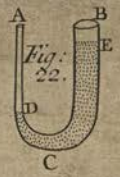

Fig. 22.

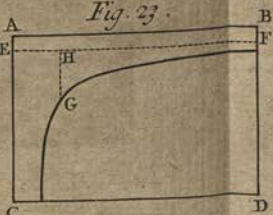

Fig. 23.

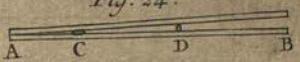

Fig. 24.

A B, and it will divide the said Perpendicular into two Parts at K: Another Line drawn from C to H will divide the Part K A (of A B) into two Parts; and since upon the Line E F, which may be produced *in Infinitum*, other Points as I, *&c.* may be taken, new Lines may still be drawn to divide the remaining Part of A B. For whatever Line is drawn from C to any Point of the Line E F, however distant, it can never coincide with the Line C D; and therefore it will still divide the Part of A B that remains after the foregoing Division. Annotat. Lect. I.

For other and more ample Demonstrations, see the said *Lect.* 3d, where he also shews the Absurdity of the contrary Opinion: And in the next *Lecture* he removes the Objections alledged against the Divisibility of Quantity; by demonstrating those very Assertions to be true, which the Objectors alledge as absurd Consequences of the Divisibility of Magnitude *in Infinitum.*

As First, *That a finite Quantity may have an infinite Number of Parts.* For if the Line A B * be divided into an Hundred Parts, all those Parts taken together will be equal to A B; and if it be divided into a Thousand Parts, all those Parts taken together will still be equal to A B. Now, a Thousand may be increased in any Proportion, nay, may receive an infinite Addition of Numbers; and since, however the Number of Parts is increased, the Sum of them can never exceed the Line A B; the said Line may, without an Absurdity be said to contain an infinite Number of Parts. * Plate 3. Fig. 3.

Secondly, *That it implies no Contradiction, but is agreeable to Geometry, to say, that a Finite may be equal to an Infinite*; shewing that a finite Space is equal to an infinite one, and that an infinite Solid is equal to a finite one.

Thirdly, *That there are Infinites, whose Magnitudes bear certain Proportions to one another*, and that some are bigger, (nay infinitely bigger) than others.

In the Circle A B F † take an Arc B F infinitely small, then the Chord B F will be infinitely smaller than the Diameter A B; and yet it will be infinitely greater than the versed Sine B G, found by drawing F G perpendicular to B A. Likewise in the Circle B F A * if the Arc B F be taken infinitely small, B E be its Tangent, F G the Right Sine, B G the versed Sine, and F H equal and parallel to the versed Sine; it is demonstrable, that C B is infinitely greater than B E, and B E infinitely greater than B G, and B G than H E. † Plate 3. Fig. 4. * Plate 3. Fig. 5.

But the most surprizing thing of this Kind, is, that if A E and A B † be drawn at right Angles, and the Parabolic Curves of different kinds, C, D, G, H, be drawn through the Point A; the Angle of Contact F A C, which is infinitely less than any rectilinear Angle, will be infinitely greater than the Angle F A D, and F A G will be infinitely less than F A D: And so there may be an infinite Series of Angles of Contact going on infinitely, of which every following one is infinitely greater than the former: Nay, between any two Angles, there may be inserted † Plate 3. Fig. 6.

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innumerable Angles infinitely greater than each other. And even between any two of these Angles, there may be a Series of intermediate Angles going on *in Infinitum*; of which every following one is infinitely greater than the former. And thus Nature knows no Bounds. See *the Demonstrations of these Propositions in the same Book*, Lect. 4.

5. [6——*There are Atoms*, &c.] Dr. *Keil* in his Introduction, *Lect.* 51. deduces the two following *Theorems* from the infinite Divisibility of Matter.

THEOREM I.

" Any Quantity of Matter, how small soever, and any finite Space, how " great soever, being given, (as for Example, a Cube circumscribed about " the Sphere of *Saturn*;) it is possible for the Matter of that small Sand " to be diffused throughout all that Space, and to fill it so, that there " shall be no Pore or Interstice in it, whose Diameter shall exceed a given " Line.

From the Demonstration of this *Theorem*, he draws the following *Corollary*, which serves for the Demonstration of the other.

Cor. " Hence there may be given a Body, whose Matter, if it be reduced " into a Space absolutely full; that Space may be any given Part of its " former Magnitude.

THEOREM II.

" There may be two Bodies of equal Bulk, whose Quantities of Mat- " ter being unequal in any Proportion; yet the Sum of their Pores, or the " void Spaces in the two Bodies, shall be almost equal.

The Doctor applies his Demonstration to an Inch Cube of Gold, and an Inch Cube of Air; the Substance of which is as follows.

* Plate 3. Fig. 7.

The Cubic Inch of Gold * A, contains near 20000 times more Matter than the Cubic Inch of Air B; but we will only suppose it to contain 100000 times more. Now let the Matter in A be reduced into a Space absolutely full, which we will suppose equal to the $\frac{1}{100\ 000}$ Part of a Cubic Inch; (which may be done by the *Cor.* of the last *Theorem*. Then if the Matter in B be reduced to a Space absolutely full, it will only take up the $\frac{1}{1\ 000\ 000\ 000}$ Part of a Cubic Inch, because B contained 10000 times less Matter than A. To compare these two full Spaces, let us make the Denominators of the two Fractions the same; and

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and $\frac{10000}{1\ 000\ 000\ 000}$ will express the solid Space in the Inch of Gold: Therefore the $\frac{999\ 990\ 000}{1\ 000\ 000\ 000}$ remaining Parts of that Cubic Inch will be void Spaces; whilst the void Spaces in the Inch of Air, (after its Matter has been reduced to a Solid) are expressed by the Fraction $\frac{999\ 999\ 999}{1\ 000\ 000\ 000}$: And consequently since the Numbers 999 990 000 and 999 999 999 are almost in the Ratio of Equality; the void Spaces in both Bodies are nearly equal.

Though these *Theorems* seem inconsistent with the Doctrine of Atoms, they do not overthrow it; because, as they are deduced from the Divisibility of Quantity, they are rather Mathematical than Physical. For though an Atom may be conceiv'd of an exceeding Smallness, yet its Diameter must be of a determinate Length, and consequently too big to answer the Conditions of the first *Theorem*, which supposes no first Parts. But then even in an Atom (or first Physical Part) Mathematicians may assign Parts smaller in any Proportion, so as to agree with the forementioned *Theorems*: For those Particles of natural Bodies, which cannot be divided in the Operations of Nature, do virtually contain an infinite Number of Parts; though those Parts are never separated from one another.

Notwithstanding, that giving Atoms a certain Bigness limits Dr. *Keil*'s *Theorems* in the Physical Sense; yet it appears from *Phænomena*, that there actually exist Particles small enough to agree with the first *Theorem*, if a Grain of Sand be made use of to fill the Sphere of *Saturn*; and with the second *Theorem*, if an Inch of Gold be compared with an Inch of Air; only supposing (which is more than probable) that a Body perfectly solid is as much denser than Gold, as Gold is denser than Light, or Æther which is still rarer than Light.

And if a Grain of Sand be supposed divided into so many Parts, as to fill the Sphere of *Saturn*, without having any Pore bigger in Diameter than an Hair; those Particles may be still bigger than those of Æther, if not bigger than those of Light.

6. [7——*These are to be imagined of an unconceivable Smallness*, &c.] Though it be surprizing to think, that Matter should be actually divided into Parts so small, as we have mentioned in the foregoing note; a few Instances of the small Parts into which it is divided by Art, or the Work of Mens Hands; and some Examples of its Subtility, as it is naturally dispersed all over the Universe, will make the Assertion very plain to any body that will afford the least Attention.

The Gold Beaters, even with coarse Tools, reduce that Metal to such a Thinness, that fifty square Inches of Leaf-Gold weigh but one Grain. Now the Length of an Inch may be divided into 200 visible Parts, as appears in

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* Plate 3. Fig. 8.

Plate 3. *Fig.* 8. where the 20th Part of an Inch is diſtinguiſhed into ten viſible Parts by ſix black Strokes and five white Interſtices *. Then multiplying 200 by 200 we have 40000, for the viſible Parts of a ſquare Inch; which are contained 50 times in a Grain of Gold; and therefore by this means it becomes divided into 2 000 000 of viſible Parts.

If we conſider the gilding of Silver, we ſhall find Gold in that caſe to contain viſible Parts, even after it has been divided above ten times more; for eight Grains of Gold will gild a whole Ounce of Silver, which is afterwards drawn into a Wire 13000 Foot long; therefore one Grain gilds a Wire 1625 Foot long. And as every Foot (by what we have ſaid above) muſt contain 2400 viſible Parts, the whole Length of the Wire contains 3 900 000 little Cylinders, which being turn'd into Cubes, will each of them have ſix viſible Sides; and conſequently by this laſt Operation, one Grain of Gold, inſtead of being divided into Two Millions of viſible Parts, will be divided into 23 400 000, which is almoſt twelve Times as many. *That one may reaſonably take Cubes for the little Cylinders, appears, when we conſider further, that all this Wire is beaten flat in order to wrap it round Silk for making Gold Lace; and that even after flattening, the beſt Microſcope cannot diſcover the Silver through the Gilding.* This ſhews that in this thin Skin, ſeveral Parts of the Gold ſtill lie upon one another; though the Thickneſs of it (as Dr. *Halley* has ſhewn, *Philoſ. Tranſ.* Numb. 194) is but $\frac{1}{12450}$ Part of an Inch, or 62,25 times leſs than the 200th Part of an Inch that we have taken, as the leaſt viſible Part of an Inch in Length.

Mr. *Boyle*, in his Book of *The Nature and Subtilty of Effluvia*, mentions, that one Grain of Copper diſſolved in Spirit of *Sal Armoniac*, will give a ſtrong blue Tincture to 105,157 Cubic Inches, or near two Quarts of Water. Now ſuppoſing no leſs a Cube of this tinged Water to be viſible, than ſuch a one whoſe Side is equal to the 100th Part of an Inch (which is making eight times more Allowance than we did in reſpect of the Gold) it will appear upon Computation, that a Grain of Sand ſo ſmall, that a Million of them may be contained in a Cubic Inch, does contain two Millions an Hundred and eleven Thouſand and four Hundred (or 2 111 400) ſuch Parts as the ſingle Grain of Copper is actually divided into.

The ſame Gentleman, having expoſed to the open Air a certain Quantity of *Aſſa Fœtida*, found it diminiſhed in Weight but the eighth Part of a Grain in ſix Days. Now if we ſuppoſe, that during all that time a Man could ſmell the *Aſſa Fœtida* at the Diſtance of five Feet, it will appear, that the Particles, into which the *Aſſa Fœtida* is divided, cannot exceed in Bigneſs the $\frac{1}{26\,250\,000\,000\,000\,000}$ Part of an Inch.

The late Mr. *Lewenhoek*, that ingenious Searcher into Nature, gives us an Account, that in the Milt of one Cod-Fiſh there were more little Animals, than there are Inhabitants upon the Face of the Earth. Now by only knowing

the Focal Length of the *Lens*, or Glass of the Microscope, we can by the Rules of Optics find the Bigness of those *Animalcula*, which cannot be so big as the $\frac{1}{36\ 000\ 000\ 000\ 000}$ Part of a Cubic Inch: And, therefore, several Thousands of them might stand upon a Needle's Point. And it appears also, that if we compare these to a Whale; they will be much less in Proportion, than a Whale is, when compared with the whole Globe of the Earth. As every Animal is an organized Body; how fine, delicate and subtile must be the Parts that make up one of these Animals? How small must be its Heart? How inconceivably little its Veins and Arteries? And much less must be the Globules of that Fluid which serves it for Blood, and which are still carried along in a finer Fluid.—It is worth while to consider the Smallness of those Globules; which we may do by making the following Allowance; namely, That the Particles of Blood of those Animals are as much less than their Bodies, as the globular Particles of the human Blood are less than a Man's Body. Annotat. Lect. I.

A Man's Body is to that of an *Animalculum*, as 17 to $\frac{3}{100\ 000}$, and the Diameter of the Globules of a Man's Blood are not bigger than $\frac{1}{79\ 200}$ Part of an Inch †, (because *Lewenhoek* found the Diameter of the Vessels through which they run to be no bigger) therefore as 17 is to $\frac{3}{100\ 000}$, so is $\frac{1}{79\ 200}$ to $\frac{3}{134\ 640\ 000\ 000}$ or in Decimals $\frac{22}{1\ 000\ 000\ 000\ 000}$; therefore the Globules of Blood in these little Creatures cannot be so big as the Cube of this Number or $\frac{8}{1\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000}$ Parts of an Inch.

But since these Numbers, expressed in Figures, do not immediately give an Idea of the Smallness of these Globules; Dr. *Keil* (in whose *Fifth Lecture* one may find at full Length the Demonstrations, that prove our two last Paragraphs) has shewn, that the smallest visible Grain of Sand would contain more of these Globules, than ten Thousand two Hundred and Fifty six of the highest Mountains in the World would contain Grains of Sand.

What we have said hitherto, shews into how many small Parts Bodies are actually divided; but there are Particles of Matter, so much smaller than the Globules above mentioned, that those Globules compared with them will not only be as Mountains, but as vast Earths. I mean the Particles of

† *The learned and ingenious Dr.* J. Jurin, Secr. *of the* Royal Society, *has not long since found Globules of the human Blood to be bigger than what is here mentioned; and having communicated his Observations to Mr.* Lewenhoek, *was by him confirmed in his Assertion; but I did not think proper to alter this Calculation, since I shall speak fully upon this Subject in another Place of this Course.*

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of Light, whoſe inconceivable Fineneſs puzzles our Conception. How amazingly little muſt thoſe Particles be which flow from ſuch a Candle, whoſe Light may be ſeen at two Miles diſtance? Since every Inſtant of Time, Particles muſt be darted out to fill a Sphere of four Miles in Diameter, ſo that a Pin's Head cannot be placed any where in that Sphere without receiving ſome Particles of Light. Dr. *Newentiit* ſhews, that the fourteenth Part of a Grain of Wax or Tallow, (that is conſumed in one Second of Time in a Candle of ſix to the Pound,) produces a greater Number of Particles of Light, than a Thouſand times a Thouſand Millions of Earths, (equal to our Earth) would be able to contain Grains of Sand. See *Religious Philoſopher*, Vol. 3. *Contemplation* 25. *Sect.* 15, 16, 17.

7. [7——*There muſt be a great deal of Vacuity*, &c.] The different ſpecific Gravity of Bodies plainly proves this Aſſertion, as will be more fully ſhewn in the ſecond Lecture. And in Fluids this is evident from their different Reſiſtance, which we have already ſhewn to be proportionable to the Quantity of Matter in Bodies.

8. [8——*All Spaces are not equally full of Matter.*] If there was any ſuch thing as a *Subtile Matter*, that wholly filled up the Vacuities of Bodies, and the whole Æthereal Space in which the Planets move; its Reſiſtance would be ſuch as far to exceed the Reſiſtance of Quickſilver. In ſuch a *Medium* as that, even a perfectly ſolid Globe muſt loſe half its Motion, before it could move thrice the Length of its own Diameter; and ſuch Globes, as the Planets are, would be ſtopt much ſooner: wherefore it is abſolutely neceſſary for continuing the Motions of the Planets and Comets, that the Places, they move in, be almoſt entirely void of Matter. That they are ſo, appears from the ſwift Motion of the Tail of a Comet, that does not appear to meet with any ſenſible Reſiſtance in the *Medium* in which it moves, though it is expanded ſo wide, and made up of ſo thin a Vapour.

9. [9——*Much more* Vacuum *than Matter*, &c.] That there is more Vacuity in Bodies than Matter, may be clearly deduced from the Properties of tranſparent Bodies; for the Rays of Light are ſpread every way in right Lines through Water, Glaſs, or a Diamond, with no more Difficulty (nay, with more Swiftneſs) than they are carried through the Air, whatſoever Side of the tranſparent Body be expoſed to the Light; therefore there is always a rectilinear Paſſage for the Light, from the leaſt aſſignable Part of the tranſparent Body to any other Part of it: And this could never happen, unleſs the Quantity of Matter in ſuch a Body was extremely ſmall when compared with its Bulk. Perhaps in a Diamond the ſolid Matter, compared to its Bulk, bears a leſs Proportion than the Diamond does to the whole Globe of the Earth: Which will not appear impoſſible, to thoſe that conſider what has been ſaid before upon this Subject. Now ſince Gold is not above

above ſix times denſer than a Diamond, how much more Vacuity muſt be in it than Matter? This ſhews the Reaſon why the *Effluvia* of the Loadſtone paſs through Gold, as eaſily as through the Air; for if a Plate of Gold, or any other Metal (except Iron) be interpoſed between a Loadſtone and a touched Needle, which is drawn out of its Poſition by the Virtue of the Stone, the Needle will in no wiſe be leſs affected than before. Nay, thoſe *Effluvia* may for a whole Day paſs through the Brain, a Body ſo tender and of ſo delicate a Contexture, without affecting the Nerves with any Senſation, or diſturbing the leaſt Thought. Annotat. Lect. I.

The Vapour of the *Aurora Borealis* (which ſome imagine to conſiſt in a great meaſure of the Magnetic *Effluvia* of the Earth) does freely paſs through Houſes and Trees, and dart through the Bodies of Animals without being felt; as appears from Obſervations made upon that *Phænomenon*, when viewed from ſeveral Places at once.

10. [15——*Any Weight may hang*, &c.] The ſame Experiment may be made, by means of a ſpiral Spring in one of the Balls within a Barrel on which a String is wound up. For the end of the String being faſtened to another Ball equal to (or double, or triple) the Ball that has the Spring; upon pulling the Balls aſunder, they will come together again with Velocities reciprocally proportional to their Maſſes. See *Plate* 3. *Fig.* 9. Where A * repreſents the Section of an hollow Braſs Ball, with a ſpiral Spring S and Barrel within it, contrived in ſuch manner as to pull back into the Ball the whole String A B, when it has been pulled out by the End B. The Ball B is ſolid and of Ivory, but of the ſame Weight as the hollow Ball A. Now if the Ball B be pulled from A to the Diſtance A B, upon letting go both the Balls at once, they will meet at C, the middle Point between them; but if B be a Ball twice as heavy as A, upon letting both go they will meet at D, D B being but half of the Diſtance A D. * Plate 3. Fig. 9.

11. [17——*For Gravity being a Virtue diffuſed*, &c.] When we compare Gravity to Light and Heat, we would not be underſtood to derive their Effects from the ſame Cauſe; or to aſſert, that all Sorts of Attractions in Bodies have the ſame Laws, ſince the Attraction of Coheſion *, and the Attraction of the Loadſtone †, do not act in that Manner: But here we only ſpeak of the Attraction of Gravity, whereby the Bodies about us are driven towards the Earth, and the Earth and Planets are driven towards the Sun; which Attraction is called alſo a *Centripetal Force:* And whatever be its Cauſe, its Laws above-mentioned have been diſcovered by Obſervations and Experiments, which have always concurred to confirm the Theory of Gravity. * L. 1. No. 19, 20, &c. † L. 1. No. 29.

In order to have a clear Notion of the Effects of Gravity, or the Centripetal Force, we muſt conſider it in three Reſpects: Either in reſpect to the Quantity of Force in the Central Body that attracts others, (or towards which Circumambient Bodies tend) which is called the *Abſolute Force*; or in reſpect to

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to the Velocity with which other Bodies move towards the Central one, which is called the *Accelerating Force*; or in respect to the Quantity of Motion in the said Bodies, when compared with each other, which is proportional to the Obstacle that they are able to remove, and is called the *Moving Force.*

The *Absolute Force* is proportionable to the Efficacy of the Cause that spreads its Virtue from a Center round about. So if the Earth had twice the Quantity of Matter that it has now, (whether it was twice as big, or only twice as dense) it would have twice the Absolute Force. Thus the Moon has near forty times less Absolute Force than the Earth, because it has almost forty times less Matter. Its Bulk indeed, (which is as the Cube of the Diameter) is almost fifty times less than that of the Earth; but then it is denser in the proportion of 21 to 17. See Sir *Isaac Newton*'s *Principia*, *Book* 3. *Prop.* 37. *Corol.* 3.

The *Accelerating Force* is expressed by that Velocity generated in a given Time, with which Bodies (considered as Physical Points) move towards the central Body attracting them by its Absolute Force. And this Accelerating Force is greater or less according to the Distance of the Center of the Force, in the reciprocal duplicate Proportion above mentioned *. Thus is the Gravity, that makes Bodies tend towards the Center of the Earth, greater in Vallies than on the Tops of high Mountains; greater at the Poles than at the Æquator, which is seventeen Miles higher; and greater at the Æquator than at greater Distances from the Globe of the Earth: For the same Body, which, near the Surface of the Earth, falls sixteen Feet in the first Second of its Fall, would fall but four Feet in the same time, if it began to fall at the Height of 4000 Miles from the Surface of the Earth, or two Semidiameters Distance from its Center. At equal Distances the Accelerating Force is the same every where: because all Bodies large or small, heavier or lighter, (abstracting from the Resistance of the Air,) are equally accelerated in their Fall †.

* Lect. 1. No. 17.

† Lect. 1. No. 8. Exper. 1.

The *Moving Force* is proportional to the Quantity of Motion, which the Absolute Force of the central Body generates in a given Time, in the Bodies that it acts upon. Though in respect of the Earth, we consider the biggest Bodies that are attracted by it (and even the Moon it self) as Physical Points; yet when we compare the Bodies with each other, we must have a Regard to their respective Quantities of Matter: for Bodies that have the same Accelerating Force, or move with the same Velocity, have their Quantities of Motion greater or less, as they have more or less Matter, or as they are more or less heavy; because the Moving Force of a Body is made up of the Sum of the Actions of the Accelerating Force of all its Parts, and consequently it is found by multiplying the Quantity of Matter into the Accelerating Force; as the Quantity of Motion in Bodies is found, by multiplying their Matter, or their Mass, into their Velocity.

Hence

Hence near the Surface of the Earth, where the *Gravitating* or *Accelerating Force* on all Bodies is the ſame, the *Moving Force* or Weight is as the Body: But if we aſcend to Places where the *Accelerating Gravity* is leſs, the Weight will alſo be diminiſhed and become as the Maſs of the Body multiplied by the *Accelerating Force*. Thus if a Weight of one Pound and a Weight of four Pounds begin to fall near the Surface of the Earth, their *Moving Forces* will be as 4 and 1; for if we take 16 for the *Accelerating Force*, equal in (and common to) both, 16 times 4, (or 64) will be juſt 4 times as much as 16, (or 16 times 1.) But if the four Pound Weight was removed to the Height of 4000 Miles, or two Semidiameters from the Center of the Earth, its *Moving Force* then would be juſt equal to that of one Pound at the Surface of the Earth; becauſe the *Accelerating Force* being four times leſs at twice the Diſtance from the Center, 4 times 4 or 16 would then expreſs the *Moving Force* of the heavier Body. Annotat. Lect. I.

If the one Pound Weight was placed at the Diſtance of two Semidiameters, its *Moving Force* would be 16 times leſs than that of the four Pound Weight at the Surface of the Earth.

A clear Idea of theſe three Forces will ſhew the Reaſon of ſome *Phænomena*, which would otherwiſe be hard to explain: As for Example, we have already ſaid, that the Earth has near forty times more Matter than the Moon; and yet Bodies on the Surface of the Moon weigh but three times leſs than they do on the Surface of the Earth, though the Moon's *Abſolute Force* be forty times leſs. But to ſhew that this is a neceſſary Conſequence of what has been ſaid, let us examine this Matter by Numbers.

Let * T A B repreſent the Earth, and L the Moon, A B a Diameter of the Earth, and *b d* a Diameter of the Moon (which are to one another as the Numbers 365 and 100) and let the Lines C E and *c e* be each equal to a Diameter of the Earth. Now if we ſuppoſe a Body placed at E, whoſe Weight or Gravity towards the Earth is there equal to 9,8427 Pounds; the ſame Body placed at *e*, juſt as far from the Center of the Moon, as it was before from the Center of the Earth, will weigh towards the Moon L but 0,25 Pound, or a Quarter of a Pound; becauſe the Maſſes or Quantities of Matter in thoſe two Bodies, and conſequently their *Abſolute Forces*, are to one another as theſe Numbers, or as 39,721 to 1, which are in the ſame Proportion. *Newtoni Prin. Lib.* 3. *Prop.* 37. *Corol.* 4. * Plate 3. Fig. 10.

Then if the Body above-mentioned be placed at A, diſtant from the Center of the Earth but a Semidiameter, it will † weigh towards the Earth four times more than it did at E, or 39,721 *lib.* and at *a* it will weigh towards the Moon 1 *lib.* or 4 times more than it did before, for the ſame Reaſon. Now if the Moon, without any new Addition of Matter, was ſo expanded or rarified, as to fill up the Sphere *m a*, which is equal to the Globe of the Earth, then the Point *a* would be on the Surface of the Moon, as the Point A is on the Surface of the Earth; and in that Caſe the Weight of Bodies on the Surface of the Earth would be to the Weight of Bodies on the Surface of the Moon, preciſely as the Quantity of Matter in the Earth, to the Quantity of Matter in the Moon; and conſequently as their † L. I. N°. 17.

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Absolute Forces. But as the Moon is less in Diameter than the Earth, when the Body that weighed 1 *lib.* at *a* comes to be placed on its Surface at *d*, it will be nearer to the Center of the Moon than it was, in the Proportion of 182,5 to 50, or 365 to 100, and therefore * it will then weigh 13,3225 *lib.* for as the Square of *c d* (100 × 100 = 10000) is to the Square of *c a* (365 × 365 = 133225) so is 1, or the Weight of the Body placed at *a* to 13,3225, the Weight of the same Body upon the Surface of the Moon; which Number being very near the third Part of 39,371, shews, that Bodies on the Surface of the Moon weigh about a third Part of what they do on the Surface of the Earth. *Which was to be demonstrated.*

* L. 1. N°. 17.

Hence it follows, That Bodies weigh more on the Surface of the small Planets, in proportion to their Quantity of Matter, than on the Surface of the larger. Thus upon the vast Globe of *Jupiter*, whose Quantity of Matter or *Absolute Force* is 220 times greater than that of the Earth, Bodies weigh but twice as much as they would do upon the Surface of the Earth. And upon the immense Body of the Sun, whose Quantity of Matter is 227512 times more than that of the Earth, Bodies weigh only 24,4 times more than they do upon the Surface of the Earth.

Hence follows also, that in respect of any Planet or central Body, as for Example, the Earth, the Weight of Bodies gravitating towards its Center is greater on its Surface, than at any Distance without it, or any where within it, though nearer the Center. For if the same Body which at A weighed 39,371 *lib.* should be brought to D, a Point within the Earth as near to C the Center of the Earth T A B, as *d* is to *c* the Center of the Moon L, it would not increase its Weight towards C 13,3225 times, as it does when removed from *a* to *d*; but it would diminish it in the Proportion of 365 to 100, because that Part of the Earth towards the Surface between D and A attracts back the Body towards A.

The just Proportion of this Decrease of Gravity is determined by Sir *Isaac Newton*, *Lib.* 3. *Prop.* 9. to be always as the Distance from the Center, going from the Surface downwards: And as the Principles from which it is deduced are very evident, I shall repeat them here.

If there be a concave spherical Surface, whose Particles attract according to the Laws of Gravity above-mentioned, as I H K L †, any little Body within it will remain at rest where-ever it is placed, the Attractions round about destroying one another. This is evident, if the Body be placed at C in the Center. And if the Body be placed at P as near again to H I as to K L, the same thing will follow; for let the Lines I K and K L, be drawn, and it will be evident that the spherical Segment between K and L will be four times greater than the Segment between I and H, because K L is equal to twice H I; therefore there are four times more attracting Particles at K L than at H I; but H I being twice nearer to the Body P, attracts it four times as much, which makes amends for the fewer Particles contained in the lesser Segment: for the Product of the *Absolute Force* of H I (1) multiplied into its *Accelerating Force* (which here is 4) is equal

† Plate 3. Fig. 11.

equal to the Product of the *Absolute Force* of KL (4) multiplied into its *Accelerating Force*, which is but 1. *This will hold good in respect of any other Part of the Surface or Position of the Corpuscle: And if the Corpuscle be put into Motion, it will go on uniformly in the concave Sphere, as if it was not attracted at all.* If instead of a Surface there was a Shell of any Thickness, as ABHIKL*, every thing else would remain as before asserted, provided that Shell was every where of the same Thickness and Density. Annotat. Lect. I. * Plate 3. Fig. 12.

If the Hollow HIKL be filled with a solid Sphere, and the Corpuscle be placed at P, it will be attracted towards the Center C only by the Force of the inner Sphere HIKL; for the attractive Forces of the several Parts of the Shell destroy one another, as has been shewn before. If the Corpuscle be removed to Q, it will be only attracted by the Sphere QR. Let us suppose the *Absolute Force* of the whole Sphere AB = 64; the Lines BC, PC, and QC, as 4, 2, and 1; and the *Accelerating Force* of the Corpuscle at B to be = 1; its *Moving Force* will of consequence be 64. Now if the Corpuscle be brought to P (as near again to the Center of the Forces) its *Accelerating Force* will be 4, but then that Number must only be multiplied by the *Absolute Force* of the Sphere HIKL, which being twice less in Diameter than the Sphere AB, is eight times less in Solidity, and therefore its *Absolute Force* will be but 8; which multiplied into the *Accelerating Force* 4, gives only 32 for the *Moving Force* of the Corpuscle at P. If the Corpuscle be brought to Q, four times nearer to C, its *Accelerating Force* will be 16; which being multiplied by 1, the *Absolute Force* of the Sphere Q (which is four times less in Diameter than the Sphere AB) will give 16 for the *Moving Force* of the Particle at Q. Now since the *Moving Forces*, whereby the Corpuscle or Particle at B, P, and Q gravitates towards the Center C, are as 64, 32, and 16, and the Distances from the Center are as 4, 2, and 1; it follows, *That going from the Surface of a Planet downwards, the Gravity decreases directly as the Distance from the Center.* Which was to be demonstrated.

To apply this to what we said before; if we conceive a Globe CD within the Earth T † equal to, and equally dense with the Moon; the Body which at A weighed 39,721 *lib.* will at D weigh just 13,3225 Pounds, as it would do upon the Moon's Surface. † Plate 3. Fig. 10.

12. [18—*If a Cube of an Inch*, &c.] As in the Bodies shined upon we only consider the Surfaces enlightened; the Experiment may be made with similar Pieces of Pasteboard exposed to the Light: As for Example, a Circle, a Square, a Pentagon, or any other Polygon of an Inch Diameter a Foot distant from the Candle, will receive the same Quantity of Light that falls upon a Circle, Square, or Pentagon, *&c.* of two Inches Diameter at two Foot from the Candle, or the like Figures of three Inches Diameter at three Foot from the Candle; where we are to observe, that the Strength of the Light is diminished in the same Proportion, as the Area of the Figures is increased: that is, here at the Distances of 1, 2, and 3 Feet, the

Annotat. Lect. I. Strength of the Light will be as 9, 4, 1; which we call a reciprocal Duplicate Proportion of the Diſtances.

13. [19—*When the Wood is more ſolid*, &c.] If we make uſe of a Glue whoſe Parts are finer in proportion to thoſe of the Wood to be joined together, it will hold as faſt as in ſofter Wood; as is experienced, when hard Wood is glued with Fiſh-Glue diſſolved in Spirit of Wine.

14. [19—*The Figure of any Portion of a Fluid*, &c.] When two ſmall Drops come to touch one another, they firſt become oval, and then immediately ſpherical. Now when the Drop is of an oval Figure, the Preſſure of an external Fluid acting upon it from all Parts cannot alter the Figure of the Drop, ſo as to bring it to be round; neither can it make it flatter, as ſome have imagined, who have aſſerted,——That if there was no Attraction of the Particles of the Liquor, but only the Preſſure of an external Fluid, an oval Drop would be more preſſed againſt the Ends of the ſhort Diameters, than againſt the Ends of the long one, or of the Drop, which would cauſe it to be lengthened. But I ſhall not make uſe of this Argument, becauſe it is a Fallacy; as will appear to ſuch as will be at the pains to conſult Sir *Iſ. Newton*'s *Principia*, *Lib.* 2. *Prop.* 19. where it is demonſtrated, that if any Portion of a Fluid be preſſed by the ſame or any other homogeneous Fluid acting from all Sides, that Portion will not have its Figure altered by that Preſſure.

15. [19—*Whereas from a mutual Attraction of the little Globules*, &c.] If two ſimilar and equal Globules A and B * attract one another and touch at C, they will remain at reſt, as they would if they touched at E or at D, or any other Point in the Circumference of either of them, becauſe the Contact would ſtill be the ſame: And therefore a very ſmall Force will remove B from its Contact at C to E, or any other Point of the Circumference (or rather of the Surface of the Sphere) C E D: becauſe in going round one Globule, it ſtill continues to touch it as much as it did before.

* Plate 3. Fig. 13.

If there be three Globules A, B, F, † touching one another in ſuch manner, that their Centers are in the Line *a b*, they will remain in that Poſition; but if any one of them be moved out of that Poſition as F *, it will not remain at F, but move on toward C, where the other Globule A will meet it; ſo that each of the three Globules will touch in two Points, being as many Points as three Spheres can touch one another in. See *Fig.* 16.

† Plate 3. Fig. 14.

* Fig. 15.

If there be four Globules in the Poſition *g* A, B, *f*, † they will for the ſame Reaſons come into the Poſition A, F, B, G, ſuppoſing their Centers all in the ſame Plane; but if any one of them, as F, be lifted up, it will not reſt till it comes to *c*, and then the Spherules will touch each of them in three Points. Hence it is, that when two Drops of Water or any other Liquor coming to touch one another, make up the * Spheroid *a c d b e f*, they

† Fig. 17.

Fig. 18.

they will not preserve that Figure, but run into the Sphere *c g d e h f*, that there may be the greatest Contact possible between the Globules of which the Sphere or new Drop is made up. Annotat. Lect. I.

Now because it may be thought unmathematical to draw Consequences from the Figure of the Parts of Fluids, without having first demonstrated that they are spherical; I shall here subjoin another Proof of the Roundness of the Drops of Liquors, without having any Regard to the Figure of the Particles of which they are compounded.

Let * A B D E be a Portion of an homogeneous Fluid whose Parts attract one another, and whose Figure is not spherical. If in such a Fluid we suppose a Syphon as A C E (or which is the same thing, if all the Fluid should be frozen, except the Canal A C E) whose Legs A C and C E are unequal, and meet at C the Center of the Fluid, towards which there is the greatest Attraction; the Fluid will run out at A in the Leg A C, till it be come down as far as *g* in the Leg C E, supposing C *g* equal to A C. But if the Leg A C be lengthened as far as *c*, then the Fluid will only come down as far as *e* in the Leg C E, and at the same time rise up to *a* in the Leg C *a*, C *a* being equal to C *e*. * Plate 3. Fig. 19.

If such another Canal or Syphon be supposed at B C D, the Fluid in it will come down from D to *d*, and rise from B to *b*. And since such Syphons may be supposed all over the Fluid A B D E, that Fluid by the Attraction of its Parts must needs be reduced to the spherical Figure *a b d e*. *Which was to be demonstrated.*

16. [20——*Having moistened*, &c. *two Glass-Planes*, &c.] This Experiment of the Drop of the Oil of Oranges, having been made by the late Mr. *Hauksbee*, in the manner that Sir *Isaac Newton* relates it in the third Part of his *Optics* (*Query the last, towards the middle*) that incomparable Philosopher has calculated the Force of the Attraction, and says; "That where the Oil of Oranges, between the two Glass-Planes, is of the "thickness of three Eighths of the ten hundred thousandth Part of an "Inch, the Attraction (collected by the Rule given in the Table of the "second Part of the second Book) seems to be so strong, as within a Circle "of an Inch in Diameter, to suffice to hold up a Weight equal to that of "a Cylinder of Water of an Inch in Diameter, and two or three Furlongs "in length.

17. [20——*The Attraction must increase continually as the Planes come nearer.*] There are six Properties to be observed in the Attraction of these Glass-Planes.

Let † the Point O (the Center of the Drop) be at equal Distances from the Glass-Planes Q *m*, Q *n*; and let the Radius O *b* express the greatest Distance at which the Places of Glass Q *m*, Q *n* can have any Effect upon the Point O. It is plain, † Plate 3. Fig. 20.

First,

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Firſt, That at its greateſt Diſtance from the Point Q, there will be no greater a Part of the Plate of Glaſs, than a Circle whoſe Diameter is *c d* within the Sphere of Attraction, or whoſe Parts are able to attract the Point O, all the other Particles being at too great a diſtance. *Secondly*, The Force of the Attraction of the Points of the Circle of Glaſs which are neareſt to the Point Q, is alſo the greateſt. *Thirdly*, The Sum of the attracting Particles placed towards Q, and which are contained in the Segment of the Circle of Glaſs, whoſe Arc has *ſ c* for its verſed Sine, is greater than that of the attracting Particles contained in the oppoſite Segment, in the ſame Proportion as the firſt Segment exceeds the other, becauſe of the Angle made by the Planes at Q. *Fourthly*, The Direction of the Attraction of the Points, that are at the ſame diſtance from O, making all along the Line Q R an Angle at O, more acute towards Q than towards R, the Point O will advance towards Q with an accelerated Velocity. *Fifthly*, As the ſaid Point O advances towards Q, the Diameter of the Circle of Glaſs whoſe Particles can attract the Point O will be increaſed, (*g d* is greater than *c d*) and conſequently the Circle alſo; which will cauſe the Drop of Oil to ſpread more and more upon the Planes of Glaſs, between which it was placed. *Sixthly*, The Drop or Point O will advance towards Q with a Velocity always accelerated with greater Forces; becauſe the Angle *g O b* becomes always more acute in reſpect to the Angle *d O q* in proportion as the Chord *g d* becomes greater than the Chord *c d*, which makes the Baſe *g b* always leſs in reſpect of the Baſe *d q*, whilſt the Sides of the two Triangles always remain equal; and conſequently the Angle *g O b*, is always leſs in proportion to the Angle *d O q*; which continually increaſes the Force of Attraction towards Q by the following Demonſtration.

* Plate 3. Fig. 21.

Let the * Angle *a b c* be divided into equal Parts by the right Line *h u*, and about the Centers *t*, *u*, taken at pleaſure upon that Line, draw the equal Circles *g h k i*, *a c m l*; I ſay, that if through the Points of Interſection, the Chords *a c*, *l m*, *g h*, *i k*, be drawn; *l m* will be greater in proportion to *a c* than *i k* in proportion to *g h*. From the Point *b* draw the Tangents *b ſ p*, *b r q*; and from the Point *u* for a Center draw between the ſaid Tangents the Circle *e o n d*, and join *e d*, *n o*.

The Segments *d e*, *g h*, are ſimilar, as well as *n o*, *i k*, as alſo are the Arcs *o p e*, *k ſ h*, which makes *i k* : *n o* :: *g h* : *d e*, and *i k* : *g h* :: *n o* : *d e*. Now *l m* is greater than *n o*, and on the contrary *a c* is leſs than *d e*; therefore *l m* compared to *a c* is greater than *i k* compared to *g h*: Which proves the ſixth Property of the two Glaſs-Planes that touch at one End and are opened to a ſmall Angle at the other.

18. [23——*The Reaſon of theſe Phænomena*, &c.] Dr. *James Jurin*, Secretary of the *Royal Society*, has made a great many curious Experiments of this kind, of which he gives an Account in *Phil. Tranſ.* Numb. 355. where he ſhews in what manner the Attraction of Coheſion operates to raiſe and ſuſtain Water in ſmall Tubes, and ſuch Spaces between ſolid Bodies as are analogous to ſmall Tubes.

19.

19. [23——*Quickſilver attracts Quickſilver more than Glaſs attracts it,* &c.] Some have been apt to imagine that Quickſilver and Glaſs repel one another; becauſe it does not in theſe and ſeveral other Experiments appear to ſtick to Glaſs; but that it is really attracted by Glaſs (though ſo much leſs than by it ſelf, as to make it ſeem to be repelled) will be ſhewn by giving an Account here of ſome more Experiments relating to the Attraction of Coheſion, which the above-mentioned ingenious and learned Gentleman made with Glaſs and Quickſilver. Annotat. Lect. I.

EXPERIMENT I.

Quickſilver is attracted by Glaſs.

If a ſmall Globule of Quickſilver be laid upon a clean Paper, and be touched with a Piece of clean Glaſs; upon drawing the Glaſs gently away, the Quickſilver will adhere to it, and be drawn away with it. And if the Glaſs be lifted up from the Paper, the Quickſilver will be taken up by it, in the ſame manner as a Piece of Iron is drawn by the Loadſtone, and will ſtick to the Glaſs by a plane Surface of a conſiderable Breadth, in proportion to the Bulk of the Drop, as manifeſtly appears by an ordinary Microſcope. Then if the Glaſs be held a little obliquely, the Drop of Mercury will roll ſlowly upon its Axis along the under Side of the Glaſs till it comes to the End, when it will be ſuſpended as before.

EXPERIMENT II.

If a pretty large Drop of Mercury be laid upon a Paper, and two Pieces of Glaſs held edge ways be made to touch it, one on each Side; upon drawing the Glaſſes gently from each other, the Drop of Mercury will adhere to them both, and will viſibly be drawn from a globular to an oval Shape; the longer Axis paſſing through the Middle of thoſe Surfaces, in which the Drop touches the Glaſſes.

The Particles of Quickſilver are more attracted by one another than by Glaſs. For the Proof of this, ſee *Lect.* I. N°. 24, 25, 26, and thoſe other Experiments of Dr. *Jurin.*

EXPERIMENT III. *Plate* 3. *Fig.* 22.

Quickſilver being poured into the inverted Syphon A C B, one of whoſe Legs A C is narrower than the other C B; the Height C E, at which the Mercury ſtands in the wider Leg C B, is greater than the Height C D, at which it ſtands in the narrower Leg C A. On the contrary, Water ſtands higher in the narrower Leg than in the wider.

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EXPERIMENT IV. *Plate 3. Fig. 23.*

A B C D represents a rectangular Plane of a Glass which makes one Side of a wooden Box. On the inside of this is another Glass-Plane of the same size, which, at the End A C, is pressed close to the former, and opens to a small Angle at the opposite End B D. When Mercury is poured into this Box to any Height as C E, it insinuates itself between the two Glass-[illegible]; and rising to different Heights between the Glasses, where the Opening is greater or less, it forms the common Hyperbola C G F; one of whose Asymptotes E F is the Line on which the Surface of the Mercury in the Box touches the inner Glass; the other is the Line A C, in which the Planes are join'd.

EXPERIMENT V. *Plate 3. Fig. 2.*

A B is a perpendicular Section through two Glass-Planes joined [illegible] and opened to a small Angle at B; C represents a pretty large Drop of Mercury, the larger the better, which (being made to descend as far as C, by holding the Planes in an erect Posture, with the End A downwards) retires from the Contact of the Planes to D, upon inclining the Planes towards an horizontal Situation; and the Distance C D becomes greater or less, as the Planes are more or less inclined towards the Horizon.

20. [30—*As the Cube and a quarter of the increased Distance—the Magnetical Attraction*, &c.] That excellent Philosopher Dr. *Brook Taylor* made some Experiments with a touched Needle and a large Load-stone that is kept in the Repository of the *Royal Society*, which, when made at some Distance from the Stone, agreed very well with this Assertion; but near the Stone, the magnetick Virtue did not seem to act according to those Laws, which might be owing to this, *viz.* That that Stone rather seems to be an Aggregate of Load-stones joined together by a petreous Substance not magnetical. For, since that Time, by some Experiments made upon it, I found that it had 15 Poles (if I may use that Expression) or Points where the Attraction was stronger than any where else; which Experiments, and some others made upon weak Load-stones, made me imagine that every Load-stone had several Poles or Points of Virtue on the *North* side, and several on the *South* side; whose Virtues being collected by the Iron wherewith the Stone is armed, made an armed Load-stone sustain much more Iron or Steel than the same unarmed. But by Experiments made since, with some good Load-stones, especially with a Stone of about six Ounces, belonging to the Right Honourable the Lord *Paizley* (one of the best in the World) I found that a good and homogeneous Load-stone has but two Poles.

Dr. *Petrus van Muſchenbroek*, the ingenious Profeſſor of *Aſtronomy*, &c. at *Utrecht*, has with indefatigable Pains and Application, made Experiments of the Attractions and Repulſions of Load-Stones, in reſpect to Iron, and to each other; but could never find any regular Proportion in the Increaſe of Attraction in their Approach to, or Decreaſe of Attraction in their Receſs from, one another —— only, that the Force of the magnetic Virtue did increaſe in the Approach to, and diminiſh in the Receſs from the Stone; but not exactly at the Diſtance, nor as the Square or Cube of the Diſtance, nor as the Square or Cube of the Diſtance reciprocally —— nor in any Proportion reducible to Numbers; and therefore he very reaſonably conjectures, that the Repulſions and Attractions diſturb one another, ſo as to confound the Proportion; nor are we to hope for any Rule concerning this Matter, till a way be found (if ever it can be) of ſeparating the attracting from the repelling Parts. I refer the curious Reader to his Diſſertation concerning the Magnet, which is very worthy every inquiſitive Philoſopher's Peruſal. It is printed at *Leyden* in *Quarto*, with ſeveral other valuable Diſſertations of the ſame Author. Annotat. Lect. I.

21. [31. *A repellent Power in Bodies*, &c.] *See* s'Graveſande's *Introduction*, Part I. from N°. 40, to 44. When Light is reflected from a poliſhed ſpecular Surface of Glaſs, Cryſtal, or Metal; the Particles of Light do not ſtrike upon the ſolid Parts, and ſo rebound from them; but are repelled from the Surface at a ſmall Diſtance before they touch it, by a Power extended all over the ſaid poliſhed Surface. *See Sir* Iſaac Newton's Optics. *Book* II. *Part* III. *Prop.* 8.

The Rays of Light are alſo repelled by the Edges of Bodies as they paſs near them, ſo as to make their Shadows, in ſome Caſes, larger than they would otherwiſe be. *See the ſame Author*, *Book* III. *Part* I. where he likewiſe proves this repulſive Force from other *Phænomena*.

22. [37.—*The ſame Part of the Tube will not ſnap or give Light twice together, in the ſame Place, without a new Friction.*] By cauſing the Tube to ſnap at the Approach of the Fingers, or any other ſolid Body near it, the Electricity of it in that Place is alſo deſtroyed; from which we may eaſily ſolve a *Phænomenon* mentioned by Dr. *s'Graveſande* in his Introduction, Vol. II. N°. 554. *Theſe are his Words:*

"There is one thing remarkable and very hard to explain in this Experiment, concerning the Direction of Attrition; when you rub the Tube, "one End is held in one Hand, whilſt it is rubbed with the other; which "if it be done from the Hand that holds towards the other End of the "Tube, the Effect will not be ſenſible; but if you rub from the free End "of the Tube towards the End held in the Hand, the contrary will happen. And this happens indifferently whether you hold the open or ſhut "End of the Tube in your Hand.

To explain this, let us examine the Experiment by help of the 10th *Figure* of *Plate* 2. A is the Right Hand that holds the Tube, and B the Left-Hand that rubs the Tube, which, after moving up and down ſeveral

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times, makes an end of the Attrition by moving in the Direction C B A, the laſt Stroke: Then the Tube brought near light Bodies (*Plate* 2. *Fig.* 11.) gives Motion to them. If the laſt Stroke in rubbing be made by the Motion of the Hand, from A to C, and the Hand B that went up to C quits the Tube in the Direction C D, without coming near it again, or letting the Coat-ſleeve come near it, the Tube will act upon light Bodies with the ſame Vigour as before. But if the Left-Hand, after it is come off at C, is brought down again careleſsly, in the Direction C E, in a Line parallel to the Tube, and pretty near it, either the Hand or the Coat-ſleeve coming down too near, cauſes the Tube to ſnap (which is not heard without Attention) and ſo deſtroys the Virtue excited by Attrition, in the whole Length of the Tube; as the Hand A (*Plate* 2. *Fig.* 13.) does in the Place A where it cauſes a Snapping, which, as I ſaid before, will not happen twice in the ſame Plate, without a new Friction.

23. [41 —*A Glaſs Globe whirled*, &c.] I cannot forbear mentioning here a very ſurprizing *Phænomenon* in one of Mr. *Hawkſbee*'s Experiments. He coated with Sealing-wax the Inſide of one Hemiſphere of one of his Glaſs Globes to ſuch a Thickneſs, as to render it perfectly opaque: Yet when this Glaſs was exhauſted of its Air, and whirled round; where the Hand was applied on the Outſide to give an Attrition to the Globe, the Wax became as tranſparent as the Glaſs it ſelf; the reſt of the Globe that had Wax remaining opaque, where it did not touch the Hand, though the Moment before it had been tranſparent as it paſſed under the Hand.

A View of the 25th *Fig.* of *Pl.* 3. will repreſent the Thing fully.

The Hemiſphere ACB of the Glaſs Globe GACB is made opaque, by being lined with Wax on the Inſide, whilſt AGB is clear Glaſs. The Glaſs is exhauſted of its Air by means of the Cock D; then being ſet between the Pillars EF, it is whirled ſwiftly round by means of the Wheel K, whoſe String goes round a Pulley P fixed to the Braſs Socket, whoſe Shank is the Axis of the Globe. *The Screw* H *draws forward the other Pulley* I, *to keep the Wheel-ſtring always tight.*

When the Hand is applied at C, its Inſide becomes viſible, through the Wax on the concave Side of the lined Globe, the reſt of the Wax remaining opaque; ſo that the Hand cannot be ſeen by an Eye at Q, but by an Eye at O looking at it through the unlined Part of the Glaſs at G.

Mr. *Stephen Gray*, who has made greater Variety of electrical Experiments, than all the Philoſophers of this and the laſt Age; has, ſince I began this Book, found out ſeveral new *Phænomena* in Electricity, the Particulars of which may be ſeen in the *Philoſophical Tranſactions*, N°. 417.

But for the ſake of ſuch, as may not have an Opportunity of ſeeing the *Tranſactions*, I will give a general Account of his Diſcoveries.

1. That all ſuch Subſtances as cannot be made electrical by rubbing, may yet receive an electrical Virtue (as is viſible by their attracting and repelling Leaf-Gold) from a rubbed Glaſs Cane (whether ſolid or hollow) not,

not only in contact with the Tube, but by the Intermediation of a String to very great Diſtances, as for Example, of above 800 Feet. Nay, if the Tube does not touch, but is only rubbed near the String, the electrical Virtue will run along. And what is very remarkable, as the Virtue runs along an hempen String, that String muſt be ſupported by Hair, or ſilk Lines; croſs Strings of Hemp or Flax, or even Wire, ſtopping the Propagation of the Virtue to a Diſtance, by receiving it laterally. Annotat. Lect. I.

2. Very large Surfaces, as Maps, Table-Cloths, *&c.* are impregnated with electrical *Effluvia*.

3. A Load-Stone, and Iron hanging at it, will receive the electrical Virtue, which therefore is not diſturbed by magnetick Effluvia.

4. The Electricity will be carried ſeveral ways at the ſame Time, without touching the String of Communication with the Tube; as to two Balls of Ivory at the Ends of a very long String, by rubbing the Tube near the Middle of the String.

5. The electrical Attraction received is not proportionable to the Quantity of Matter in Bodies, a ſolid and an hollow Cube of Wood attracting the one as much as the other.

6. The Effluvia are alſo carried round in a Circle, and communicated from one Circle to another.

7. This Virtue will be communicated to Leaves of Trees; nay, even to Fluids, as to Bubbles of ſoaped Water.

8. Animals alſo will receive this Virtue. A Man ſuſpended horizontally by two Hair-Lines, attracts and repels Leaf-Gold with his Face and Hands, if the Tube be rubbed near his Feet; nay, at the End of a long Fiſhing-Rod held in his Hands, a Ball will attract and repel. It is remarkable, that the Virtue is ſtrongeſt at that Part of the impregnated Body, which is fartheſt from the Tube; for if the rubbed Tube be held over the Head, not the Head, but the Face will then attract, and ſo *vice verſâ*. If the Man but juſt touch the Floor with a ſmall Cane or Wire, the Virtue runs to the Floor, and the Man is no longer electrical. So hanging by an hempen Rope, carries away the Virtue to the Cieling.

9. If two Boys be by Hair-Lines ſuſpended as above, and at a conſiderable Diſtance from one another, the Virtue may be communicated from the one to the other by a String, which both hold, or tied to their Clothes; the Electricity being always ſtrongeſt in him, who is at the greateſt Diſtance from the Tube.

10. Mr. *Stephen Gray* has found ſince, that if a Man ſtands upon a Cake of Roſin, or of Glaſs, or of ſome other Subſtances, which are of themſelves electrical, or become ſo by rubbing; the Effect will be the ſame as if he hung from a Hair-Line. So that if a whole Regiment of Soldiers were in a Line, and each Man ſtanding upon a Cake of Roſin held the next by the Hand, or only communicated with him by a String, I do not doubt, but they might all be ſo impregnated with electrical Virtue, that when the Tube ſhould be rubbed near the firſt Man, the laſt would with his Hand, Face, or Clothes, attract and repel Leaf-Gold.

Annotat. Lect. I.

11. It has been alſo found, that Electricity will be communicated through denſe and large Bodies.

24. [42.—*Though the Cauſes of theſe Cauſes are not known*, &c.] When Genealogiſts, in ſearching into the Original of Families, are got as far as they can, and have found the firſt of the Family ſo called, becauſe they cannot diſcover who were his Parents; it would be very abſurd to ſay, that becauſe the Father of this Firſt (which for example we will call *John*) is not known, therefore *John* is not the Father of *Peter*, the Grand-father of *William*, and Great Grand-father of *Nicholas*, &c.——Which was proved before, by ſuch Evidence as is proper in that Caſe. So when it appears by Obſervation that Gravity is the Cauſe of the Fall of heavy Bodies, which obſerve certain Laws in their Motion——That a heavy Body by its Deſcent moves the Axis of a Wheel, that carries round another by its Teeth, which by the Intermediation of other Wheels and Pinions, carries round a Hand upon a Dial-Plate to meaſure Time, or for other Uſes, it would be very unphiloſophical to ſay——That our Reaſoning about the Cauſe of the Motion of the Hand is falſe, as being founded upon *occult Qualities*; becauſe we can go no higher than Gravity, whoſe Cauſe we don't pretend to know.

LECTURE II.

1. THE *Momentum* or *Quantity of Motion* in Bodies (sometimes called simply *Motion*) is that Force with which Bodies change their Place.

I DON'T mean the Stroke, Pressure, Traction, or any other Action which causes this Change of Place in a Body; but the Force which it has all the while it is moving from one Place to another.

2. THIS *moving Force* may always be known by the Effect which it is able to produce; that is, by the Stroke which the moving Body can give, or by the Resistance or Obstacle which it is able to overcome.

3. THIS Quantity of Motion, which is the Measure of the Force, is made up of the Quantity of Matter and the Velocity taken together. That is, when we compare the *Momenta*, moving Forces, or Quantities of Motion in Bodies, we multiply the Mass or Quantity of Matter in each Body by its Velocity *. * Ann. 1.

4. *Velocity* or *Celerity*, is the Swiftness with which a moving Body changes its Place; and may always be known by the Space that the Body goes through in a given Time.

5. THE Quantity of Motion may be increased; either by increasing the Quantity of Matter, which is moved with a determinate Velocity; or keeping the same Quantity of Matter, and increasing the Velocity; or by increasing both.—And in the three Cases it is done by applying more Force; for here * *Force* and *Motion* mean the same thing. * Ann. 2.

6. THE Motion of any Whole is the Sum of the Motion of all the Parts; and therefore (as we said before) it becomes doubled in a double Body moved with equal Velocity, and quadrupled in a double Body moved with a double Velocity.

7. IF a Man with a determinate Force throws from him a Weight of 50 Pounds to the Distance of ten Feet; he must apply twice the Force to throw a Weight of 100 Pounds to the same Distance, or to throw the 50 Pound Weight twice as far: but if he uses no more Force than he did before, he will throw the 100 Pound Weight only

to

Lect. II. to the Distance of 5 Feet, and then the two Bodies will have the same Quantity of Motion; because 50 multiplied by 10, or 100 multiplied by 5, give the same Product, *viz.* 500.

EXPERIMENT I. *Plate* 4. *Fig.* 1.

8. ABCDE is an Instrument contrived for illustrating what has been said, and distinguishing Motion and Velocity, which some Authors have confounded. The rabbeted Cheeks BD, CE, are so contrived that the smooth cylindric Weight K or L (the one of 8, the other of 4 Ounces) may move between them with very little Friction. Let the † Spring AB be bent to a certain Degree, by slipping the Knot G of the String fastened to B upon the Catch of the Iron F; then lay on the Weight L at B, which upon letting go the Spring, by lifting up the Knot, will be shot from B to the Point I, which is 24 Inches from B. If the Cylinder K be shot in the same manner, it will go but to H 12 Inches from B.

† Plate 4. Fig. 1.

THAT the Quantity of Motion is the same in both Bodies, is evident, because the Spring is equally bent in both Cases; and that those Quantities of Motion are made up of the Masses multiplied into the Velocities, is also evident, because L = 4 Ounces × by BI (24) its Velocity gives 96, equal to K (8 Ounces) × BH (12) its Velocity.

BUT if you would have K driven as far as the Point I, to which L was driven; the Spring must be bent with a double Force, and then K will have double the Motion that it had. N.B. *This Experiment is made use of rather to illustrate this matter than to prove it.*

CARE must be taken not to draw Consequences from this Experiment; because the Success of it will be different according to Circumstances. If the Cylindric Weights * be placed quite close to the Spring when it is bent, the Weight K (of 8 Ounces) will be shot farther than half the Distance that L (of 4 Ounces) is shot to; and that will happen for the Reasons given in the 4th Annotation of Lect. 5. But if both the Weights be set at some Distance from the bent Spring, so that the Spring may give a sudden Blow, and not act long upon the Cylinders; then the Experiment will succeed. N. B. *That Distance must be found by Trials.*

* Plate 4. Fig. 1.

9. HENCE it follows, that any little Body may have as much Motion as a great one, be their Disproportion what it will; provided that the Velocities that are given them be reciprocally proportionable to their Masses: that is, if the little Body has as much more Velocity than the great one, as it has less Matter.

THIS

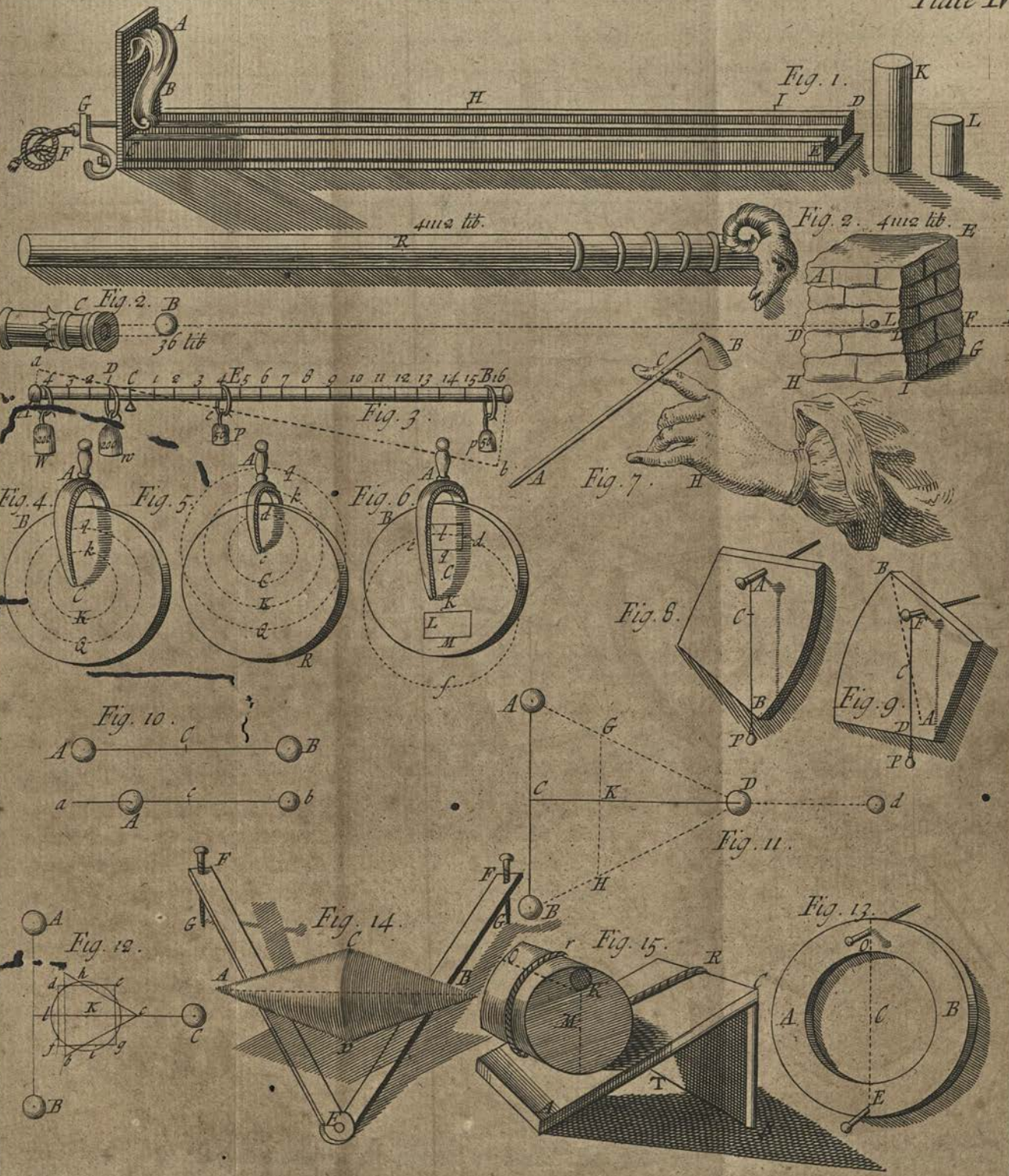


Fig. 1.
Fig. 2. 4112 lib.
Fig. 2. 36 lib
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.
Fig. 14.
Fig. 15.

Lect. II.

THIS is the Reason why, since the Invention of Gun-powder, battering Rams have been disused in War; for those and other heavy Machines managed by a great many Hands, and moved against a Wall with little Velocity, did no more than now is performed by a small Cannon-Ball, three or four Men only being employed to manage the Cannon. If the Ball B* weighing 36 Pounds be shot out of the Cannon C, against the Wall AHGE, so as to strike it at L, it will produce the same Effect as the battering Ram R, which weighs 41112 Pounds; provided that the Cannon-Ball moves as many times swifter, as it has less Matter than the Ram. *See the Calculation of it in the Notes* †.

* Plate 4. Fig. 2.

† Ann. 3.

IF a small Pistol-Bullet should move with the same Velocity as Light, it would strike as strongly against an immoveable Obstacle as a Cannon-Ball 700000 times as big; because Light moves 700000 times faster than a Cannon-Ball. Light is about 8 Minutes coming from the Sun, and a Cannon-Ball would spend ten Years in going through the same space.

10. As the Quantity of Matter in a moving Body, multiplied by its Velocity, gives us the Quantity of Motion; so the Quantity of Motion divided by the Velocity, will give us the Quantity of Matter: but if it be divided by the Quantity of Matter, it will give us the Velocity. If several Bodies of different Weights move with equal Velocity, their Motions will be to one another as their Quantity of Matter.

11. HENCE may be deduced an unanswerable Argument for a *Vacuum*. For if all Bodies, abstracting from the Resistance of the Air, move downwards with the same Velocity, as has been proved *, their Motions compared will be respectively as their Quantities of Matter; but the *Motion downwards*, or Force which drives them to the Earth, is their *Gravity*: therefore we find the Quantity of Matter in any Body by its Gravity, which must always be proportionable to it. Now if two Bodies of equal Bulk weigh differently, as we find by Experience they do, there must be Vacuities interspersed in the lighter: As for Example, if there be two Inch Cubes A and B †, and the Cube A be of Silver, whilst the Cube B is of Cork; it will be found that A weighs 40 times more than B, therefore B has 40 times less Matter, and ought to be 40 times less in Bulk, if it had no Vacuities. For if it be answered, that the Voids or Pores of the Cork are filled with Air and subtile Matter; that Air and subtile Matter * together with the Cork ought to weigh as much as the Silver; or else the two Cubes cannot be equally full.

* L. 1. N°. 8, 9.

† Plate 3. Fig. 7.

* Ann. 4.

12. THE

Lect. II. 12. THE whole Effect of mechanical Engines (whereby Motion is given or ſtopped, or a Reſiſtance is overcome) depends upon what we ſaid above, N°. 9.

IF a great Weight is to be ſuſtained by a little one, we muſt contrive to give the little one ſo much more Velocity than the great one as it has leſs Matter, and then its Force being equal to that of the great one, it will ſuſtain it, if they move in contrary Directions; becauſe then equal Forces will deſtroy each other. This is done by the Contrivance of the Engines, and Manner of applying them. For if the Velocity of the great Weight be determined as well as its Quantity of Matter, and the Quantity of Matter in the little one: then the Velocity of the little one (which in this Caſe is called the Power) muſt be increaſed in the Proportion above-mentioned. But if the Velocity of the Power be determined, then that of the Weight muſt be diminiſhed in the ſaid Proportion: the Engines for thoſe Purpoſes being always ſo contrived, that the Weight or Power may be applied in ſuch manner, as to render their Velocities reciprocally proportionable to their Maſſes.

EXPERIMENT II. *Plate* 4. *Fig.* 3.

Plate 4. Fig. 3. 13. LET AB be a Leaver or Balance divided into 20 equal Parts, whoſe Center of Motion is at C; if the Weight W of 200 Pounds hangs by an immoveable Hook at A, and we would ſuſtain or keep it *in Æquilibrio* by means of the Power or ſmall Weight P of 50 Pounds; it is plain that it has not Force enough at E to ſuſtain the Weight W, becauſe in the Motion of the Leaver the Point E and A will deſcribe not only ſimilar but equal Arcs E *e* and A *a*; ſo that when W moves 1 Inch, P will only move 1 Inch; and ſince the Velocities being equal, the Quantities of Motion or Forces are as the Maſſes *, the Weight W will always over-power, as having 4 times the Maſs of P. But if P be removed to B, then it will deſcribe an Arc ſimilar to, but 4 times greater than that which W deſcribes; that is, it will deſcend 4 times faſter than W riſes. If then its Velocity be 4, and that of W only 1, *p* or 50 multiplied by its Velocity 4 will give 200; which is equal to the Product of W, or 200 multiplied by its Velocity 1.

* N°. 10.

14. BUT if the Power P had been immoveable at E, and the Weight W moveable; an *Æquilibrium* would have been had by diminiſhing the Velocity of the Weight, in bringing it forward to D,

D, where being 4 times nearer to the Center than P, it would have had 4 times less Velocity. Lect. II.

15. If both the Weights had been moveable, and P had been but 12 Pounds and an half, then the Velocity of P must have increased by removing it to B the 16th Division, at the same time as that of W was diminished by bringing it to D the first Division on the other Side of the Center; for then W (200) multiplied into DC (1) which is proportionable to its Velocity, would have given 200, which is equal to the Product of *p* (12 $\frac{1}{2}$) multiplied into BC (16) its Distance from the Center, which expresses its Velocity. If therefore the Power has a little more Velocity given it, than in a reciprocal Proportion of the Masses, it will have more Force than the Weight, and consequently raise it, whereas it only sustained it before.

16. Thus by means of a Leaver, a Man, whose natural Strength does not exceed 200 Pounds, shall acquire so much relative Force as to raise a Stone of 2000 Pounds by applying his Leaver so as to render the Velocity of the Stone ten times less than that of his Body at the opposite End of the Leaver, which in that Case will be ten times farther from the *Fulcrum* or propping Point, than the Place where the Stone is applied. For if a Man by his natural Strength can raise 200 Pounds with a determinate Velocity, there is no Engine in the World that shall enable him to raise 2000 Pounds with the same Velocity; but he must do it with the 10th Part of that Velocity. If he must employ 10 Seconds of Time to raise 200 Pounds 10 Foot; and would raise a Stone of 2000 Pound Weight by a Leaver whose *Brachia* (or Lengths on each Side the Center of Motion) are as 10 and 1; he must move 10 Foot at the End of the long *Brachium* of the Leaver whilst the Stone moves 1 Foot, which comes to the same as if the Stone being cut into 10 Pieces, each of them was successively lifted up one Foot by the same Man, who would do it just with the same Labour as when he raised it all at once with the Leaver*.

We cannot alter Nature; where we would gain Strength, we must lose Time; and where we would gain Time, we must employ more Strength†.

* Ann. 5.

† Ann. 6.

17. If the Velocity of the Weight as well as its Quantity of Matter, be determined, and likewise the Velocity of the Power;

Lect. II. then if the Power is not sufficient to raise the Weight, there must be more Matter added to it till the Product of its whole Mass multiplied into its Velocity be equal to the Product of the Mass of the Weight into its Velocity. If, for example, the Weight * W still supposed equal to 200 Pounds be fixed at A, and the Power *p* equal only to 12 $\frac{1}{2}$ Pounds be fixed at B; it will not be possible for the said Power to sustain the Weight, till its Quantity of Matter be quadruple to make it 50 Pounds, and then 50×16 will be equal to 200×4; because here the Distances are as the Velocities.

* Plate 4. Fig. 3.

Sometimes it is required to give a heavy Body or Weight a considerable Degree of Velocity, as when the Ancients used to throw great Stones with those kind of *Balistæ* which they called Scorpions *; and then the Power must be considerably greater than the Weight; for as it is applied nearer the Center of Motion than the Weight to be thrown, it must be heavier in a reciproeal Proportion of those Distances when the Weight is only sustained, and much heavier to give the Projectile Body a sufficient Velocity *. But these Things will be better understood as we come to consider the *mechanical Powers*, and the Uses of simple and compound Engines; in order to which we must explain some Terms, and take notice of some Truths necessary to be known by every Engineer.

Ann. 7.

Ann. ibid.

Definitions.

18. A *Weight* is any Body to be sustained, raised or depressed, pushed or drawn, or moved in any manner; so that the Expression *to raise a Weight* is very extensive in the mechanical Sense; as sometimes it is applied to the driving a Pile into the Ground, sometimes to the stopping of a Body in Motion, as the running of Water, *&c.*

19. A *Power* is whatever is made use of to raise a Weight in the Sense above-mentioned, whether the Power it self be a heavy Body, a Spring, the Motion of Water, Air, Smoke, Flame, or Pressure of Stream excited by Fire from Liquids; or the Force of any Animal, acting by its Strength or Weight, or both.

20. The *Intensity of a Power* is its absolute Force, that is, its Force, supposing its Velocity equal to that of the Weight; for its *Moving* or *Acting* Force may be greater or less, according as its Velocity is increased or diminished, in respect of that of the Weight. As for Example,

If

Lect. II.

If a Man be the Power, and can raiſe from the Ground a certain Weight, that Weight will expreſs or be equal to the *Intenſity of the Power*; for in this Caſe, whatever Engine be made uſe of, that Part of the Engine, where the Weight is duly applied, will move juſt as faſt as that which a Man acts upon with his whole Force. So if a Man can preſs upon the Point E, at the 4th Diviſion of the Balance A B * with the Force of 200 Pounds, he will juſt ſuſtain the Weight W, or 200 Pounds, hanging preciſely as far on the other Side of the Center of Motion.

* Plate 4. Fig. 3.

21. The *Line of Direction* is that Line in which a Weight or Power acts, or endeavours to act *. Ann. 8.

22. A Power may act in any Direction whatever; but a Weight has but one Direction, which is towards the Center of the Earth, in which Direction all heavy Bodies endeavour to deſcend, and actually do, when no Obſtacle hinders: So that the *Line of Direction of a Weight* is a Line drawn from its Center of Gravity to the Center of the Earth.

23. The *Center of Motion* is that Point round which a Body or a Machine moves, or endeavours to move when it cannot or does not turn quite round; and in that Caſe, all the Points of the Body deſcribe Circles, or Arcs of Circles, about the Center of Motion. *This Center may be taken any where according to the Make of the Engine.*

24. The *Center of Gravity* is a Point about which all the Parts of a Body are *in Æquilibrio*. It is conſidered as the Middle of the Weight of the Body, though often it is not the Middle of the Body it ſelf; and if the Body be ſuſpended by that Point, it will hang in any Poſition; otherwiſe the Center of Gravity will deſcend as low as it can.

Experiment III. *Plate* 4. *Fig.* 4.

25. BQ is a round Board ſuſpended by its Center C on the Points of the ſpringing Calibers A. In turning the Board round, the two Marks made upon it K and Q, deſcribe the Circles K *k*, and Q *q* round the Center of Motion C, which is here the Center of Magnitude: If the Center of Motion was taken in any

Lect. II. Point which is not the Center of Magnitude, as at *c* *, still K and Q would describe Circles about the Center of Motion, though the Center of Magnitude would not then be their common Center, but it self would describe a Circle as C *d* about *c*, the Center of Motion. If the Body suspended does not go quite round, as here the Side R stops against the Top of the Calibers at *q*; instead of Circles, the Points Q, K and C describe only Arcs of the Circles Q *q*, K *k*, C *d*.

* Plate 4. Fig. 5.

26. If C* be the Center of Gravity of the Body, and the Body be suspended by it, and made to turn round that Point, it may be stopt in any Position of the Points K and Q, and it will then remain at rest; but if the Body be suspended by *c* *, which is not the Center of Gravity, then the Center of Gravity C will descend as low as it can to C.

* Plate 4. Fig. 4.

* Plate 4. Fig. 5.

* Plate 4. Fig. 6.

But if *c** *be the Center of Gravity, and placed directly over the Center of Motion* K, *the Body will remain in that Position*; because as the Center of Gravity endeavours to descend in the Line *c* K, which is the Line of Direction of the Body (in which Line the Point K is supported) it presses directly upon the Point K, which is sustained by the Calibers; *but if the Body be moved ever so little,* so as to bring the Center of Gravity *c* towards *d* or *e*, *the Body will* turn and *not rest till the Center of Gravity comes to* M directly *under the Center of Motion,* the Body falling into the Position *e d f*.

Experiment IV. *Plate* 4. *Fig.* 8.

27. Hence may be deduced a Method for finding the Center of Gravity of any Body mechanically *. Let A B † be a Body whose Center of Gravity is to be found. If it be suspended by any Part as A, so as to move freely upon the Pin at A, and a Plumb-Line A B hangs from the same Pin, its Center of Gravity C must be under, or rather behind that Line; because it will fall below the Center of Motion A: Let that Line A B be marked upon the Body, as in *Fig.* 9. and then suspend the Body by any other Part, as F; provided that the Center of Motion be not in the above-mentioned Line A B. Hang on the Plummet at F, and the Line F D under the Plumb-Line F P will cut the Line A B, and shew the Center of Gravity to be at C; for since it must be both in the Line A B and in the Line F D, it can only be in the Point C where they intersect.

* Ann. 9.
† Plate 4. Fig. 8.

N. B.

N. B. *We have not here considered the Thickness of the Body; but if we suppose it a Piece of Board as the Figure represents; then we must only make the same Experiment on the other Side, and we shall find another Point* C *just opposite to the first Point* C. *The Line which joins these two Points will be the* Axis of Gravity, *and the Middle of that Line the Center of Gravity.*

28. HENCE also follows, That let the Figure of a Body be what it will, the Body cannot fall when its Center of Gravity is sustained; and when any Body is *in Æquilibrio*, its Center of Gravity must be in a Line which goes through the Center of Motion and the Center of the Earth; which is the Line of Direction of heavy Bodies mentioned before *. Thus in *Fig.* 7. † where the Tobacco-pipe is sustained *in Æquilibrio* upon one's Finger, the Point C just over the Finger is the Center of Gravity.

* 22.
† Plate 4.

EXPERIMENT V. *Plate* 4.

29. IN Bodies that are both regular and homogeneous, the Center of Gravity is just in the Middle of the Body; that is, in its Center of Magnitude; as appears in the Body A B *, which being suspended by its Center of Magnitude C, then turned round, will remain in any Position in which it is stopped; but in *Fig.* 5. * where the same Body is suspended by another Point *c*, it will not rest but when the Point C is come down below *c*, or is directly above it, as we said before. But if the same Body, which was supposed a circular Piece of Wood (as for Example of Oak) be denser in one Part than another, or be made so by letting a Piece of Lead into the Wood as at M (*Fig.* 6. *) then the Center of Gravity will no longer be at C, but at K; about which Point of Suspension only, the Body will remain in any given Position. If the Body was suspended by the Point C, it would be at rest only in two Positions, *viz.* when the Lead being carried up to *l* the Center of Gravity is at *c* *, or when the Lead is at M, and the Center of Gravity at K †. If the Body was homogeneous but not regular, the Center of Gravity would also then be different from the Center of Magnitude. As for Example, If you take the Pipe of *Fig.* 7. * and break it at C its Center of Gravity, you will find by weighing the two Pieces successively, that there is more Matter in the half C B, than in the other half or Shank A C.

* Plate 4. Fig. 4.
* Plate 4.
* Plate 4.
* 24.
† 24.
* Plate 4.

30. As

Lect. II. 30. As any Body that we consider in Mechanics, is only an Aggregate of several other Bodies or Parts; so the Center of Gravity of a Body is only the common Center of Gravity of all its Parts: and consequently if several Bodies are united in any Machine, or if there be any Combination of Bodies to be sustained, regard is no longer had to the particular Centers of Gravity of the several Bodies which make up the Compound, but only to the common Center of Gravity of the Whole. Thus a Wind-mill must be supported under the common Center of Gravity of all its Parts, and its Line of Direction must fall along the Axis of the Post round which it moves: And a Crane upon a Wharf or a Dock (where the whole Machine turns round) must have the Line of
Ann. 10. Direction in its Axis *.

EXPERIMENT VI. *Plate* 4. *Fig.* 10.

31. Let the Line AB represent an even Rod or Wire divided into two equal Parts at the Point C; its Center of Gravity will
29. be at C *. And if two equal Bodies equally heavy are thrust on upon its Ends, so as to have their Centers of Gravity at the same Distance from C, they will be *in Æquilibrio* about the said Point, which will then become their common Center of Gravity, and continue so whether the Bodies approach nearer to, or recede farther from it in proportion to their Masses. The same will happen if the Bodies are unequal as A and *b*, whose Masses are to each other as Two and One, provided that the greater Body be at A twice nearer to the common Center of Gravity *c* than the lesser Body *b*: And *c* will still be the Center of Gravity of those Bodies, though they should move to immense Distances from each other, provided their Distances from the said Point are reciprocally as their Masses, as we said before.

32. So that when two Bodies approach towards or recede from each other, with Velocities reciprocally proportionable to their Masses, their Center of Gravity will remain at rest. And if the Bodies being made fast upon the Wire, the Center of Gravity be sustained by a Pivot or pointed Broach, how swift soever the Bodies be made to turn round the Center of Gravity and each other, the
* Ann. 18. Center of Gravity will remain at rest *, and the Bodies will describe similar Circles about it and about each other, the one
* 25. never over-powering the other *. If they be carried forward in any

any manner by any external Force acting upon them in proportion to their Masses, their Center of Gravity will go forward uniformly in a Right Line, and move just as if the two Bodies were united into one at the said Center: And if they be projected, their Center of Gravity will move in the same Curve as all Projectiles do, whether in their Motion they turn round each other or not. This is evident in the Motion of an Arrow, of Chain-Shot, or Bar-Shot, and of a Stick thrown from the Hand, the Center of Gravity of any of these Bodies moving in the same manner as a single Ball would do. So the Moon and Earth, in their Motion round the Sun, do neither of them describe the *Magnus Orbis*; but their common Center of Gravity describes it, in the same manner that they would do if they were both united in that Point, or in the same manner that the Earth alone is supposed to do it, when these Inequalities of Motion are overlooked: And provided that their Distances from their common Center of Gravity be reciprocally proportionable to their Masses; their Distances from each other may be greater or less in any Proportion: If there were no other Bodies in our System but the Earth and Moon turning round each other, their Center of Gravity would always remain at rest. Lect. II.

EXPERIMENT VII. *Plate* 4. *Fig.* 11.

33. IF to the two Bodies A and B there be added a third at D * equal to one of the other, let A and B be reduced to their common Center of Gravity, and be considered as a Body equal to both placed at C; then the common Center of Gravity of C and D will be found at K *, as much nearer to C as the Mass of the Body or Bodies at C exceeds that of the Body at D. If the new Body weighed but half as much as one of the others, it must be removed to *d*, so as to have the Distance K *d*, quadruple the Distance K C. Now if C D be a Wire, and it be supported under K, the three Bodies, whether D or *d* be made use of, will be sustained by that means; only in taking the Center of Gravity exactly at K, we must not consider the Weight of the Wire, whose Thickness we are to suppose diminished *in Infinitum*, so as to look upon the Wire only as a mathematical Line without Substance or Weight.

* Plate 4. Fig. 11.

* 31.

EXPERIMENT VIII. *Plate* 4. *Fig.* 12.

34. IF upon this Center of Gravity of the three Bodies, be placed a flat triangular Body *a b c*, with its Center of Gravity just over K, then

Lect. II. then a ſquare one as *d e f g*, then a circular one as *h l i*, all in the ſame manner; ſtill the whole will be ſuſtained by ſupporting the Point K. Whence it appears, that if the whole Weight of a Body be reduced into its Center of Gravity, it will (act as a heavy Body, that is) *gravitate* juſt in the ſame manner as it would before.

35. If theſe three Bodies, united to or acting upon one another in any manner proportionable to their Maſſes, be carried round their common Center of Gravity; that Point will be at reſt, for the ſame Reaſon as it will happen in reſpect of two Bodies; becauſe any Number of Bodies may in this reſpect be reduced to two. Thus in our Syſtem, where the Sun and all the Planets move round their common Center of Gravity, that Center is at reſt in the middle of the Syſtem. Though we commonly conſider the Sun as immovable in the middle of the Syſtem; becauſe as it has vaſtly more Matter than all the Planets together, that Center will always be very near the Sun's Center.

36. It often happens that the Center of Gravity of a Body, or of a Syſtem of Bodies, is not within the Body itſelf, or any one of the combined Bodies; yet we are to have the ſame Regard to its Support, Deſcent, or Motion in any Direction, as if it was. As for example, let us ſuppoſe the Bodies A and *b* * to be at the Diſtance A *b* from each other, and that *a b* is no longer a Wire, but A *b* is a Line repreſenting their Diſtance; we ſhall find their Center of Gravity to be at *c* without the Bodies. And if inſtead of the Wire C D we ſuppoſe the Body D * joined to the two Bodies A and B by the Wires A D and B D, the Center of Gravity at K will be neither in the Bodies nor Wire: ſo that if we would ſuſtain the ſaid Bodies, we muſt ſupport ſome one of them, or ſome Part of the Wire as G; which being made the Center of Motion, the Center of Gravity K will (if the Bodies hang freely) come juſt under it; or if we ſupport the Point H, the Bodies will be at reſt, by reaſon of the Center of Gravity being juſt over H the Center of Motion. In the Ring A B H * the Center of Gravity is in no part of it, but it may be ſupported by any other Point, as O or E, &c. Upon this account the Ring of *Saturn* has its Center of Gravity in the Body of *Saturn*. And though the common Center of Gravity of the Sun, Moon, and Earth is within the Sun's Body, yet the common Center of Gravity of the Moon and Earth is in neither of the Bodies, but between them.

* Plate 4. Fig. 10.

* Plate 4. Fig. 11.

* Plate 4. Fig. 13.

35. Since

37. SINCE OE is the Line of Direction, it is evident that in any Position of a Body or of a Combination of Bodies, *if the Point of Suspension or Center of Motion be in any Part of the Line of Direction, the Body or Bodies will remain in that Position; otherwise the Center of Gravity will descend as low as it can, and by its Motion alter the Position of the Body.* Lect. II.

EXPERIMENTS IX *and* X.

38. SEVERAL odd *Phænomena* depend upon this Principle; as for example, the double Cone or Spindle ACBD * being laid on at E upon the lower Part of the Rulers EF, will of itself move towards FF, though those Ends are raised up to the Heights FG by the two Screws FG; and by that means will seem to move upwards: And the Cylinder M *, whose Center of Gravity is in the Mid-way between K and Q, will actually roll up the inclined Plane AC, provided it be hindered from sliding by the Rope R*r*. How high FG † may be in proportion to the Bigness of the Spindle ABCD; or BC * in proportion to the Cylinder M, is demonstrated in the Notes †. * Plate 4. Fig. 14. * Plate 4. Fig. 15. † Fig. 14. * Fig. 15. † Ann. 12.

39. BUT before we mention any more of these *Phænomena*, it will not be amiss to shew how the common Center of Gravity of two or more Bodies may be found.

IN two Bodies it is in a Line which joins their particular Centers of Gravity, and this Analogy gives it:

As the Mass (or Quantity of Matter in) the two Bodies:
Is to the Mass of one *of the Bodies: :*
So is the Distance of the Centers of Gravity of the Bodies:
To the Distance of the common Center of Gravity from the Center of Gravity of the other *Body.*

FOR example, if we suppose the Bodies A and B * to weigh 2 Pounds each, and to have their respective Centers of Gravity 4 Foot distant from each other; then, * Plate 4. Fig. 10.

As 4 *the Mass of the two Bodies:*
Is to 2 *the Mass of the Body* B: :
So is AB, or 4, *the Distance of their two Centers of Gravity:*
To AG, the Distance of the common Center of Gravity, from the Center of Gravity of A, *which is* 2 *Foot.*

Lect. II. OR to express it short by Algebraical Notations, A+B : B :: AB : AC; but if we take A of 2 Pounds and *b* of 1 Pound, and A*b*, equal to 3 Feet, the common Center of Gravity will be at *c* within 1 Foot of A, or twice nearer its Center than the Center of *b*; because A+*b* (3) : *b* (1) :: A*b* (3) : A*c* (1.) Thus is the common Center of Gravity of the Moon and Earth found, when once we know their Masses and Distances. The Earth weighs about 40 times more than the Moon, and the Center of the Moon is about 61 Semi-diameters of the Earth distant from the Earth's Center; therefore the common Center of Gravity of both is distant from the Center of the Earth almost one Semi-diameter and a half, or near 2000 Miles above the Surface of the Earth; for as the Mass of the Earth and Moon (41) : to the Mass of the Moon (1) : : so is their Distance (61) : to the Distance of the common Center of Gravity of both from the Center of Gravity of the Earth, that is, $1\frac{20}{41}$ Semi-diameter of the Earth.

40. IF there be more Bodies than two, as for example, the three Bodies A, B, and D *; first find the common Center of Gravity of two of them by the foregoing Rule, and it will be at C: then say,

* Fig. 11.

As the Mass of the two Bodies considered as united at C, *together with the Body* D:
Is to the Body D : :
So is CD *the Distance of the common Center of Gravity of* A *and* B *from the Center of Gravity of* D:
To CK, *the Distance of the common Center of Gravity of the three Bodies from the common Center of Gravity of the two first.*

OR in short, A+B+D : D : : CD : CK. And if there be a Combination or System of any Number of Bodies; their common Center of Gravity may be found, Step by Step, in the same Manner *.

* Ann. 13.

41. HENCE follows, that one may alter the Place of the Center of Gravity of a System of Bodies, by adding one or more Bodies to it, or by taking one or more of the Bodies away. And the Center of Gravity of a single Body may be removed at pleasure, by adding to or taking from its Mass. And this is of singular Use in Machinery, or that Part of Mechanics that relates chiefly to Engines, because of the several Powers that are combined

in a Machine, and the ſeveral Poſitions that they muſt have in reſpect to each other in their Motions. Lect. II.

EXPERIMENT XI. *Plate* 5. *Fig.* 1.

42. A B * is a rolling Lamp that has within it the two moveable Circles D E and F G, whoſe common Center of Motion is at K, where their Axes of Motion croſs one another, in which Point alſo is their common Center of Gravity. If to the inward Circle you join within-ſide the Lamp K C made pretty heavy and moveable about its Axis H I, and whoſe Center of Gravity is at C, the common Center of Gravity of the whole Machine will fall between K and C, and by reaſon of the Pivots A, B, D, E, H, I, will be always at Liberty to deſcend; and therefore let the whole Lamp be roll'd along the Ground or moved in any manner, the Flame will always be uppermoſt, and the Oil cannot ſpill. Thus is the Compaſs hung at Sea; and thus ſhould all the Moon-Lanterns be made, that are carried upon a Pole before Coaches or Carriages that go in the Night. * Plate 5. Fig. 1.

EXPERIMENT XII. *Plate* 5. *Fig.* 2.

43. WHEN inclined Bodies, ſuch as the oblique Cylinders ABED, * *a b e d*, are ſet upon an horizontal Plane; they will fall the Way that they incline, if in their Motion that Way, their Center of Gravity can deſcend without firſt riſing *. Thus *c*, which is the Center of Gravity of the Body *a b e d*, will deſcend in the Ark *c k*, whoſe Center is the Point *e*, which is the Center of Motion of the Body when it falls. But C the Center of Gravity of the Body A B E D moving round the Center of Motion E in the Arc C K, cannot come down to K without firſt riſing up to F, which is above C; therefore the Body cannot fall by its own Gravity, becauſe the Center of Gravity will not riſe of itſelf *. * Plate 5. Fig. 2. * 24. * 24, 28.

44. HERE it is obſervable that *c* O, the Line of Direction * of the Body *a b e d*, falls without its Baſe; and C O the Line of Direction of A B E D falls within its Baſe: whence it follows, that inclined Bodies ſet upon an horizontal Plane will fall, when their Line of Direction does not go through their Baſe: but they will ſtand when the Line of Direction falls within their Baſe *. * 20. * Ann. 14.

Lect. II.

THIS is the Reaſon why an inclined Tower, ſuch as that of *Piſa* or *Bologna*, does not fall, though its Top hangs ſo far over the Baſe as to appear dangerous to thoſe that walk at *o* * near its Foot, and don't know upon what Principle it is ſafe.

* Plate 5. Fig. 3.

45. IF H I *e d* the lower Part of the Body *a b e d* † be equal, ſimilar, and alike inclined, to the Body A B E D, it will not fall for the Reaſon above-mentioned, its Center of Gravity being then at Q; but as ſoon as the upper or Cylinder *a b* I H is ſet upon it, and made faſt by the Pins *f f*, it brings up the common Center of Gravity to *c*, and then the two (which are now become one Body) fall towards *o*: but if H I *e d* being held faſt, the upper Part *a b* H I be ſet upon it, that Part will ſtand faſt without the Help of the Pins *f f*; for as its Center of Gravity is at G, its Line of Direction will go through the Plane H I, which is now the Baſe of the Body *a b* I H. But upon letting go H I *e d*, the upper Part will bring down the lower, and they will fall together, beginning to move round the Center of Motion *e*, the common Center of Gravity *c* deſcending towards *k* *.

† Plate 5. Fig. 2.

* Ann. 14.

EXPERIMENT XIII. *Plate* 5. *Fig.* 4.

46. IF in the Hand H * be held upright a Needle or ſharp-pointed Broach as C, the Fork D, whoſe Center of Gravity is at D, being ſet upon the Point of this Needle, will be ſupported †, though it will be difficult to place it right; becauſe the Point of the Needle is ſo ſmall a Baſe, that it will require a nice Hand to bring the Center of Gravity ſo directly over that Baſe, as to make the Line of Direction go through it *. But if another Fork, as B, be ſtuck into the Handle of the firſt, and a third of equal Weight with the ſecond, as A, be ſtuck upon the Points of the firſt, all the three will be ſuſtained.

* Plate 5. Fig. 4.

† 26.

* Ann. 15.

THE Line A B, which goes through the Centers of Gravity of the Forks A and B, being biſſected at *c*, ſhews that Point to be the common Center of Gravity of thoſe two Bodies: The Addition of the Fork D alters the Center of Gravity *, and cauſes the Point C (which is as near again to *c* as to D, the Center of Gravity of D) to become the common Center of Gravity of the three Bodies *. In this caſe the Bodies will be all ſuſtained by means of the Needle under D, becauſe the Center of Gravity is as low as it can be: Only with this Difference, that in the Caſe of the ſingle Fork D, the Center of Gravity (which was then over the Center of Motion) would

* 41.

* 33.

would descend upon the least Shake, and throw down the Fork; but now no Shake, but what is strong enough to make D jump off from the Point of the Needle, can cause the Bodies to fall: for if the Center of Gravity be raised out of its Place, it will always return to C the lowest Point it can descend to *. Lect. II. 24.

47. SINCE the Line of Direction runs through the supported Point under D, it follows, *That a Body, or System of Bodies will be sustained* (that is, their common Center of Gravity won't descend) *when any Part of the Body which is in the Line of Direction is supported; but will fall when no Part which is in the said Line is supported.*

48. IF by a Force impressed upon the Bodies A or B *, or both, they be made to turn round each other and round the Center *c*, in a Circle whose Diameter is A B; they will still be supported as before: whether they turn fast or slowly round *; and whether D be heavy or not, in which latter case the common Center of Gravity will return to *c*, and whether D, C, or *c* be the Point supported: Nay, if the Hand that carries the Bodies stands still, or moves in a right or a curve Line, the same will hold; that is, the Action of the Bodies upon each other (or in respect to each other) will not be altered, tho' they are carried along with their Center of Gravity: Neither will the Alteration of the Plane in which the Bodies move (that is here the rising of A and sinking of B, or the rising of B and sinking of A, as they turn) have any Effect upon the Motion of their Center of Gravity. Thus whether the Moon and Earth move fast or slow about each other, and their common Center of Gravity; and whether the Plane of the Moon's Orbit be more or less inclined to the Plane of the Ecliptic, and whatever may be the Change of that Inclination, the Motion of their common Center of Gravity (which describes the *Magnus Orbis*) will be no way affected. * Plate 5. Fig. 4. * 32.

EXPERIMENT XIV. *Plate* 5. *Fig.* 5.

49. IF such a Body as A B * be set upon the Pedestal N D P, it will fall, because its Center of Gravity can descend (in the Arc C *q*) or, which is all one, no part of it that is in its Line of Direction C O is supported; but if the two Awls L, M, be stuck into the said Body, their common Center of Gravity being at *k* (in the Mid-way between L and M) will bring back the * Plate 5. Fig. 5.

Lect. II. common Center of Gravity of all the three Bodies to K, and then K *o* will become the Line of Direction of the Bodies, in which the
* 45. Point K being ſupported, the Bodies cannot fall *.

EXPERIMENT XV. *Plate* 5. *Fig.* 6.

* Plate 5. Fig. 6. 50. LIKEWISE the Body A B * being ſet upon the Pedeſtal A E would fall, its Center of Gravity moving round the Point M in the Arc C *q*. But if an heavy Body, as D, be joined to it ſo as to bring the Center of Gravity to K, then C O will ceaſe to be the Line of Direction: and in the Motion of the Bodies round M, the Center of Gravity K muſt deſcribe the Arc K *k* moving upwards, which cannot be *; therefore A B will be ſupported by adding another heavy Body to it.

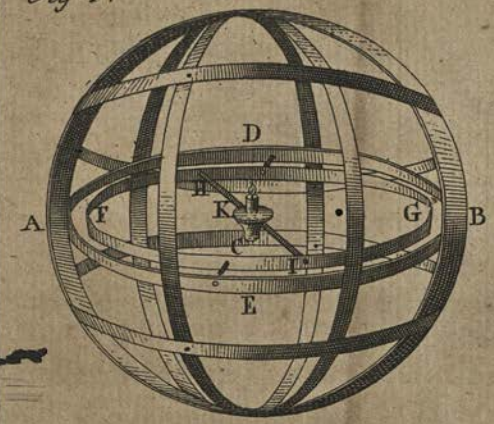
24.

51. WHEN Bodies are laid upon an inclined Plane, they will come down, though the Line of Direction falls within their Baſe. In ſuch a Caſe their Center of Gravity will not move in their Line of Direction (which only happens when Bodies fall freely) but will move in a Line parallel to the Plane, the Body ſliding all the while. But if the Line of Direction of the Body falls out of its Baſe, which is applied to the Plane, the Body will tumble or
† Ann. 15. roll along the Plane *.

EXPERIMENT XVI. *Plate* 5. *Fig.* 7.

HENCE follows, that the ſame Body, that in one Poſition would ſlide along an inclined Plane, will roll down in another. Thus
* Plate 5. Fig. 7. the Body A B C D *, when ſet upon the Plane *c* M N, will ſlide in the Poſition a b c d, becauſe its Center of Gravity k cannot fall in the Line of Direction of k o (the Body being ſtopped by the Plane) nor move in the Arc k c about d for a Center of Motion; becauſe
* 24. in this laſt Caſe the Center of Gravity muſt riſe, which cannot be *: And therefore the Center of Gravity will deſcend in the Line k s. But if the ſame Body ſhould be laid on in the Poſition *a b c d*, it would tumble towards M, its Center of Gravity deſcending in the Arc K *q*. For this Reaſon a Column may be drawn up an Hill when laid along in a Waggon, which would tumble backwards if ſet upright in the ſame Waggon: And a Load of Hay would be overthrown going along the Side of an Hill, where the ſame Waggon would go along ſafely, if loaded with an equal Weight of Iron;

Fig. 1.



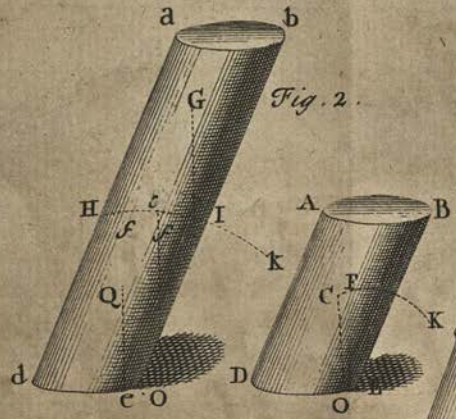


Fig. 2.

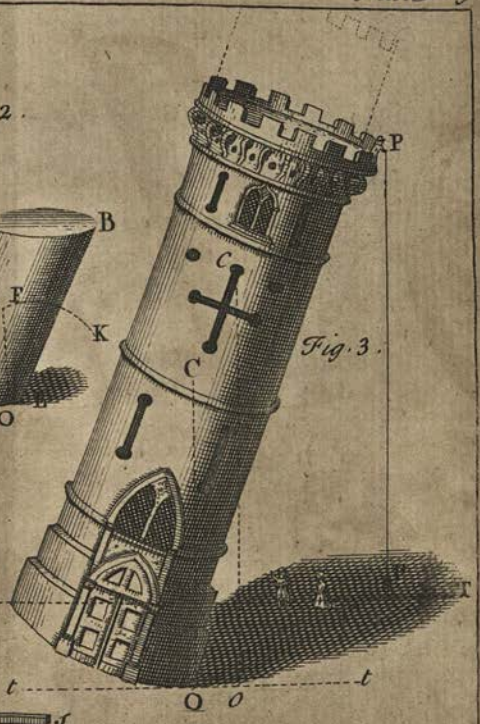


Fig. 3.

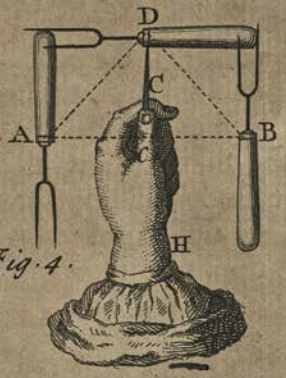


Fig. 4.

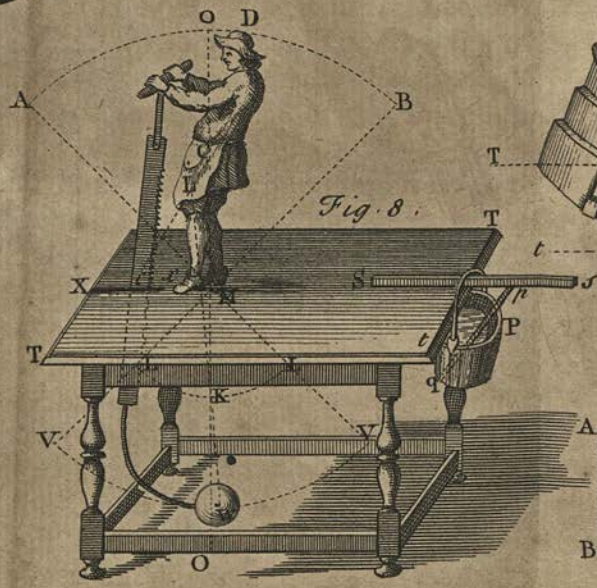


Fig. 8.

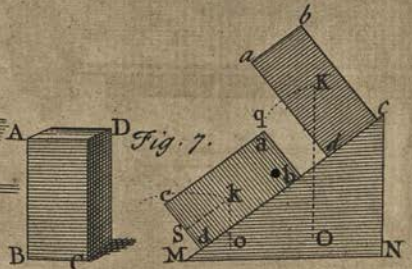
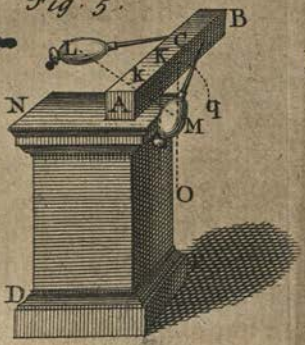


Fig. 7.

Fig. 5.



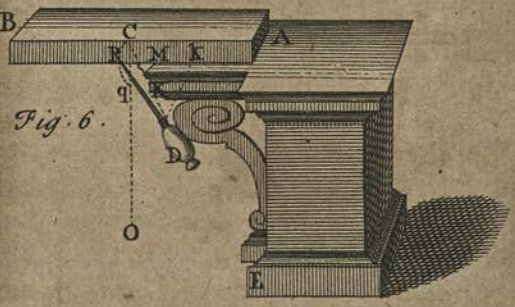


Fig. 6.

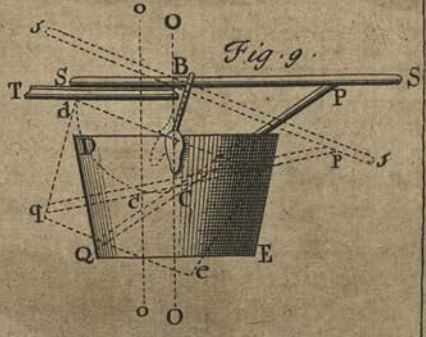


Fig. 9.

Iron; only becauſe in the Load of Iron the Center of Gravity is low; but very high in the Load of Hay*. What has been ſaid in the three laſt Paragraphs †, will be farther confirmed by the following Experiments. Lect. II.

* Ann. 17.
† 47, 48, 49.

EXPERIMENT XVII. *Plate 5. Fig. 8.*

52. UPON the Table T *t* T*, which has a Slit from X to *x*, ſet the little Image D M in ſuch manner, that the Saw *c* (which is faſtened at one End to the Hands of the Image, and has a Weight W fixed at the other End) may paſs through the Slit X *x*, and the Image will ſtand in an upright Poſture: Then if the Head of the Image be brought down to A or B, it will imitate the Motion of Sawing, and vibrate ſeveral times in the Arc A *o* B, whilſt the Weight W does in the ſame manner deſcribe the Arc V W V, the Center of Motion of the whole, (that is, of the Image, Saw, and Weight) being at M: the common Center of Gravity K does likewiſe deſcribe the Arc L K L, till (after having ſeveral times deſcended from L on either ſide) it comes to ſettle at K, juſt under the Center of Motion. If the Image had no Saw, it would ſtand upright when ſet on the Table, becauſe its Center of Gravity C would then be juſt over the Center of Motion M*; and ſome Part of the Image which is in the Line of Direction *o* O would be ſupported †, but the leaſt Alteration of Poſition that ſhould move C from over M would throw down the Image*. Then if the Saw *c* be added; ſince its Center of Gravity is at *c*, the common Center of Gravity of the Man and Saw will be at L, and in that Caſe the Image with its Saw will fall towards X: but if by means of a curved Wire the heavy Weight W be joined to the Saw, the common Center of Gravity of the Man, Saw, and Weight, will be at K*, and the Line of Direction will again be at *o* O; therefore the Image will ſtand in its upright Poſition. If now the Image be inclined forwards or backwards, it will after ſeveral Vibrations return to its firſt Poſition, becauſe the Center of Gravity always endeavours to deſcend to K, in doing which it will bring the Image upright. N.B. *This Experiment ſucceeds beſt with two little Points in the Heels of the Shoes to bear on the Table.*

* Plate 5. Fig. 8.
* 26.
† 45.
* 26.
* 38.

EXPE-

EXPERIMENT XVIII. *Plate* 5. *Fig.* 8.

53. UPON the Stick S *s* * (which of it ſelf would fall from the Table, becauſe its Center of Gravity hangs over) ſuſpend the Pail P, fixing another Stick *p q*, one End in a Notch at *p*, and the other againſt the Inſide of the Pail cloſe to the Bottom, and the Pail without any other Help will be ſupported on the Stick S *s*; which will not fall from the Table though the Pail be afterwards filled full of Water, provided the Bale or Handle of the Pail be pretty near the Table, and the Stick *p q* long enough to puſh the Pail a little out of the upright.

* Plate 5. Fig. 8.

WHEN the Stick S S * is horizontal on the Table T *d* B, C is the common Center of Gravity of the two Sticks S S, P Q, the Pail D Q E, and the Water contained in it, all which taken together are to be looked upon as one Body whoſe Line of Direction is O O; and as the Part of S S, which is a little behind the Bale B, is in the Line of Direction, and ſuſtained upon the Edge of the Table, the whole Body above-mentioned cannot fall: for if it did, the Part B S muſt riſe at the End S, into the Poſition B *s*, and P S deſcend into the Poſition *p s*; which cannot happen unleſs the Pail riſes into the Poſition *d q e*, bringing up the common Center of Gravity to *c* in the Arc C *c* D, which is impoſſible * from Gravity alone, without the Action of an extrinſecal Agent. But if the Pail D E be lifted up under the Table, and the Stick S S inclined above it, ſo that the whole Machine comes into the Poſition *s s p e q d*: If then it be left to itſelf, and the Stick *s s* is ſo ſmooth as well as the Table under B, that there is little or no Friction between them, the whole Machine will ſlide down and ſo fall from the Table; *s s* moving in the Direction *s p s*, and the common Center of Gravity *c* in the Line *c* E, Tangent to the Arc D *c* C. Here it is obſervable, that as *o o* is now the Line of Direction, no Part of the Body in the ſaid Line is ſupported *.

* Plate 5. Fig. 9.

* 24.

* 45.

N.B. *The Experiment is repreſented in Figure* 8.

ANNO-

ANNOTATIONS upon the Second LECTURE.

[3.——*By its Velocity.*]

SOME Authors have confounded *Velocity* with *Motion*, tho' it is but an Adjunct of it, imagining that one Body must have as much Motion as another, whenever it moves as fast, and more Motion when it moves faster; but this is only true when the Bodies have equal Quantities of Matter, or when we compare the Motion which the Body has at one time with the Motion that it has at another. The general Definition indeed (*viz.* that *Motion is the Translation of a Body from one Place to another*) takes no notice of the Quantity of Matter; but when we compare moving Bodies, we have a Regard to their Quantity of Matter: for Velocity alone, without considering how much Matter is mov'd, will never enable us to determine the Force, which we call the *Quantity of Motion*. For example, shou'd a Dog and an Horse moving with equal Swiftness run against a Wall of the Thickness of a single Brick; the Dog would be beaten back, whilst the Horse that carried 40 times more Matter wou'd beat down the Wall, *&c.* Annotat. Lect. II.

2. [5.——*Force and Motion mean the same Thing.*] See Sir *Isaac Newton* in the Beginning of the first Book of his *Principia*, Def. 2. and 8.

3. [9.—— *Less Matter than the Ram.*] If the Battering Ram R * * be 28 Inches in Diameter, and 180 Foot long, made of several Pieces of Timber, as for example, of Oak join'd together, it will contain 750 Cubic Feet of that Timber, which at 50 Pound a Cubic Foot will weigh 37500 Pounds: If the Head of it made of Cast Iron weighs a Ton and a half, that will be 3360 Pounds; then if the five Iron Hoops about it are one Inch thick, two Inches deep, and 94 Inches in Circumference, they will weigh about 50 Pound each, which with two Pounds of Nails allow'd to keep them tight to their Places will make 252 Pounds. Now all these Weights added together will give us 41112 Pounds for the whole Weight of the Ram, which if it be mov'd by 1000 Men employ'd only to make it strike against the Point L of the Wall AHIGE (supposing it to swing suspended by its Center of Gravity from a moveable Gallery or only a Tressel) each Man will move a Weight of 41 Pounds. The Quantity of Motion produc'd by this Action, when the Ram moves one Foot in a Second, may be expres'd by the Number 41112; which Motion or Force compar'd with the Quan- Plate 4. Fig. 2.

Annotat. Lect. II.

tity of Motion in the Iron Ball B shot out of the Cannon C, will be found equal to it: For a Cannon Ball is known to move as fast as Sound for the Space of above a Mile, and if you multiply 36 Pound the Weight of the Ball by 1142 (the Number of Feet which Sound moves in one Second) you will have the Number 41112 to express the Force or Quantity of Motion in the Ball B striking at L. And if, after a few Strokes given by the Battering Ram, the Mortar or Cement is so loosen'd that the Piece of Wall ADDFE is at last by a Stroke of the Ram carried forward from F to K, and so beaten down; the same thing will be performed by one Cannon Ball after the same Number of equal Strokes given before by others, as we had supposed by the Battering Ram: and then the Quantity of Motion in the Piece of Wall ADDFE carried from F to B will be just equal to the Shock of the Ram, or of the Bullet B.

This shews how advantageous the Invention of Gun-powder is; since we are thereby enabled to give such a prodigious Velocity to a small Body, that it shall have as great a Quantity of Motion as a Body immensely greater, and therefore perform as much by its Percussion; tho' we use but few Hands in the Management of it, for three Men are able to manage a Cannon which shall do as much Execution as the Battering Ram above mention'd. *Those that would have a more ample Account of the Battering Rams and warlike Machines of the Ancients, may find them describ'd in several Authors, especially in* Justus Lipsius *. *The Ram which I have considered here is taken at a Mean, being bigger than some and less than others that we read of.*

* Poliarch. Lib. 3.

4. [11.——*Subtile Matter*, &c.] The *Cartesians*, in order to maintain their *Plenum*, suppose a certain *subtile Matter* to fill all Spaces between, and Pores within Bodies; and that this *subtile Matter* by being continually divided, becomes so fine a Dust as to have neither Weight nor Resistance, and yet that it is the Cause of Gravity: but any one that considers this Assertion will find it inconsistent with itself. For *first*, if such a Matter fill'd all the Interstices between the Parts of Bodies, it wou'd render them equally full: *Secondly*, Wherever it fill'd any Space that had no other Bodies in it, it must be more solid than Gold and harder than Diamonds, and consequently could not be a fine Dust, as is imagin'd; for a Body, when solid, differs from the same Body in Dust, only because its Parts are separated from one another, so as to have a great many Voids between: as a Pound of solid Gold differs from as much Gold Dust, only because the Parts of the Dust are more separated, mix'd with Vacuities, and do not touch in so many Points as the solid Lump; the Dust easily becoming a solid Lump when the Vacuities are driven away by Fire, which turns the Dust into a Fluid that afterwards (upon the Removal of the Fire) is chang'd into a solid Lump without losing any of its Weight. *Thirdly*, As the *Cartesians* affirm—That of *subtile Matter* the Earth and Air (heavy Bodies) are compounded, it is absurd to suppose the Matter of which they are made to be without Weight; since the Gravity of any whole Body is made up of the Gravity of all its Parts taken together. *Fourthly*, When the *Cartesians* make their subtile

Matter the Cause of Gravity, they seem to forget what they have said of it before, namely, *that it has no Resistance*; for if a Fluid, by its Motion about the Earth, impels all the Bodies near it to fall down to the Earth, it cannot be void of Resistance, because whatever *impels* must *resist*: And to say, that solid Bodies can move thro' this subtile Matter without suffering any Resistance, but that the subtile Matter when it runs against solid Bodies drives them out of their Places, is an Absurdity unworthy of a Philosopher. Yet because the *Cartesians* cannot deny the Experiment of a Piece of Gold, and a Feather falling equally fast in a Glass Receiver out of which the Air has been pump'd * (and the Experiment has been made in a Receiver 10 Foot high; see *Phil. Trans.* N°. 354.) rather than give up their *Plenum*, they say, that when the Air is pump'd out, the Receiver is as full as before, but that it is fill'd with such a subtile Matter as makes no manner of Resistance: So that upon the whole, the *Cartesian* Account of the Cause of Gravity, and the Non-resistance of their subtile Matter, are inconsistent with each other: Many more are the Contradictions which they fall into, by endeavouring to solve *Phænomena* by the Powers and Motions in all Directions which they attribute to this subtile Fluid; but I shall say no more on this Subject, till I come to speak of the Motions of the heavenly Bodies.

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* L. 1. 3 9.

5. [16. *At once with the Leaver.*] It was upon this Principle that *Archimedes* proposed the lifting of the whole Earth, in case there could be found *a fix'd Point*, or Place to support his Instrument, δὸς πᾶ στῶ, καὶ τὸν κοσμὸν κινήσω. By which he meant—That the least Power, by increasing its Velocity, might raise the greatest Weight; and that in this respect there are no Bounds, where we can get a Place for a fix'd Point, and a due Distance for the Power. Now, tho' raising the Earth is a Proposition purely Mathematical, and not to be reduc'd to Practice; yet for Curiosity sake we'll consider it a little here.

If we take the common Center of Gravity of the Earth and Moon for the fix'd Point (*Fulcrum*, or propping Point) of our Leaver, which we will suppose 240000 Miles long, that is, reaching from the Center of the Earth to the Center of the Moon; then the Moon, or a Weight equal to it made use of as a Power, will be able to support the Earth at the other End of the Leaver; and if it be removed but one Inch farther, it will raise it up.* Here the Distance of the Center of Gravity of the Earth from the fix'd Point of the Leaver is 6000 Miles, and that of the Center of Gravity of the Moon or Power is almost 40 times as far; and if the Moon be supposed to move with the same Velocity that it wou'd fall to the Earth by the Force of Gravity, if its projectile Force did not hinder it; then the Earth wou'd be moved one Inch out of its Place, by the Moon's moving about 40 Inches.

* N°. 15.

Now if, instead of the Moon, we make use of a Power whose Intensity is equal only to a Weight of 200 Pound, as for Example, the Force of a Man, as *Archimedes* proposes: Then still supposing the Earth at the Distance of 6000 Miles from the fix'd Point, that *Brachium* of the Leaver to which

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the Power is applied, must be lengthened in the Proportion that the Weight of the whole Earth bears to 200 Weight. In this Case the End of the Leaver will reach quite out of our System among the fix'd Stars, above fifteen thousand Millions of Millions of times farther than the Distance of *Saturn*. And if *Archimedes* (or the Power) be suppos'd to press upon the Engine with the Velocity of a Cannon Ball, he wou'd be in Motion at the End of the Leaver above 26 and near 27 Millions of Millions of Years to raise the Earth one Inch in that Time, and wou'd go thro' a Space above 39 thousand Millions of times greater than the Periphery of *Saturn*'s Orbit.

For the sake of such as wou'd examine this Calculation, we subjoin the Numbers that we made use of.

The mean Diameter of the Earth is equal to 19 688 725 Paris Feet, (*Newton, Princ. Lib.* 3. *Prop.* 20. *Pag.* 387.)

Supposing the Diameter to the Circumference as 7 to 22; and multiplying the Diameter by the Circumference, you will have 1 218 315 660 966 250 Square Feet for the Surface of the Earth: which last Number, multiplied by the 6th Part of the Diameter, will give 3 997 847 001 180 744 647 897 $\frac{1}{2}$ for the Cubic Feet contained in the whole Earth.

Now if we suppose a Cubic Foot of Earth to weigh 100 Pound, multiplying by 100 we shall have the Weight of the whole Earth in Pounds, *viz.* 399 784 700 118 074 464 789 750.

Then, As 200 Pound *(or the Intensity of the Power):*
Is to 399 784 700 118 074 464 789 750 *(or the Intensity of the Weight::*
So is 6000 *Miles (the Distance of the Weight or of the Center of the Earth from the fix'd Point):*
To 11 993 541 003 542 233 943 692 500 *Miles (or the Distance of the Power).*

This last Number not only expresses the Distance of the Power, but the Number of Miles that the Power must move to raise the Earth one Mile; because the Velocities of the Power and Weight must be reciprocally as their Masses. But if we wou'd raise the Earth but one Inch, we must divide by 66 360 (the Number of Inches in a Mile) and we shall have the Miles gone thro' by the Power whilst the Earth moves 1 Inch, *viz.* 189 291 996 583 668 465 020 Miles.

To have a clearer Idea of what has been said, let us compare the Distance of the Power and Space that it must go thro', with some great Distance that we know, and with the Space describ'd by some Body that we can observe; as for Example, with the Distance of the Planet *Saturn*, and the Space which it describes in going thro' the whole Circumference of its Orbit.

Saturn, at its mean Distance from the Earth (which is equal to its Distance from the Sun) is above 9 times and a half (or 9,51 times) farther from the Sun than the Earth, which last we suppose 81 Millions of Miles from the Sun; and therefore will be express'd by this Number of Miles (*viz.*) 770 310 000, by which dividing 11 993 541 003 542 233 943 692 500, we shall have a Number which shews us that the Power must be applied farther

farther from the Fulcrum or Center of the Motion than *Saturn*'s Distance, Annotat. 15 569 745 951 035 731 times. Then if the Space that the Power must go thro' (or 189 291 996 583 668 465 020 Miles) be divided by 4 841 948 571 the Number of Miles in the Periphery of *Saturn*'s Orbit, the Quotient will shew us that the Power will go thro' a Number of Miles 39 094 177 438 times greater. Lect. II.

Again, if we suppose the Power, or *Archimedes*, pushing forward the farthest End of the Leaver; we shall find, that tho' he shou'd move as fast as a Cannon Ball, he wou'd spend 26 978 123 942 460 Years in moving the Earth one Inch. For if we suppose a Cannon Ball to move with the same Velocity as Sound, or a Mile in 4 Seconds and an half (as Experiments have confirm'd it) it must move 800 Miles in an Hour: And as one Year contains 8766 Hours, 800 times that Number is the Number of Miles which a Body with the Velocity of a Cannon Ball would move in a Year; by which Number (7 012 800) if you divide 189 291 999 583 668 465 020 (the Number of Miles that the Power goes thro') you will have the Number of Years which it must take up to go thro' the said Space, namely, 26 978 123 942 460 Years; *which was to be proved.*

Bishop Wilkins *in his* Mechanical Powers *mentions a compound Engine of Wheels, whereby the Earth might be rais'd, without supposing the Power to be applied at any considerable Distance from the Earth; but what he says depends entirely upon the same Principle: for even in that Case, the Power (if equal to what we have suppos'd) must go through a Length equal to as many Miles as we have mention'd above, in order to raise the Earth but one Inch; though the Line in which it mov'd should be but a Circle of a Foot in Diameter; for let the Machine be made in any manner whatever, the Difference of the Velocities of the Power and Weight will always be reciprocally as their Masses (or their Intensities) considering the Power as a Quantity of Matter in Motion) when they balance one another; and the smallest Addition of Velocity (still suppos'd in these Calculations) will make the Power overpoise.*

6. [16.—*Where we would gain Time, we must employ more Strength.*] For want of rightly considering this, a great deal of Time and Money has been fruitlessly spent in mechanical Works, by such as imagine that Force might be generated by the Figure of a Machine; whereas Mechanics teach us not to make, but to apply Powers, such as we find them in Nature; for we deceive ourselves, if we think that by means of any Engine whatever, One Man shall do the Work of Two in the same Time, supposing the Men to employ the same Strength. But yet the Science of Mechanics is not to be rejected as useless; for to the practical Arts that are deriv'd from that Science we are indebted for a great many of the Necessaries as well as Conveniences of Life. In the Performance of several Works, where we have sufficient Strength, we often want Time; and sometimes where we have Time to spare, we want Strength. In such Cases the Skill of a good Mechanic is to be exerted in directing the Application of the Powers according to Time. Thus in making Harbours, and carrying on Digues, Moles

or

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or Banks, where at every Tide the Sea may damage the Work, and a Spring-Tide over-ſet it: the greateſt Number of Hands *muſt be employ'd that can work by one another.* In ſome Caſes, as raiſing Blocks of Marble or other heavy Goods out of Ships to lay them upon a Wharf, many Hands cannot be employ'd; either becauſe they cannot well ſtand by one another about the ſame Block, or becauſe they cannot lift all at once; or when they have got up their Burthen, they cannot conveniently walk with it; or if they could, the Planks over which they go, or Ladders which they muſt climb, could not ſupport them and the Weight; then an Engine muſt be uſed (as for Example, a Crane) where one Man ſhall do the Work of 10 or 20 Men, but he ſhall be 10 or 20 times longer in performing it. Yet the Engine is abſolutely neceſſary, becauſe without it the Work could not be done; therefore a ſufficient Time muſt be employ'd, without which a great Strength would be of no uſe. Thus likewiſe in Building we muſt uſe Engines to raiſe great Stones and large Pieces of Timber, where ſo much more Time is employ'd according as the Force of the Men working at the Engine is leſs than the Force that would be requir'd to raiſe the Stone if the Hands were applied directly to it, but then the Engine takes up leſs Room, and the reſt of the Men may be otherwiſe employ'd. In draining Mines we are always confin'd in Time, becauſe the ſubterraneous Springs ſupply the Water whilſt a Force is employ'd to draw it out; and in ſuch a Caſe the Power (that is, the Intenſity of it) muſt be ſuperior to the Quantity of Water to be rais'd in a certain Time; that is, the Power muſt be able, without any Engine, to draw from the Bottom to the Top of the Pit (ſuppoſe an Horſe drawing up a Bucket faſten'd to a Rope that runs over a ſingle Roller) a Weight greater than the Weight of the Quantity of Water that runs in, during the Time that the Power goes through a Space equal to the Depth of the Pit. Engines are applied for the Conveniency of delivering the Water, and not to gain any Degree of Force; for we always loſe ſome——So much as is employ'd to move the Parts of the Engine, which cannot be apply'd to one another without a Friction that ſpends ſome of the Power; ſo that the beſt Engine is that which conſiſts of the feweſt Parts *. And he that by Machinery pretends to gain by bringing up a greater Weight, or a larger Veſſel of Water with the ſame Power, does not conſider, that it will riſe ſo much the ſlower, and give Time to the Springs to ſupply more Water in proportion; or if, by adding any Part to the Engine, the Power (as for Example, an Horſe or Horſes) is made to go eaſier, then leſs Water will be rais'd at a Time. *This ſhou'd* give a Caution to thoſe that have any Concerns in Mines, Water-works, Mills, or other Manufactures; that they might not be impos'd upon by Engine-makers, that pretend to (and often fancy they can) by ſome new-invented Engine out-do all others, and make one Horſe do as much as three or four. This proceeds from their being unſkill'd in Mechanical Principles, the Know-

* *All that relates to raiſing Water ſhall be particularly conſider'd, when we come to treat of Hydroſtatics.*

Knowledge of which would keep them from attempting Impossibilities. It were to be wish'd that our Engine-makers, who often abound with Invention, and are generally quite ignorant of Mathematics, wou'd apply themselves to that Science; at least to know so much as wou'd direct them in their Works; or that some of our best Mathematicians wou'd not think it below them to direct Workmen, and consider Engines a little more than they do, which wou'd render their Speculations more useful to Mankind. There are some, who, being too clumsy, and wanting a nice Hand to make Experiments, are unwilling to own it, and therefore ridicule and despise Mechanical Performances; forgetting that the incomparable Sir *Isaac Newton*, whom They, with all other Philosophers admire, has made as many as (if not more Experiments than) any Man living; and look'd upon Geometry as no farther useful than as it directs us how to make Experiments and Observations, and draw Consequences from them when made; so that the Improvement of Philosophy must be the Result of mix'd Mathematics, that is, of Mechanics and Geometry. A Man that shou'd learn to fence by Book, wou'd be as much at a loss if he was call'd to fight, as another that shou'd prefer brutal Courage to the whole Art of Fencing; only with this Difference, that the latter wou'd be much more likely to kill his Adversary; as Men quite illiterate have often produc'd wonderful Engines. The Machine at *Marly*, which was made by an ordinary Man at *Liege*, who was entirely ignorant in Mathematics, has a great many excellent Contrivances; but does not raise all the Water that it shou'd do, because the Workman did not know how to calculate, so as to give the Power of the River *Seine* its utmost Advantage. Annotat. Lect. II.

When great Works or Manufactures are carried on in such a Manner that a great Part of the Intensity of the Power is uselessly spent, and but little of it employ'd in doing real Services; as for example, when there are unnecessary Frictions arising from the ill Contrivance of the whole Engine or the wrong Figure of some of its Parts, or the bad Performance of the Workman; or if Men or Horses, *&c.* exert but a small part of that Strength which they might apply without Weariness or Inconveniency: Then the Skill of a good Engineer may be advantageously applied in changing the Form or altering the Parts and Motions of a Machine. An Instance of this may be seen in the winding of Thread or Silk, in which Business if 50 Men being employ'd they should only move a Weight or overcome a Resistance equal to half a Pound, as they carry their Hand round (whereas one Man can easily raise 25 Pound, with the same Velocity as his Hand moves during the Space of 10 Hours in a Day) a Machine may be contriv'd whereby one Man applying his whole Strength shall do the Work of the fifty Men in the same Time. Thus in any other Case, Machines may afford us great Profit, by rendering effectual that Force or Intensity of the Power or Powers, which was before misapply'd or not employ'd at all. This has been perform'd at *Derby* in a very ingenious Manner by Messieurs *Thomas* and *John Lombe*, who have employ'd the Force of a Water-Wheel to the working of *Italian* Silk, so as to lose or mispend no Part of the Power.

As

Annotat. Lect. II. As the Proprietors of this curious Machine are not willing to have a Deſcription of the whole Engine or any of the Movements of it made publick, I ſhall only here give the general Account of it, *viz.* That there are

26 586 *Wheels.*
97 746 *Movements.*
73 728 *Yards of Silk wound every time the Wheel goes round, which is three Times every Minute.*
318 504 960 *Yards of Silk in one Day and Night,* and conſequently
99 373 547 550 *Yards of Silk in one Year.*

One Water-Wheel communicates Motion to all the reſt of the Wheels and Movements, of which any one may be ſtopp'd ſeparately and independent on the reſt. One Fire-Engine conveys Air to every individual Part of the Machine, and one Regulation governs the whole Work.

7. [17.—*Scorpions*, &c. *give a ſufficient Velocity*, &c.] Scorpions were Machines to throw Arrows, Fire-Balls, or great Stones. A Deſcription of them may be found in *Vitruvius* and the above-mention'd *Lipſius* *, from whom I have taken the Figures repreſented in Plate 6. The 1ſt Figure repreſents one of theſe Machines charg'd †. The Point A of the longeſt Brachium AC, which in the natural Situation is kept uppermoſt by the Boxes of Stones or Weights BB, having been brought down to A (by the Rope RR and Loop *a*, drawn by help of the Wheel W, and Pinion at I, round the Rollers M and L) is kept from riſing up again by the Pin HH, made a little taper. Then the Loop *a* being taken off from A, and the Sling S being charg'd with the Ball or Stone T, the Scorpion is ready to be diſcharg'd; which is done by a ſmart Blow of an Hammer on the End H *i* of the Pin, or a ſudden drawing it out with a Rope; for then A being no longer kept down, riſes with great Velocity by the Deſcent of the Weights BB, and one of the Loops of the Sling ſlipping off of the Point A, made conical for that purpoſe, the Stone flies out as repreſented in the ſecond Figure, which is another Scorpion little differing from the former. All the Difference is, that in this laſt Figure, as the diſcharging End A is nearer to the Axis of Motion D D, than the ſame End in the former Figure, the Pulley * L is applied in ſuch a manner, as to cauſe the Handle I of the Pinion leading the Wheel W, to move as eaſy again as in the former Caſe: And as to the Effect, ſuppoſing the Weights B B equal in both Scorpions; the latter will throw a Ball of greater Weight, but then it will have leſs Velocity than the former Projectile. In both Caſes the Scorpion turns upon the Pivot C, and the whole Frame H I round the upright Shaft C *c*, that the Machine may be directed any way. The Hook H in the ſecond Figure does the Office of the Pin H in the firſt.

* Polior. Fig. 3. Dial. 4.
† Pl. 6. Fig. 1.
* Pl. 6. Fig. 1.

However powerful theſe Machines were, and however numerous, they are not to be compared with a Battery of Cannon, either for Force or Expedition. Biſhop *Wilkins* had not preferr'd them to our Artillery, as ſome others

others have done, if he had examin'd them less superficially, and calculated their Force. Annotat. Lect. II.

To prove this, we will here consider the Force of one of these Scorpions, even supposing it much larger than could be conveniently carried about; (for that they were portable, appears from the Account that *Cæsar* had great Numbers of them in his Camp) and we shall see how much short it will fall of the Force of a Cannon.

Let * AD the Tail of the Scorpion, or the End that throws the Stone or Bullet, be supposed 24 Foot long; and the shorter *Brachia* D B, which carry the Boxes B B, 8 Foot long each. When the Boxes are fill'd with Stones so as to weigh, for example, 1000 Pound each, and the Tail brought down to A, in order to throw the Stone T; the utmost Velocity that can be given to it can never exceed 96 Feet in a Second, because the Weights at B cannot fall faster than 16 Feet in a Second, tho' they shou'd not move the Machine in their Descent. But as we may reasonably suppose that throwing up the End A loaded with the Projectile T, must retard them one half, it will follow that the Body T will be thrown forward only at the Rate of 48 Feet in a Second, which is about 24 times slower than the Motion of a Cannon Ball; and therefore the Effect of the Scorpion will be 24 times less than that of a Cannon throwing a Ball of the same Weight. Besides, there must be more Men to manage the Machine, and a great deal more Time spent in drawing down A by means of the Wheel W, than in charging a Cannon: for if we suppose the Force requir'd to raise the Boxes loaded with Stones, and to overcome the Friction, to be equal only to 2500 Pound, and the Engineer who turns the Handle at I to move his Hand thro' a Space of $3\frac{1}{2}$ Feet in a Second (which is the utmost he can do) if the Force he applies be equal to 25 Pounds, he must spend 7,6 Minutes, or above $\frac{1}{8}$ of an Hour, to carry the Handle thro' a Space of 1600 Feet, in order to raise the Boxes 16 Feet in Height: This, besides the Time spent to put in the Pin H and to fit the Stone T into the Sling S, will so retard the Operation, that the Scorpion cannot discharge its Projectile above 6 times in an Hour, whereas a Cannon may be fired with great Ease twice as many times in an Hour.

* Plate 6. Fig 1.

Now if we consider how much such a Scorpion as we have describ'd must weigh, it will appear more troublesome to carry than a Cannon. The upright Shaft C must be 30 Foot long; and that it may have sufficient Strength, we will suppose it of $13\frac{1}{2}$ Inches in Diameter, which will make it contain 30 cubic Feet of Timber, which if of Oak will weigh about 50 Pound a Foot; let the Body and Tail of the Scorpion ABB with the Boxes B B, contain 40 Foot more of the same Timber; the Frame KH 60 Foot, and the Wheel W with the Pinion on the Handle I, the Pulleys, Ropes and Iron-work of the Machine weigh as much as 60 cubic Feet more of Oak: All this together will make 190 cubic Feet, which multiplied by 50 Pound will give 9500 Pound, a Weight which will render the Machine very inconvenient, even tho' it be taken to pieces. We may therefore suppose the Scorpions much less than what I have describ'd: So that there can be no Comparison between their Effect and that of our Artillery. Whoever will be at the pains

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of calculating the Force of any other of the Machines used by the Ancients, will find that they fall very short of the Effects of Gunpowder: Especially if we consider what Force it exerts in the springing of a Mine, whereby prodigious Rocks are rent in pieces, and such Quantities of Earth and strong Walls lifted up, that all the Machines made use of in a *Roman* Army, if they cou'd be apply'd at once to one Part of a Fortification, wou'd produce nothing like this new Invention of a portable Powder, which contains such an immense Force in so small a Compass.

8. [21.—*Line of Direction, &c. endeavours to act.*] Though an heavy Body as A *, may by a Force or Forces impress'd upon it, be made to move in any Direction, yet (as it always retains its Tendency towards the Center of the Earth, which, if no Obstacle hinder'd, would carry it the nearest Way) its *natural* Line of Direction always goes thro' the Center of the Earth; and if, whilst the Body is going down by its own Gravity, it be hinder'd by one or more inclin'd Planes, as BG, GD, and carried down afterwards in a Curve (46) in its Descent, on account of the Alteration of the Direction of its Motion, or carried up again along another Plane as DE; we are not to call the Line 12346, or 12345 which it describes, its *Line of Direction*; but when the Body comes from the Point 1 successively to the Points 2, 3, 4, its Lines of Direction are 1 *c*, 2 *c*, 3 *c*, 4 *c*, which are all directed to the Center of the Earth, and by reason of their great Distance from it, may be consider'd as parallel, or indeed as one and the same Line.

* Pl. 7. Fig. 1.

Now the Line of Direction of a Power varies according to the Application of the Power, whatever Line the Body acted upon moves in. Thus when the heavy Body A * (whose Line of Direction is *c* C) suspended by a Rope, is held by the Hand at H, the Line of Direction of the Power is the same with that of the Weight; but if the Rope be brought over the Pulley B, the Lines of Direction of the Power may be any of the Lines BG, BF, BE, BD, whilst the Line of Direction of the Weight still continues the same. Nay, if the Body A*, being acted upon by the Power at I, be made to move in the Line *c* D, along the Plane MB, *c* D is not the Direction of the Power, but AI. And if the said Body be raised from E to F by means of the Wedge KFL drawn under it (whilst a Board or immoveable Plane at HG keeps it from going out of the Line EF) the Line of Direction of the Power will be LB: But if the Body having been plac'd at K upon the horizontal Plane L, the Wedge or inclin'd Plane F be suppos'd immoveable, and the Plane HG, to move from K to G, and to push up the Body in the Line KG, then does the Power acting in the Line of Direction KG cause the heavy Body to rise the Height EF, whether it moves in the Line KG or directly up in the Line EF to get that Height.

* Pl. 7. Fig. 2.

* Pl. 7. Fig. 3.

Hence it follows that the Velocity of a Power is not to be consider'd in the same Manner as the Velocity of a Weight, (unless when a Power ascends or descends directly from or towards the Center of the Earth;) for the *Velocity of a Power* is *the Space that goes through in a certain Time*,

which may be greater or less to perform the same Operation, according to the manner in which it is applied; as the Power B * moves only the Length LK, when it raises the Weight up through the Line EF, by drawing the Wedge LFK through EF; but if it pushes it up along FK supposʼd immoveable, it must move the whole Length of the Hypotenuse FK of the Triangle LFK. But whether the Weight rises from K or from E to the Point F, its Velocity must only be callʼd EF, because whatever Line it runs throʼ, it only rises or removes farther from the Center of the Earth the Height EF. So likewise the Velocity of A * is only the Line 1 *c*, when the Body runs through a much longer Space, namely from 1 to 6 throʼ the Points 2, 3, 4 along the Planes BF, GD. Therefore *the Velocity of a Weight is always to be measurʼd by the Line of its upright Ascent or downright Descent, which shews how much it is got nearer to or farther from the Center of the Earth.*

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* Plate 7. Fig 3.

* Plate 7. Fig. 1.

We are to observe, that this Definition relates chiefly to that Part of Mechanics which considers the Actions of Bodies upon one another by the Application of Instruments.

9. [27—*A Method for finding the Center of Gravity, &c. mechanically.*] If a Board, of equal Thickness all over, be laid upon the Edge of a Triangular Prism P *p* *, or upon the sharp Edge of any straight Body placed in an horizontal Situation, so as to be *in Æquilibrio*; whatever Bodies are laid upon such a Board in such a manner that they do not alter its *Æquilibrium*, must have one Plane of Gravity (that is, a Plane in which their Center of Gravity is) directly over the Edge which supports the Board. Another Position of the Body, if the *Æquilibrium* be still preservʼd, will give another Plane of Gravity, the Section of which Plane will give an Axis of Gravity, or a Line which has the Center of Gravity in it. A third Position of the Body may be obtainʼd so as to find a third Plane of Gravity, which shall cut the other two at right Angles, or any large Angle, and the Intersection of the three Planes will give the very Point which is the Center of Gravity. If the Body, whose Center of Gravity you would have, be long and flexible, so as not to lie across upon the Board above-mentionʼd, a second Board must be laid upon the first with a Pin in its Center, so that it may turn quite round without altering the *Æquilibrium*; and then long Bodies laid upon this last Board may easily be movʼd on the Board, so as to find their different Planes of Gravity. Thus one may find the Center of Gravity of an human Body, or of any Animal. In relation to the human Body, it is observable, that whether a Man be fat or lean (nay, in a Skeleton) the Center of Gravity is always near the same Place, *viz.* in the *Pelvis*; between the Hips, the *Ossa Pubis*, and the lower Part of the Back-bone. Raising up the Arms and Legs will raise up the Center of Gravity a little; but still it is always so placʼd that the Limbs move freely round it, the Center of Gravity at the same time moving much less than if it was in any other Part of the Body. A Statue, though it represents a Man, has not its Center of Gravity in the same Place as a Man; for if it be hollow, the Hollows are not in the same Places as in a Man's Body, and the Center of Gravity in a

* Plate 7. Fig. 4 & 5.

Plate 7. Fig. 4. & 5.

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solid naked Statue is higher than in a Man. I mention this, because in setting up a Statue, and fixing it (especially in a Place exposed to the Wind) great regard is to be had to place the Center of Gravity over the middle of the Base; or, if the Attitude of the Statue does not permit it, then the Statue is to be secur'd most strongly on that Side which is farthest from the Center of Gravity.

Mathematicians, in order to settle Rules for finding the Center of Gravity of Bodies, first give Methods for finding the Center of Gravity of 2 or more Lines, then of the Periphery of Figures, then of Planes; for tho' Lines and Surfaces do not exist separate from Bodies, yet they consider Lines as slender homogeneous Bodies, and Planes as extremely thin Solids: and, from that Consideration, more regularly proceed to find the Center of Gravity of Solids. Dr. *Wallis* has fully treated of this Subject in his Mechanics, in his Chapter *de Investigatione Centri Gravitatis*, and *Monſ. Ozanam* in the third Chapter of his *Statics*, in the fourth Volume of his *Course of Mathematics*.

I shall give here some of the most easy and useful Methods, and must refer the more curious Readers to the Authors above-mention'd, and other Mathematicians that have particularly consider'd the Center of Gravity.

If a Line be consider'd as an homogeneous Wire infinitely diminish'd, its Center of Gravity will be in its middle Point; as the Point I in the Line A B *. Let there be another Line as C D † in any Position in respect of A B; then if from the Center of Gravity I of A B, a Line (supposed without Gravity) be drawn to K, the Center of Gravity of C D, you will have their common Center of Gravity at G by this Analogy,

* 29. † Plate 7. Fig 6.

‖ 38.

‖ A B + C D : C D : : K I : I G

And if the second Line had been less than A B, as for Example, if F E had been taken instead of C D, the common Center of Gravity would have been at H; because ‖ A B + F E : F E : : K I : I H.

If there be three Lines (whether they include a Space, so as to make the Periphery of a Triangle, or not) their common Center of Gravity may be found after the same Manner as that of three Bodies *. So likewise may be found that of 4 or more Lines, and consequently of Polygons.

* 39.

It is to be observ'd that the Center of Gravity of plane Surfaces is not the same as the Center of Gravity of their Peripheries, unless when they are regular. Thus in the Triangle * A B C, which is not equilateral, the Center of Gravity of the Periphery will be found at H, nearer to the Angle B than I, which is the Center of Gravity of the Triangle. For (by 39.) D and E being the Centers of Gravity of the two Lines A B and B C, F is found to be their common Center of Gravity, and H the common Center of Gravity of the three Lines A B, B C, and C A.

* Plate 7. Fig. 7.

To find the Center of Gravity of a Triangle, draw a Line from the middle of any Side to its opposite Angle as G B: Let G I be taken = $\frac{1}{3}$ of the said Line, and the Point I will be the Center of Gravity of the Triangle. Now as every rectilineal Figure may be divided into Triangles, the common Center of all the Triangles will be the Center of Gravity of the Figure.

Let * A B and C D be two Quantities (whether Surfaces or Solids) whose particular Centers of Gravity are at F and G, the Center of Gravity of their Sum, or of A D, will be at † E; but if you would have the Center of Gravity of the Difference of two Quantities, their particular Centers of Gravity being known, (as for Example, the Center of Gravity of C D, which is the Difference of the two Quantities A B and A D, whose particular Centers of Gravity are F and E;) draw F E, and produce that Line towards G, and you will find the Point G the Center of Gravity requir'd by this Analogy C D : A B : : F E : E G. That is, *As the Difference is to the least Quantity; so is the Line* F E: *to the Line* E G, *or the Length of the Production of the Line* F E. Annotat. Lect. II. * Plate 7. Fig. 8. † 39.

The Center of Gravity of a Cone is in its Axis, at the Distance of one fourth Part from the Base; as for Example, in the Cone A B C *, whose Axis is D C, the Center of Gravity will be at F, F D being equal to $\frac{DC}{4}$. But in a Conic Surface the Center of Gravity will be distant from the Base $\frac{1}{3}$ of the Axis; that is, D F will be equal to $\frac{DC}{3}$. * Plate 7. Fig. 9.

If A B I K be a truncated Cone, to find its Center of Gravity, let the Cone be compleated, which will then be A C B; then having found the Center of Gravity of the Cone A B C (namely, the Point F) and the Center of Gravity of the Cone I C K (which is the Point E) join those Centers together by the Line F E; then considering that A B K I is the Difference of the two Quantities A C B and I C K, you will find its Center of Gravity by the Rule above-mention'd, which will be at G; for A B I K : I K C : : E F : F G.

If a Bucket be made of Copper, Tin, or Wood, in the Shape of a truncated Cone, the Center of Gravity of such a Vessel will not be in the same Place when it is empty as when the said Vessel is full; which Consideration is useful in several Cases of Mechanics in general, and Hydraulics in particular. For by that means Vessels made of the Shape above-mention-ed, which being suspended and moveable upon Pins or an Axis (passing between the Center of Gravity of the empty and the Center of Gravity of the full Vessel) shall turn with the Bottom upwards when empty, will be drawn directly up with the Bottom downwards when full; or on the contrary have their Mouth upwards when empty, and turn over and empty themselves as soon as they are quite full.

Let A B E D * be the Section of a truncated hollow Conic Vessel, whose Mouth is A D. Its Center of Gravity by the Rules above-mention'd will be found to be at *c*; but because the Bottom or Bottom-plate B E is of some Weight, the Center of Gravity will be brought down to C. The Center of Gravity of the full Vessel (which will be a solid truncated Cone) will be at K. If therefore the Axis of Suspension be placed between those two Centers, as at O, [*Fig.* 11.] * such a Vessel when empty may be drawn up and down hanging with the Mouth downwards, but come up when full, with the Mouth upwards. This is of use in a Chain of Buckets going round * Plate 7. Fig. 10. * Plate 7. Fig. 11.

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round an Axis or Rag-wheel to draw Water from a Depth and deliver it in a Trough above.

But if *a b e d* be ſuch a Veſſel, only with this Difference, that the Bottom is fix'd to the narrow Part *e d*, and the Mouth is at *a b*; the Center of Gravity of the empty Veſſel (without conſidering the Bottom) is at *c*; but by the Weight of the Bottom brought to C, the Center of Gravity of the full Veſſel will be at K. If ſuch a Veſſel be ſuſpended between thoſe Centers, as at O (*Fig.* 13.) it will continue with the Mouth upwards when empty; but turn over, as ſoon as it is full. Such a Bucket may be of uſe to raiſe Water by a Machine made with a Couple of Buckets fix'd to a Beam which moves upon a Center unequally diſtant from its Ends, in ſuch manner that a Bucket fix'd to the ſhorter Brachium raiſes up a Bucket at the other End, ſo as to make it empty its Water in a Ciſtern above. --- But a ſhort Deſcription and Figure will make the Thing more plain.

Plate 7. Fig. 12.

Plate 7. Fig. 13.

Plate 7. Fig. 14.

A A, are the two Spouts running from a Brook or Spring of Water into the two Buckets D and E, D containing about 30 Gallons, and being call'd the *loſing Bucket*; and E, the *gaining Bucket*, containing leſs than a quarter part of D; as for example, 6 Gallons.

D E is a Leaver or Beam moveable about the Axis or Center C; which is ſupported by the Pieces F F, between which the Bucket D can deſcend when the contrary Bucket E is rais'd up. D C is to C E, as 1 to 4.

G L is an upright Piece, through the Top of which the Leaver K I moves about the Center L, ſometimes reſting on the Prop H, and ſometimes rais'd from it by the Preſſure of the Arm C E on the End I.

The Bucket D, when empty, has its Mouth upwards, being ſuſpended as above-mentioned. The End D with its Bucket is alſo lighter than the End with the Bucket E, when both are empty. By reaſon of the different Bore of the Spouts, D is fill'd almoſt as ſoon as E, and immediately preponderating, ſinks down to D, (*Fig.* 15.) and thereby raiſes the contrary End of the Leaver and its Bucket E up to the Ciſtern M, where it diſcharges its Water; but immediately the Bucket D becoming full, pours out its Water, and the End of the Leaver E comes down again into its horizonal Situation, and ſtriking upon the End I of the loaded Leaver I K, raiſes the Weight K, by which means the Force of its Blow is broke. If the Diſtance A B, or Fall of the Water be about ſix Foot, this Machine will raiſe the Water into the Ciſtern M 24 Foot high. Such a Machine is very ſimple, and may be made in any Proportion according to the Fall of the Water, the Quantity allowed to be waſted, and the Height to which the Water muſt be raiſed.

Plate 7. Fig. 15.

" Some Years ago a Gentleman ſhew'd me a Model of ſuch an En-
" gine, varying ſomething from this, but ſo contriv'd as to ſtop the run-
" ning of the Water at A, A, when the Leaver D E began to move.
" According to this, he told me that he had ſet up an Engine in *Ireland*
" which raiſed about half a Hogſhead of Water in a Minute 40 Foot high,
" and did not coſt 40 Shillings a Year to keep in Repair; and that it was
" not very expenſive to ſet it up at firſt."

10. [30.——*A Windmill must be supported*, &c. *and a Crane.*] This is not strictly true in Practice; because a Regard is to be had to the Force with which the Wind pushes the whole Windmill back with part of its Pressure, whilst it turns the Sails with the rest; and therefore the Line of Direction passing through the Center of Gravity, must fall before the Axis of the Post nearer the Sails. Likewise in a Crane (I mean such a one as wholly turns round with the Weight) Regard is to be had to the Weight which is to be lifted by it; and the Center of Gravity of the Crane placed so much back from the Weight, that the Line of Direction may only pass through the Middle of the Shaft, when the Weight (which brings the Center of Gravity forward) is hanging upon the Crane. Annotat. Lect. II.

11. [32.——*The Center of Gravity will remain at rest*, &c.] See Sir *Isaac Newton*'s *Principia*, Corol. 4th, of the Laws of Motion, 2d Edition, Page 17.

12. [37.—*Demonstrated in the Notes.*] Let the Spindle or double Cone of *Plate* 4. *Fig.* 14. be represented here seen endwise (*Plate* 8. *Fig.* 1.) A F is one of the rising Rulers upon which the Body is to roll, A G the horizontal Line, B the Vertex of one of the Cones. Let F G the lower part of the Screw S be made equal to *e E*, which is somewhat less than the Semidiameter of the common Base of the two Cones; or (which is the same thing) let E F be another horizontal Line passing a little under B the Axis of the Cones, and B F will be the Way of the Center of Gravity of the Body, which Line having a Declivity towards S, the Center of Gravity of the Body must descend; and consequently bring the Body along, more or less swiftly, as that Declivity is quicker or slower. Plate 4. Fig. 14. Plate 8. Fig. 1.

The Cylinder of Figure 15. Plate 4. is made of light Wood, with a small Cylinder of Lead at K, going quite through it near the curve Surface, and parallel to the Axis of the great Cylinder, to the Intent that the Center of Gravity of the compound Body may be remov'd from the Axis M into the Line K O; and then the Cylinder must be so laid on the inclin'd Plane A C, that the Center of Gravity of the said Cylinder may descend whilst it is rolling towards R, which will make it go up the Plane till the Center of Gravity is fallen as low as it can: Supposing always a String fix'd to the upper part of the Plane, and going round the Cylinder, to keep it from sliding when the Plane is not horizontal, as in the Figure. Plate 4. Fig. 15.

As the Length of the Cylinder does no way relate to its rising on, or being supported by, the inclin'd Plane; in considering the Motion of the Cylinder on the Plane differently posited, we shall in the second *Figure* of *Plate* 8. only consider the Sections of the Cylinder, Plane, and Horizon. P T A is the Section of the wooden Cylinder, C A that of the leaden one, C the Center of Gravity, M the Center of Magnitude, and P Q the Section of the Plane at first supposed horizontal. Plate 8. Fig. 2.

I say, *first*, If P *a* be taken upon P Q equal to P T A, the half Circumference of the Cylinder, the Point *a* will be the farthest Place to which the Cylinder will roll.

When

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When the Diameter P A going thro' the Center of Gravity C is perpendicular to the horizontal Plane, as in the Figure, the Cylinder will ſtand ſtill, becauſe the Center of Gravity is directly over the Center of Motion at P *; but as ſoon as C is ever ſo little inclin'd towards Q, the Body will roll till the Point A comes to *a*, deſcribing the Semi-Cycloid A *a*, whilſt all the Points of the Semicircle apply themſelves ſucceſſively to the Line P *a*, which is the Baſe of the Cycloid. That the Cylinder will go quite to *a*, is evident by obſerving C *c* the Way of the Center of Gravity, which is not in its loweſt Place till it is come to *c*, and muſt afterwards riſe towards *x*, if the Body roll'd on farther: and therefore if by the Velocity acquir'd the Body ſhould go on towards Q, the Center of Gravity in going down again from *u*, will bring back the Body to *a*; the Diameter P A being again perpendicular to the Horizon, but in the inverted Poſition *a p*. *Q. E. D.*

* N° 26.

N°. 24.

I ſay ſecondly, that if the Plane be inclin'd to the Horizon in any Angle, whoſe *right Sine* is equal to M C the Diſtance of the Center of Magnitude from the Center of Gravity, the Semidiameter of the Cylinder being *Radius*: the Cylinder laid upon ſuch a Plane will neither aſcend or deſcend when the Center of Gravity is directly over the Point T, where the Cylinder touches the Plane, provided it be kept from ſliding by a String going under it *, in the Manner repreſented in the 15 *Fig.* of *Plate* 4.

* L. 2. Plate 4. Fig. 15.

Turn the Cylinder till the Center of Gravity is at K in the ſame horizontal Line with the Center of Magnitude; or (which is the ſame thing) 'till the Semidiameter M A becomes M α; from K drop the Perpendicular K T, which cuts the Circle at T, and draw the Radius M T, to which the Plane $\Pi\ \pi$ being made perpendicular, you will have the Angle π D Q made by the Plane with the Horizon, equal to the Angle M T K, whoſe *Sine* is M K equal to M C. For by producing M α to L, it is evident (by 8. 6. of *Eucl.*) that the Angle M T K = K L T, but (by 29. 1. *Eucl.*) K L T = L D Q.

In this Situation it is evident by Conſtruction, that the Center of Gravity cannot deſcend, becauſe the Line of Direction is ſupported at T *, where the Plane touches the Cylinder.

* N°. 45.

For if the Body was roll'd up any higher on the Plane to bring K towards π, T the touching Point would advance faſter towards π than K does, and therefore the Line of Direction would cut the Plane below T towards D, ſo that the Center of Gravity would deſcend and bring back the Cylinder to bear on T. The other way the Body would roll down, upon moving K ever ſo little towards D, the Line of Direction then advancing faſter towards D than the Point of Contact T. This may be made plainer by conſidering the Cylinder as a Balance; as for Example, if M *w* be a Balance, ſuſtaining on its End M a Weight, equal to the Weight of the Cylinder without the Lead, and on the other End *w*, a Weight equal to the Exceſs of Weight of the Lead, above the Bulk of Wood whoſe Room it takes up. Let K be their common Center of Gravity, found as has been taught *: Then conſidering K T as an inflexible perpendicular Prop, ſuſtaining

38.

taining the Balance at K, the Balance will continue *in Æquilibrio*, whilst the Prop is supported by bearing on T, the Place where the Plane touches the Cylinder. If the Plane should make a greater Angle with the Horizon, the Point T being remov'd farther towards L, it would be the same, as if the Prop should endeavour to support the Balance between K and *w*, in which Case the Weight M wou'd preponderate, and carry the whole Cylinder towards P; but if the Plane makes a less Angle with the Horizon, T will go towards D, and the Balance then being propp'd between K and M, the Weight at *w* will preponderate, and carry the Cylinder towards L. *Q. E. D.* Annotat. Lect. II.

COROLLARY.

Hence follows also, that there must be an Angle *d* D Q less than π D Q, which will be the Inclination of the Plane, on which the Cylinder abovemention'd can roll to its greatest Height on the Plane. For if the Angle π D Q be diminish'd a little, the Cylinder will roll towards π going upwards; and if the Plane D Q be rais'd a little, so as to make a small Angle with the Horizon, the Cylinder will rise on that Plane, tho' it will not roll so far upon it as in the horizontal Situation. If this Angle be increased, the Cylinder will rise higher, as its Way on the Plane shortens, but not beyond a certain Number of Degrees of Elevation, at which Elevation the Cylinder will go no higher above the Horizon, tho' its Way measured on the Plane will continually shorten till it is contracted to a Point when the Inclination is in the Angle π D Q. Likewise as the Angle π D Q (by lowering the Plane D π round the Center) diminishes, the Cylinder will go higher as it rolls farther on the Plane, till the Angle is diminished to a certain Number of Degrees, after which the Cylinder will rise less; but its Way measured on the Plane will increase, till the Plane comes to be horizontal. Therefore there is an Angle of the Plane *d* D Q, which is a *Maximum*, as to the rise of the Cylinder on the Plane.

I say, *Thirdly*, that the Inclination of the Plane being given on which the Cylinder will rise to the greatest Height (or any other Inclination of a Plane on which it can rise at all) the length which the Cylinder will go on the Plane will be equal to the length T υ = T V, which is equal to half the Circumference *minus* the Arc A V, which Arc A V contains twice the Number of Degrees of the Plane's Inclination, together with the Degrees of twice the Difference of the Angle at the Center of two rectangular Triangles, which have M N the Sine of Inclination for Radius, but their Secants are M R the Semidiameter of the Cylinder, and M C the Distance of the Center of Gravity from M the Center of the Cylinder. Moreover, T υ being the Length of the Cylinder's Progression on the Plane, the Height of the Plane at υ, *viz.* the Perpendicular υ Z, will be the Cylinder's Rise above the Horizontal Line. Plate 8. Fig. 3.

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PREPARATION.

P Q is the Horizon and P *q* the Plane of given Inclination. Since P *q* cuts the Circle P V A, the Cylinder cannot rise upon it, therefore we must take another Plane parallel to it. The Diameter P G being drawn perpendicular to the Horizon, and R T thro' the Center of Gravity C or S parallel to P G, draw the Diameter T A, making the Angle P M T equal to the Inclination of the Plane, and draw Π π perpendicular to that Diameter at T, and Π π will be a Tangent Plane parallel to the former; (for P M T being equal to M T R, because of the Parallels P G, R T, and the common Angle R T υ being taken away from the two right Angles M T υ and R T Z, υ T Z the Angle of Inclination of the new Plane Π π will appear equal to the given P M T) draw M R; thro' C draw M *f*; make the Angle X C M equal to M C T, produce X C to V and draw M V. Because X C V is one straight Line and at the same Distance from the Center M as R T, therefore C R and C V are equal (by 7. 3. of *Eucl.*) since the Triangles C M *d*, C M *e* are equal (by 6. 7. 8. 1. *Eucl*) the Angle R M V is bissected. Draw M N the Sine of the Angle of Inclination, and the Degrees of the Arc R V, or the two Differences mention'd will be found by comparing together the Angle of Inclination and the Triangles R M N and C M N.

The Cylinder being laid on the Plane in the Position describ'd in the Figure, will not only remain at rest, if the Center of Gravity be at S, but will return to that Position when moved out of it towards Π or towards υ, because in each of those Cases the Center of Gravity must rise, and therefore S T must be the Distance of the Center of Gravity from the Plane, measured upon the Line of Direction of the Center of Gravity when nearest to the Plane, over that part of the Plane, where the Cylinder rolling upwards or downwards will stop its Motion. Now if the Center of Gravity be brought to C, it is evident that C R will be equal to S T, and that C V will also be equal to it, because by Construction it is equally distant from M *f* a Line going thro' the Center, and consequently = C R; also that no other Point of the Circumference will be equally distant from C. Then if the Center of Gravity from the Point C in the Line R T, where it is exactly over the Point of Contact, be ever so little mov'd towards υ by pushing the Cylinder that way, the Cylinder will roll on the Plane, and if it advances the Length of a Semicircumference till the Diameter T A is inverted and becomes *a t*, the Center of Gravity will be got to *c* the Chord T R being now *t r*; but since *r* does not touch the Plane, the Center of Gravity must descend again and bring back the Cylinder till *u* (which was the Point V) returns on the Plane to υ, where X K υ will be the Chord which was mark'd X C V in the first Position of the Cylinder; K being the Center of Gravity, and K υ which is equal to C V being in the Situation of S T, and the whole Cylinder bearing on υ just as it did on T, when we suppos'd S the Center of Gravity, T A and T R being turn'd into T α and

T ϱ.

T ϱ. Therefore the Distance T υ on the Plane will not be equal to the Semicircumference, but want of it the Arc $a u = \alpha \upsilon = A V$. Consequently the Length run on the Plane by the Cylinder will be equal to the Arc T V, which may be found by the Thread applied to the Cylinder from T to V. Annotat. Lect. II.

Now the Number of Degrees of this Arc T V *may be found in the following Manner.* Plate 8. Fig. 3.

The Angle of Inclination is $\upsilon TZ = MTN = MRN = TMP = AMG = GMR$.

In the rectangular Triangles M R N and M C N, N M the Sine of Inclination of the Plane is the Radius common to both; M R the Semidiameter of the Cylinder is the Secant in the Triangle M R N, and M C the Distance of the Center of Gravity from the Center of the Cylinder is the Secant in the Triangle M C N. The Angle R M N = C M N = R M C; therefore R M V = 2 R M C, which may be found by Trigonometrical Tables, and consequently it is known. But the Angle of Inclination being given, its double is given, therefore A V and T V are known. *Q. E. I.*

To apply what has been said, let us suppose the Center of Gravity of the Cylinder to be distant from its Center of Magnitude $\frac{2}{3}$ of the Radius, that is, to be at the Point C in the Figure*. * Plate 8. Fig. 3.

The Angle *p l g* of the Plane, on which the Cylinder cannot rise is thus found, by Analogy;

As M *b* is to M *k* : : (3 : 2 : :) So is the whole Sine: To the Sine of the Angle M *l k* = *l* M P = *p l g* = 41°. 48′ +, *&c.*

The *Maximum* of the Angle of the Plane on which it will rise most is in this Case found to be 26°, and the Height of that Rise is υ Z = 42,85, such Parts of which the Circumference of the Cylinder contains 360.

N. B. *Hence it appears that the* Maximum *is not in the middle of the Angle p l g, or at* 28°. 54′, *as one might at first imagine.*

Now let us suppose the Angle of Inclination from the Plane to be 15°, the Length which the Cylinder will roll on the Plane, and the Height to which it will go above the Plane's Base will be found not only in the Manner above-mention'd; but also two other Ways, which for Variety-sake I set down here.

P R E P A R A T I O N.

The Center of Gravity being in the Point C, *at* $\frac{2}{3}$ *of the Radius of the Cylinder* M f b *from its Center* M; *making the Arc* f V = R f, *and drawing thro' the Center of Gravity* C *the Line* V C X, *the Segment* V b X *is* = *to the Segment* R ż T, *and consequently when the Cylinder has trac'd on the Plane* T υ *the Arc* T ż V, *it will rest and be æquipois'd (on the Plane) upon the Point* V, *for the same Reason that it is so upon the Point* T *before its rolling.*

As the Angle of the Plane υ T Z is = 15°, that Angle is equal to the Angle P M T = *b* M R; therefore there only remains the Angle R M V

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to be found, or its half R M *f*; and the two following Ways give the same Value for it.

1. Since the Sine G R for the Angle *b* M R, when the Radius is the Radius of the Cylinder, is equal to *y* C, the Sine of the Angle *b* M *f* for the Radius of *o* C *k*, the Circle of the Center of Gravity: I say,

As the Radius of the Center of Gravity:
To the Radius of the Cylinder:: (or as 2 to 3)
So is *n o*=2588190, the Sine of 15° for the Radius of the Center of Gravity:
To 3882285=G R or *y* C, the Sine of the Angle *b* M *f*, which is found to be 22°, 50′ +.

From which therefore subtracting the Angle *b* M R = 15°, there will remain 7°. 50′ for the Angle R M *f* = *f* M V. Therefore the whole Angle R M V is = 15°. 40′ +; to which adding 30° for the two equal Angles *b* M R, P M T each of 15°, the whole makes 45°. 40′ + to be subtracted from 180, equal to half the Circumference *b* V *z* T P of the Cylinder, and there remains the Arch V T for the length run by the Cylinder rolling on the Plane T *v*.

2. The other Way to find the Angle R M *f* is thus;

In the Triangle R M C are given R M, M C, and the Angle M R C = R M *b* = 15°; therefore I say,
As the Radius of the Center of Gravity:
To the Radius of the Cylinder:: (or as 2 to 3, or M C M R::)
So is the Sine of the Angle M R C of 15°=2588190:
To 3882285 the Sine of the Angle N C M, or R C *f* = 22°. 50′ +.

And that Angle R C *f* being equal to the two opposite Angles M R C, R M C, or R C *f*, if from 22°. 50′ be subtracted 15° for the Angle M R C = R M *b*, there remains the Angle sought R M *f* = 7°. 50′ +, as before.

Having therefore subtracted 45°. 40′ + from 180°, there will remain for the Arc V *z* T 134°. 20′ for the Way of the Cylinder on the Plane T *v*, that is, $134\frac{1}{3}$ of such Parts, of which 360 make the Circumference of the Cylinder, by which one may find the Height *v* Z to which it rises on the Plane, thus;

As the whole Sine:
To $134\frac{1}{3}$::
So is the Sine of the Angle *v* T Z:
To *v* Z=34,77—. *Q. E. I.*

Mr. *Charles de Labelye*, having further consider'd the Motion of the loaded Cylinder (see *Plate* 4. *Fig.* 15.) as it rolls up on an inclin'd Plane, communicated to me several Propositions relating to that Motion, which I thought would not be unacceptable to the curious Reader.

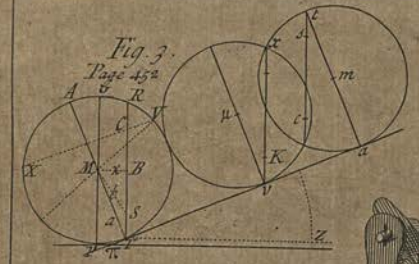
Fig. 3.
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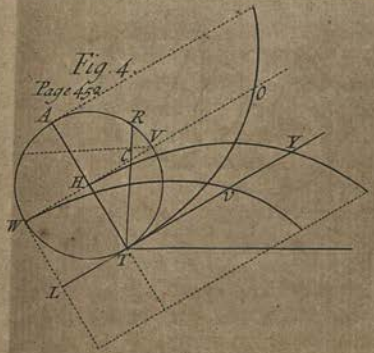
Fig. 4.
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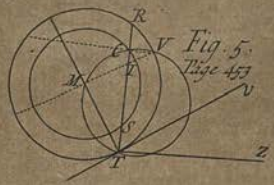
Fig. 5.
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Fig. 6.
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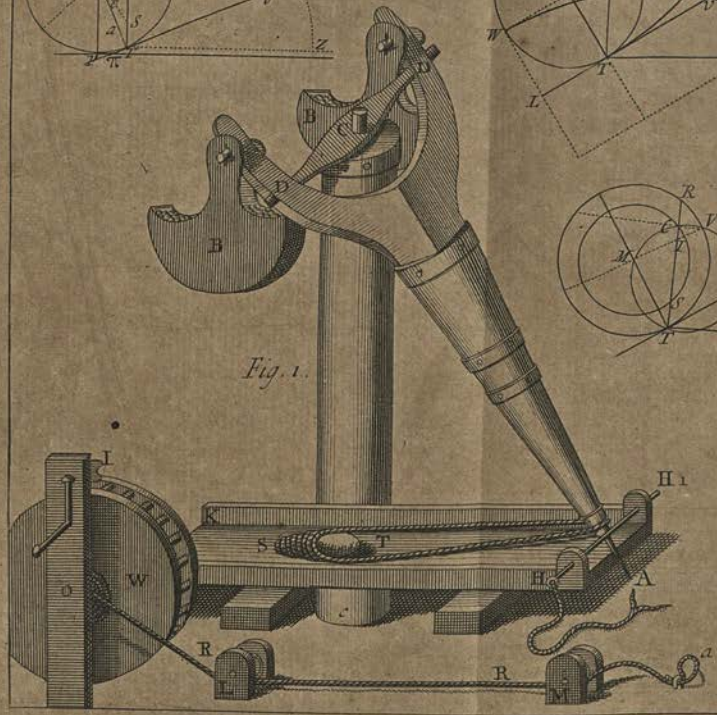
Fig. 1.

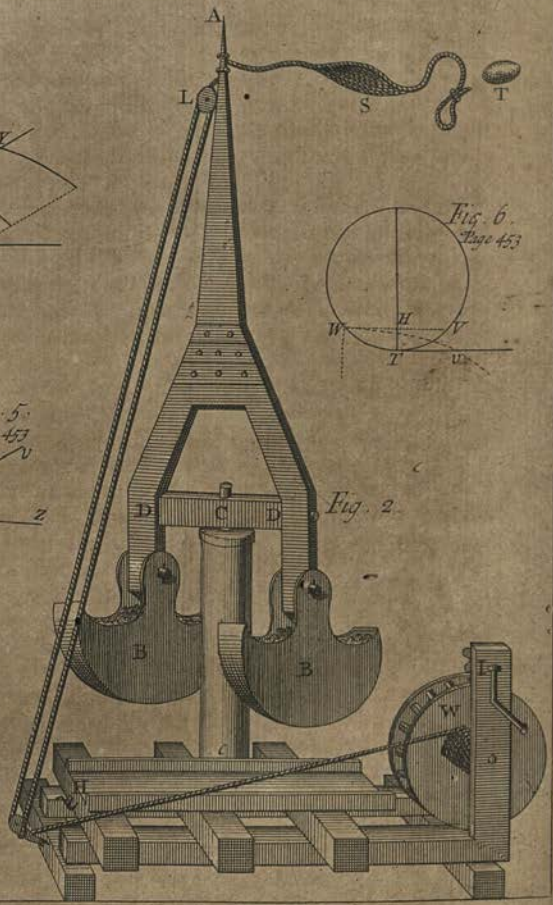
Fig. 2.

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PROP. I. *Plate 6. Fig. 3.*

To find T S *universally.*

Let M T=a, M S=b, M B=x. Then we shall have $\surd\overline{a\,a-x\,x}$ = B T, and $\surd\overline{b\,b-x\,x}$=B S. Now, since B T—B S=T S, it will follow, that $\surd\overline{a\,a-x\,x}-b\,b-\surd\overline{b\,b-x\,x}$ = T S; which consequently will be known by making x = to the Sine of the Angle of Inclination of the Plane with the Horizon, supposing M T *Radius*.

Another Way, without extracting Roots.

As M S ($=b$): is to the Sine of the Angle M T S:: so is M T ($=a$): to the Sine of the Angle M T S; which being known, M S is also known. Then say,

As the Sine of the Angle M T S: is to M S ($=b$):: so is the Sine of the Angle T M S: to T S required.

Now, supposing the Center of the Gravity in C, it is evident, that the Cylinder will roll up till the Point V in the Circumference (having made C V = C R = T S) comes to υ; for then the Center of Gravity will be in the same Position, as when at S over T. And to find in what Point of the Line T a the Point V will be applied, it is evident, that the Line T υ will be equal to the Arc T V: therefore supposing a Cycloid * whose generating Circle is equal to the Base of the Cylinder, and Vertex at T, and Axis upon T A, if you draw V O parallel to the Plane T F; by a Property of the Cycloid, the Line V O, being part of an Ordinate, will be equal to the Arc T V. Therefore making T υ equal to V O, you will have υ the Point required. *Pl. 6. Fig. 4.

Another Way.

Since the Ordinate H O is made up of V O equal to the Arc sought, and H V equal to the Sine of the Angle of the said Arc; by placing the same Cycloid, so as to have its Vertex at H, and Axis on H T, it will cut the Plane in Y, so that H O will be equal to T Y; from which subtracting Y υ=H V, the Remainder will be T υ = V O = the Arc T V required.

Another Way.

Draw through V, V H parallel to the Plane; and placing the Cycloid with its Vertex at W, and its Axis on W L parallel to H T, it will cut the Plane at υ; for L υ = T Y = H O, and L T being equal to W H = H V = υ Y, there will remain T υ = V O = the Arc T V required.

PROP. 2. *Plate 6. Fig. 5.*

It may be farther ask'd, *What Degree of Inclination to the Horizon, a Plane must have for the Cylinder to roll up that Plane the Length of any given Arc of the Circumference of its Base*; as for Example, suppose the Arc given to be T V.

To

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To find this, we muſt conſider, that when the Center of Gravity is over V, after the Cylinder has roll'd, the Line C V, drawn through that Center of Gravity V, will make the ſame Angle with the Line V M (drawn through the Point V and M, the Center of Magnitude) as the Line C T makes with T M, when the Center of Gravity is at S over T. Now, ſince the Angle C V I or C V M is equal to C T M, and C I V = / to its Oppoſite M I T; it muſt follow, that T M V given, is equal to T C V ſought. Whence we conclude, that a Circle paſſing through T, M and V, will alſo paſs through C. From whence we obtain this following Method:----*The Arc* T V *being given, to find the Center of a Circle paſſing through* T, M, V; which Circle will cut a Circle deſcrib'd round the Center of Magnitude M (with the Diſtance of the Center of Gravity from it) at C: then making C R = C V, draw R C T, and where it cuts the Circle of the Baſis of the Cylinder at T, draw M T, making the Angle M T S; and this Angle will be equal to υ T Z, the required Angle of Inclination of the Plane T υ, as is evident from what has been ſaid.

PROP. 3. *Plate* 6. *Fig.* 5.

It may be alſo demanded, *How many Degrees of the Circumference, or what Part of an Arc will the Cylinder roll upon, to deſcribe a given Space on the inclin'd Plane*; that is T υ, being given to find T V. This may be done two Ways; for having the Diameter, you muſt ſay--- As 113 : is to 355 :: ſo is the Diameter : to the Circumference, half of which will be the longeſt T υ, which the Cylinder can deſcribe. So by the Rule of Proportion ----As half the Circumference : is to 180 Degrees : : ſo T υ (meaſur'd in Parts of the ſame Bigneſs, as thoſe of the half Circumference:) will be to the Number of Degrees in the Arc T V.

But if it be requir'd to find the Point V by Conſtruction; upon the Point T of the Line T υ* or the Plane given, erect perpendicularly the Line T A or Diameter of the Cylinder, and deſcribe the Circle equal to the Baſis of the Cylinder; then placing the Cycloid made uſe of before, ſo that the Axis being kept parallel to the Diameter of the Circle (which is here equal to the generating Circle) when the Vertex of the Cycloid is in the Circumference of the Circle, and the cycloidal Curve falls at the ſame Time on the Point υ; you will have given you, by the Vertex of the Cycloid, the Point W: then taking T V equal to T W, it will be alſo equal to T υ, from what has been ſaid before, *&c.*

* Pl. 6. F. 6.

As for finding the true Maximum *of the Riſe of the Cylinder, it cannot be obtain'd without finding ſome algebraick* Formula, *or Expreſſion, for the Riſe of the Center of Magnitude, as well as another* Formula, *without any more unknown Quantities, than in the former Way of expreſſing the Deſcent of the Center of Gravity of the Cylinder whilſt it is rolling. Now for a due Conſideration of the Figure, it appears, that theſe* Formulæ *require the Rectification of the Circumference of a Circle, or of ſome Arc thereof; which, not being poſſible to be done without the infinite* Series's, *has made me leave the Subject.*

13.

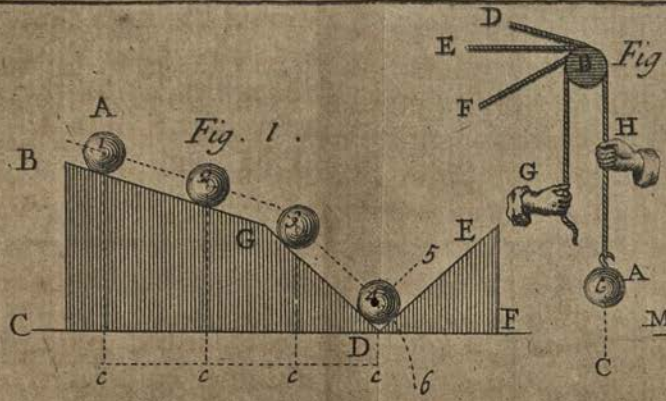

Fig. 1.

Fig. 2.

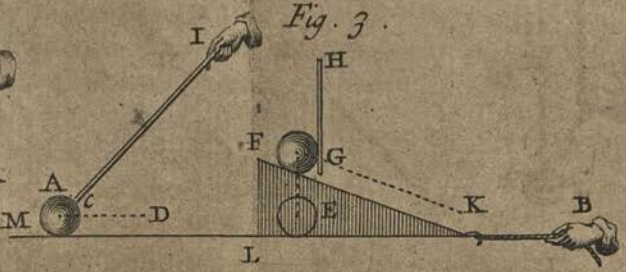

Fig. 3.

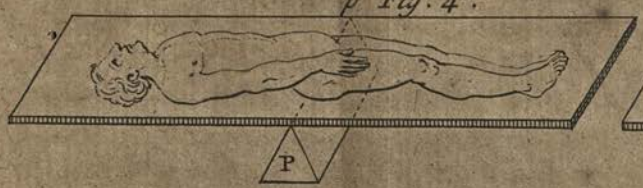

Fig. 4.

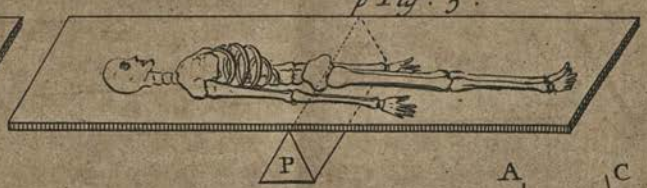

Fig. 5.

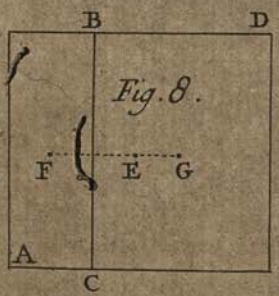

Fig. 8.

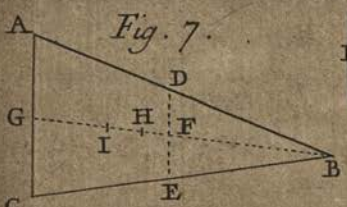

Fig. 7.

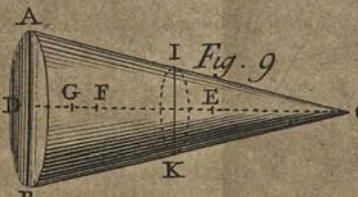

Fig. 9.

Fig. 6.

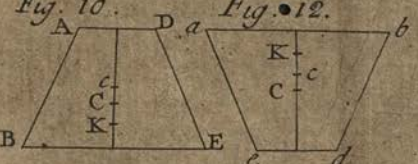

Fig. 10.

Fig. 12.

Fig. 11.

Fig. 13.

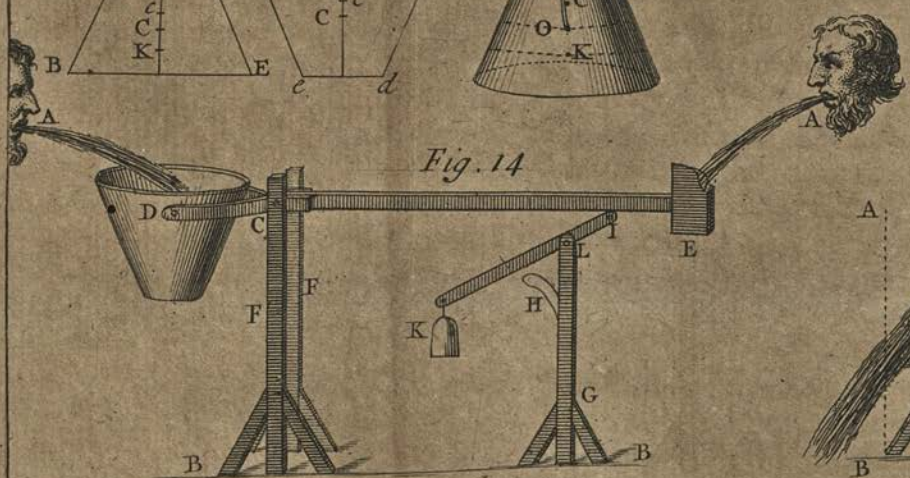

Fig. 14.

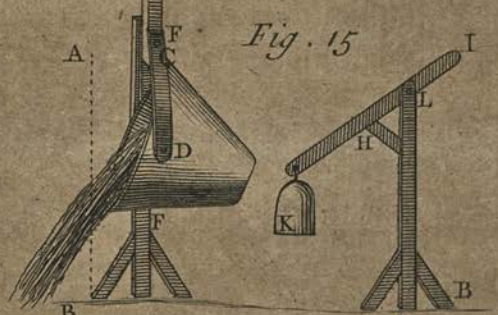

Fig. 15.

13. [19. ——— *Common Center of Gravity may be found*, &c.] By this means may be found the Center of Gravity of our System, in any Position of the Planets *. Let us first suppose them all on one Side and in a Line as S C A, going thro' the Sun's Center C: Or, to speak like an Astronomer, let all the superior Planets be in Opposition, and the inferior ones in their inferior Conjunction. *First*, The common Center of Gravity of *Mercury* and the Sun will be very near the Sun's Center; because the Quantity of Matter in the Sun is more than a Million of times greater than in *Mercury*, and *Mercury* is not 82,5 Semidiameters of the Sun distant from the Sun's Center. *Secondly*, *Venus* (supposing it, as very probable, to have about the same Quantity of Matter as the Earth) taken into Consideration, will bring the common Center of Gravity of the three Bodies but a little forwarder, *viz.* to ♀, because its Mass is to the Sun's but as about 1 to 169282, and its Distance from the Sun's Center but about 145 Semidiameters of the Sun. *Thirdly*, The common Center of Gravity of the Earth, and the three Bodies above-mention'd, will be brought but to ⊖, or a little forwarder. *Fourthly*, The common Center of Gravity of *Mars* and the other four Bodies will be brought still a little nearer to the Surface of the Sun, as to ♂, but not half way from the Sun's Center to its Surface. *Fifthly*, The Quantity of Matter in *Jupiter* being to the Quantity of Matter in the Sun, as 1 to 1067; and *Jupiter*'s Distance from the Sun compar'd to the Sun's Semidiameter in a Ratio sometimes greater; the common Center of Gravity of *Jupiter* and the Sun will be a little way without the Sun's Surface; and therefore the common Center of Gravity of *Jupiter* and the other five Bodies will come to ♃, a little farther out. *Lastly*, As the Matter in *Saturn* is to the Matter in the Sun as 1 to 3021, and the Distance of *Saturn* from the Sun is to the Semidiameter of the Sun in a Ratio something less; their common Center of Gravity, without the other five Bodies, would be in a Point as ♄, a little within the Sun's Surface; and therefore the common Center of Gravity of all the seven Bodies will be at I, still a little farther out from the Surface of the Sun, but hardly a whole Diameter distant from the Center of the Sun. When *Jupiter* and *Saturn* are on different Sides of the Sun, this common Center of Gravity will always be within the Body of the Sun, let the other Planets be in any Position, because of their Nearness, and the small Quantity of Matter which they contain. It is this common Center of Gravity of our System which is at rest, and not the Center of the Sun; for the Sun has a kind of wabbling Motion about this Center. *The little Difference made by the Comets and Satellites of the primary Planets is not worth mentioning here.*

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* See the last Edition of Sir *Isaac Newton*'s *Principia*, B. 3. Prop. 12. Plate 8. Fig. 4.

Pl. 6. Fig. 4.

14. [43.——*Line of Direction falls within their Base.*] The Tower of *Pisa* is a round Tower 138 Foot high, whose Top over-hangs the Base 15 Foot, as represented in the fifth Figure: And the Tower of *Bolonia* is square, 130 Foot high, its Top over-hanging the Base only nine Foot. See the sixth Figure.

Plate 8. Fig. 5, and 6.

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Animal Motions are always subject to these Rules, which we observe without thinking on them. When we stand upright with our Feet, as represented in *Fig.* 7. the Line of Direction goes thro' the Point C, and passes between our Feet and D, and we may move our Head from F to E or G, and our Bodies forwards, backwards, or side-ways, as far as I or H, without Danger of falling or stirring our Feet, as long as the Line of Direction traverses no farther than I A or H B, and falls any where within the Space A B; which in this Situation of our Feet makes a pretty large Base. But if we set one Foot before the other, as in *Fig.* 10 *, a little Push side-ways will make the Line of Direction (which went thro' C) fall out of the Base to the right or left, towards E or B; in which Case a Man must fall, if he does not quickly remove his Feet to the Position of *Fig.* 7, or 9. When we stand upon either Leg we must bring our Body so much over the Foot A B or D E*, that the Center of Gravity being directly over it, the Line of Direction may go thro' *c* or K; and in walking, the Line of Direction must travel thro' every Place where each Foot is set down; going successively thro' the Points † E, A, D, B, whilst the Center of Gravity goes thro' the Points G, C, F, *&c.* so that unless a Man, in walking straight forward, sets one Foot directly before the other, the Line of Direction will not describe a straight Line upon the Plane where the Man walks, but an indented Line; that is, Angles to the right and left, whilst the Body of the Man goes on in a waddling Motion. This we see in the walking of fat People, and all others that straddle in their Gait ‖. The Line of Direction

Plate 8. Fig. 7.

* Plate 8. Fig 10.

Plate 8. Fig. 7, and 9.

* Plate 8. Fig. 9.

† Plate 8. Fig 8.

‖ It is not strictly true, that any Man in his common Walk sets one Foot so exactly before the other, as to carry on the Bottom of his Line of Direction in a straight Line, as represented in *Fig.* 10. and 11. Because if a straight Line be drawn with Chalk, it is difficult to walk straight along it; but the plainest Proof is the Observation of two upright Sticks, of about the Height of a Man, the one painted white and the other black, and set up about ten Yards beyond one another, in the same Line that a Man walks towards them: for in such a Case, tho' he keep one Eye shut, the last Stick will appear sometimes on the right and sometimes on the left of the first; and the more so, the nearer the Man comes to the Sticks. Rope-dancers *, indeed go in a straight Line; but it is what they have learn'd by Art, and inur'd themselves to by long Practice; yet they must even after all have Helps to keep their Center of Gravity over the Rope. They generally fix their Eyes on some distant Point in the same Plane as the Rope. They have commonly a long Pole loaded at the Ends with the Balls of Lead B, *b*, by the Motion of which they can alter the Position of the common Center of Gravity of their Body and the Pole; as for Example, the Center of Gravity of the Rope-dancer CA being at A, his Line of Direction should go thro' *a*, off from the Rope: but by moving the Pole towards B, the common Center of Gravity of the Man and Pole is brought to C, in which Case the Line of Direction C D goes thro' the Rope. Those who are well skill'd in this Art will sometimes use their Arms only, instead of a Pole; and it is very common for several of them to dance with a Flag, with which they strike the Air the same Way that the Center of Gravity goes when the Line of Direction does not go thro' the Rope; and, by the Re-action of the Air, the Center of Gravity is brought back to its proper Place.

* Plate 8. Fig. 13.

Those who would enquire farther into this Matter, may consult J. A. Borelli, *in his Book* De Motu Animalium, *Chap.* 18, 19, 20. *and* 21. *In the said* 21*st Chapter he gives an Account of the Motion of an Horse, part of which (as it is very curious) I shall repeat here.*

The Ancients, observing that Horses and other Quadrupeds, in Galloping, lift up their two fore Feet, and then their hind Feet, as soon

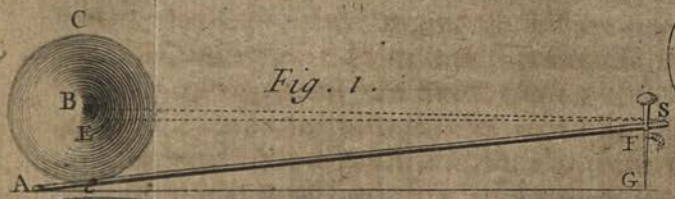


Fig. 1.

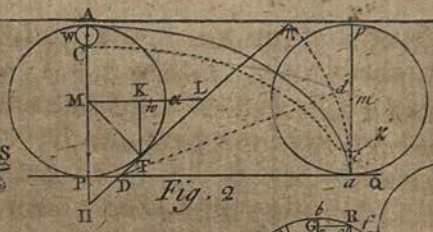


Fig. 2

Fig. 3

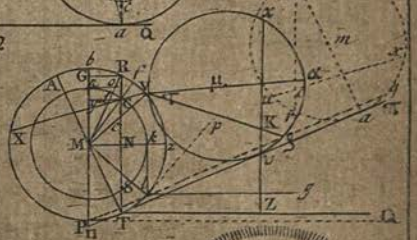


Fig. 4

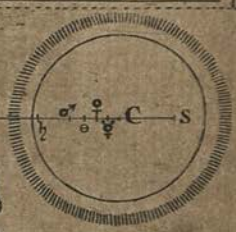


Fig. 5

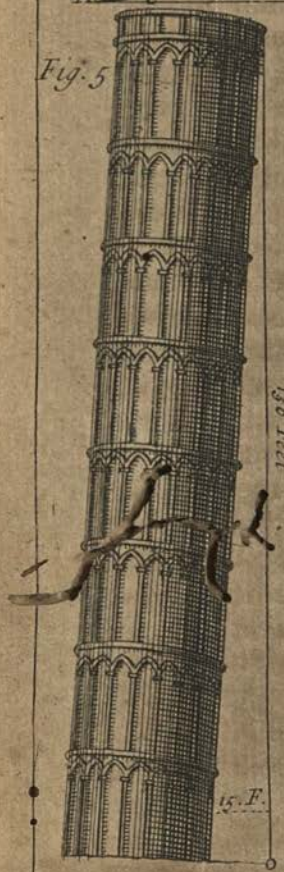


Fig. 6

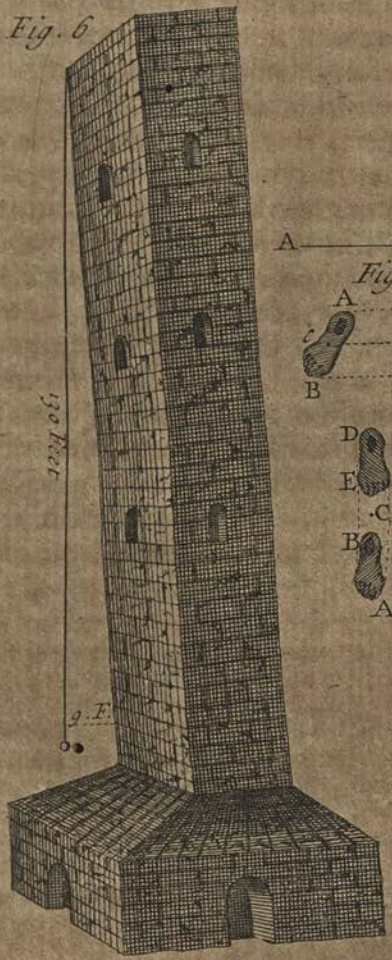


Fig. 9

Fig. 10

Fig. 10

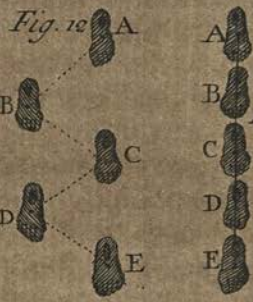


Fig. 11

Fig. 8

Fig. 17

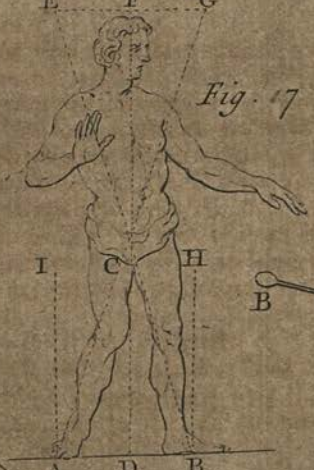


Fig. 14

Fig. 15.

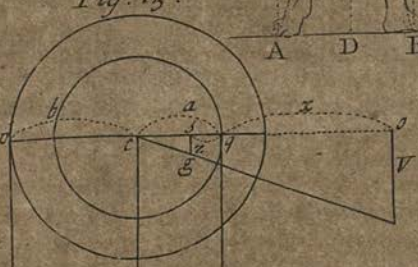


Fig. 13.

rection going thro' the Points A, B, C, D, E, deſcribes a ſtraight Line in *Fig.* 11, where the Feet are ſet before one another; but when the Motion of one Foot is in a parallel Line with the Motion of the other, an indented Line is deſcrib'd by the Center of Gravity above, and the Line of Direction as it cuts the Ground at A, B, C, D, E, *Fig.* 12. Ducks, Geeſe, and the greateſt Part of the Water-Fowl, whoſe Legs are ſet wide aſunder for the Conveniency of their ſwimming, and turning quick in the Water, have always a waddling Motion upon Land; but a Cock, a Stork, an Oſtridge, and moſt other Birds that are not web-footed, walk almoſt directly forward, without waddling (eſpecially when they walk ſlow) having their Legs ſo plac'd as to put one Foot before the other with greater Eaſe. Thus Quadrupeds ſeldom or never waddle, becauſe they have commonly three Feet upon the Ground at a Time: So that however the Baſe receiving the Line of Direction alters from a quadrangular to a triangular Figure: that Part of it, in which the Line of Direction falls, is always in or near the ſame Line.

Annotat. Lect. II. Plate 8. Fig. 10, 11, and 12.

When a Man ſtands in a firm Poſture *, AB, the Diſtance of his Feet, is the Length of a quadrilateral Figure, whoſe Breadth is nearly the Length of the Feet, and D is the Point under the Center of Gravity C, where the Line of Direction falls. Let the Lines AC and BC be drawn, then let thoſe two Lines and DC be continued to the Points E F G, ſo as to make the Triangles E C G and A C B equal and ſimilar. As long as the Line F D (or a Plane going thro' it) cuts the whole Body of the Man into two equal

* Plate 6. Fig. 7.

ſoon as the fore Feet are ſet down, did imagine, that in *Walking*, as well as *Pacing* and *Trotting*, an Horſe has two Feet off of the Ground at one Time; and accordingly in their Braſs or Marble Statues, they have repreſented their Horſes with two Legs off of the Ground diagonally oppoſite, as the right before and the left behind, or left before and right behind. The modern Statuaries have alſo fallen into the ſame Error, becauſe in the quick walking of an Horſe the Eye cannot well diſtinguiſh; and therefore he has ſhewn from Mechanical Principles, that the Motion of raiſing two Feet at once in Walking cannot be conſiſtent with the Wiſdom and Simplicity of Nature. To *Borelli* therefore I refer the Reader, who wou'd know what the Motion is not, and only copy from him, here, what the Motion is.

* Plate 8. Fig. 14.

'Let us conſider an Horſe * as 'an oblong Machine ſuſtain'd by 'the four Legs, as four Props or 'Columns, reſting on the Points 'A B C D, which make a rectangular quadri-'lateral Figure; then the Line of Direction 'will fall perpendicularly on E, a Point in 'or near the Center of the quadrilateral Figure, which will make the Station or Standing 'of the Horſe the moſt firm. The progreſſive 'Motion begins by one of the hind Feet, as 'for Example, the left hind Foot C, which by 'ſtrongly preſſing back the Ground moves for-'ward the Center of Gravity, and conſequently 'carries on the Line of Direction from E to G, 'as itſelf moves from C to F. This done, 'immediately the Foot B is rais'd, and carried 'forward as far as H, which Motion of the 'Foot is eaſy, becauſe the Line of Direction 'firſt falls within the Triangle ABD; ſecondly, 'within the Trapezium A B F D; that is, 'the Body of the Horſe is ſuſtain'd by three or 'by four Columns. Laſtly, the three Feet A, D, F, 'remaining firm, and taking in the Line of 'Direction at G, immediately the left fore Foot 'B is carried forward to H; and, by the Im-'pulſe already made, the Center of Gravity is 'alſo carried over I, namely the central Point 'of the Rhomb A H F D. The Motion of 'the two left Feet being compleated, the Im-'pulſe and Motion of the right hind Foot D 'begins, and then that of the right fore Foot; 'and ſo on in the Manner above deſcrib'd, a' 'the Animal moves forward.'

Annotat. Lect. II.

equal Parts, the Center of Gravity will be at C, and C D will be the Line of Direction. But if the Body be inclin'd towards the left Hand H, the Center of Gravity will move from C to H, the Line of Direction will become H B, and the right Foot being easily remov'd from A, may be carried on beyond B, by which means the Man will go on towards the left. In the like Manner by inclining towards I, the Line of Direction will be removed to I A, and the Man go to the right. When a Man stands upon one Foot, it is with some Difficulty. For Example, let the Line of Direction be C D; by the Motion of the Blood, and Lungs, and other animal Motions, the Center of Gravity will be apt to vacillate or totter towards F or G on either Side about the Center of Motion D, where now the Base is but small. If the Line of Direction comes to B, the Man must fall forwards, backwards if to E; and tho' A be under the Heel of the Foot, yet in the Motion of the said Line of Direction from D to A, the Body will be apt to go too far towards E, and so bring the Line of Direction beyond the Base. This will more probably happen in the side Motion of the Body; so that the Body will be in Danger of falling, unless the right Foot be put down towards that Side where the Body inclines. Birds stand upon one Foot much more easily than Men, because their Line of Direction being much shorter, and the Base of one Foot a large Rhomboidical Figure made by the four Claws, the Line of Direction cannot go out of that Base, unless the Center of Gravity rises, which is impossible without a violent Motion *.

Plate 8. Fig. 8.

* 28, 43.

When a Porter carries a Burthen upon his Shoulders, he must stoop, because if he should stand upright, the common Center of Gravity of the Man and Burthen would be so far brought back, that the Line of Direction would fall behind the Feet. For the same Reason, when a Woman with Child is very near her Time, she bends backwards as she goes, by reason of the Burthen before, which otherwise would cause her to fall forwards.

15. [49.——*Roll along the Plane.*] If the Ball * F E be laid upon a smooth horizontal Plane A B, it will stand still, tho' it touches but in one Point as O, because the Line of Direction *c* O goes thro' the said Point; but if the Plane be ever so little inclin'd to the Horizon, as in the Position C D, the Ball will continually roll forward towards D, because then the Line of Direction will always fall before the touching Point.

* Plate 9. Fig. 1.

Plate 9. Fig. 2. & 3.

If such a solid Body as G, contain'd under twelve rectangular Parallelograms, and two opposite, parallel and equal Dodecagons, be laid upon the inclin'd Plane B A C, as it will slide from A to C, the Center of Gravity moving in the Line *e g* parallel to the Plane A C, and *i o* one of the parallelogram Planes of the Body always touching the said Plane A C But if the Plane be more inclin'd, as in the Position D E, it will appear, by drawing the Arc *e f* with the Distance *i e* about the Point *i* (the only touching Place of the Body then) that the Center of Gravity can descend; and when the Point *o* is applied to the Plane D E, the Line of Direction will fall beyond the

the said Point *o* towards E; and therefore the Body will roll or tumble to- Annotat.
wards E. In the same manner all the angular Points, or rather the Edges Lect. II.
of the plain Surfaces, will successively apply themselves to the Plane D E, till the Body has roll'd quite down.

16. [49. —— *In the Load of Iron the Center of Gravity is low; but very high in the Load of Hay.*]

K * is the common Center of Gravity of the Load of Hay, and the Car- * Plate 9.
riage P M, whose Line of Direction is K E, whilst the Plane P M (on Fig. 4.
which the Carriage is drawn) is horizontal; but if C D be a horizontal Line, the Plane P M will be inclin'd to the Horizon in the Angle P B D, and the Line of Direction being chang'd from K E to K P (because K P is the only Perpendicular from K to C D) will fall out of the Base Q M towards C, and consequently the Waggon will be thrown over that Way; which also appears by drawing round the touching Point Q with the Distance Q K, the Arc K R, shewing the Way of the Center of Gravity, which can in this Case descend without first rising. For the same Reason, if the Carriage was drawn along an horizontal Plane, whose Section is represented by C D or P N, and the Wheel M should meet with a Rub of the Height of N M, the Load of Hay would also be overthrown upon that account.

But as a Load of Iron * lies much lower upon its Carriage, the Center * Plate 9.
of Gravity must also be lower, and therefore the Line of Direction will Fig. 5.
fall within the Base on the same Inclination of the supporting Plane, as would occasion it to fall out of the Base in a Load of Hay, as is evident from the Figure. Let C D also here represent an horizontal Line, and Q M the Road or the Base supporting the Carriage, the Angle of Inclination M Q N in this Figure being equal to M Q N in *Fig.* 4. The Line of Direction K P will here fall within the Base Q M, and cannot fall out of it till the Angle of Inclination is increas'd to B Q Y, by making X Y the horizontal Line; or, which is the same thing, unless the Wheel M meets with a Rub of the Height of *y* M much higher than the Rub which would overturn the Load of Hay.

LECTURE III.

1. *Simple Machines* or *Organs*, call'd by some *Mechanical Faculties*, or *Mechanical Powers* *, are such Instruments as are of one Piece (or considered as such) by means of which the † *Powers* described in the last Lecture act upon Weights, in order to give or stop Motion; to overcome, make, or stop Resistance.

* Annot. 1.

† L. 2. 19, 20.

2. All Engines (however compounded) for the Uses of Life, are made up of various Combinations of the simple Machines. Sometimes all of them may be found in one Engine; sometimes two or three of them; and sometimes only one of the Mechanical Powers multiplied.

3. The *Simple Machines* are the seven following; *viz.* the *Balance*, the *Leaver*, the *Pulley*, the *Axis in Peritrochio* (or Axle in the Wheel) the *Inclined Plane*, the *Wedge*, and the *Screw*. N. B. *Authors differ in their Enumeration of the* Mechanical Powers, *some making them to be* Six, *excluding the* Inclined Plane *from those which I have named. Others have reduced them to* Five, *considering that the* Screw *is only a* Wedge *carried round a Cylinder: And others again have made the* Leaver *and* Balance *to be the same* Power, *from their near Resemblance. But since the same Principle is only applied differently (as may be shewn by reducing all the* Mechanical Powers *to the* Leaver, *or explaining all their Operations by that of the* Leaver *) *and we are to give an Account of the Instruments contrived for that different Application, the* Mechanical Powers *will appear to be* Seven.

* Annot. 2.

4. Before we explain these Powers or Organs severally, there are some general Things to be considered relating to all Engines, which will facilitate our Calculations concerning them, and render the Execution of them in Practice, as perfect as the Nature of the Materials of which they are made will allow.

SUPPOSITIONS.

THOUGH the Earth be ſpherical, yet we are to ſuppoſe it flat, when we conſider Mechanical Engines; becauſe the largeſt Machine covers ſo ſmall a Part of the Earth's Surface, that to allow it any ſenſible Roundneſs in ſo ſmall a Compaſs, would be to allow too much.

6. THOUGH none of the Bodies which we handle are perfectly hard, or truly of the Figure which we intend to give them, yet we are to ſuppoſe every thing perfect in all our Engines; as for Example, that ſtraight Bodies, as the *Beam of a Balance*, a *Leaver*, *&c.* are Mathematical *Rigids*, and without Thickneſs or Lines altogether inflexible; that the *Mechanical Powers* (whether Simple or in Compoſition in complex Engines) are without Weight, whatever Materials they are made of; that Bodies are perfectly hard and ſmooth; that the Parts of Engines move one another without Friction; that Cords are exactly pliable; that the Center-pins of Pullies, or Axes of Motion of them, of Balances, Leavers, Axes *in Peritrochio*, *&c.* are only Mathematical Lines.

7. THOUGH * the Lines of Direction of all heavy Bodies tend towards the Center of the Earth, and conſequently converge together in a ſmall Angle; yet we muſt conſider them as parallel, becauſe they are ſo, as to Senſe; their Point of Convergence being near 4000 Miles off. For this Reaſon the Walls of a Building, when exactly adapted to the Plumb Rule, are nearer at Bottom, than at Top; though in Practice we muſt always look upon them as parallel. * L. 2. 22.

8. NOTWITHSTANDING the Falſity of the Suppoſitions, we ſhall not be led into any Error by them; becauſe, by a ſecond Conſideration, we are to have regard to the Imperfection of Engines and Materials, and the Quantity of *Stickage* or Friction; which differ according to the Number and Combination of Parts and Nature of the Materials, of which the ſeveral Engines conſiſt: And having made uſe of the beſt Methods we can to diſcover the Imperfections abovementioned, in each particular Machine; we are to take care to allow enough

Lect. III. enough to be deducted from the Calculation made concerning an Engine supposed Mathematically true.

N. B. *Several Methods of finding the Quantity of Friction in Engines will be considered hereafter* *.

* Ann. 3. and L. 4.

DEFINITIONS.

9. When equal or unequal Quantities of Matter are so applied to a Mechanical Organ, or Engine, that their *Momenta*, or Quantities of Motion, or moving Forces, destroy one another; they are said to be *in Æquilibrio* *.

* L. 2. 12.

10. When Powers, whose Intensities are equal or unequal, are so applied to an Engine as to destroy each other's Actions, they are also said to be *in Æquilibrio* *.

* L. 2. 12.

11. All Bodies that act against each other by means of Engines, may be considered as *Powers* and *Weights*, already described in Lect. 2. Numb. 18, 19.

12. Bodies *in Æquilibrio* are said to *æquiponderate*.

13. When Powers and Weights have their Velocities reciprocally as their Masses, or as their Intensities *, their *Momenta* are equal; therefore they are in *Æquilibrio* †.

* L. 2. 20.
† 9.

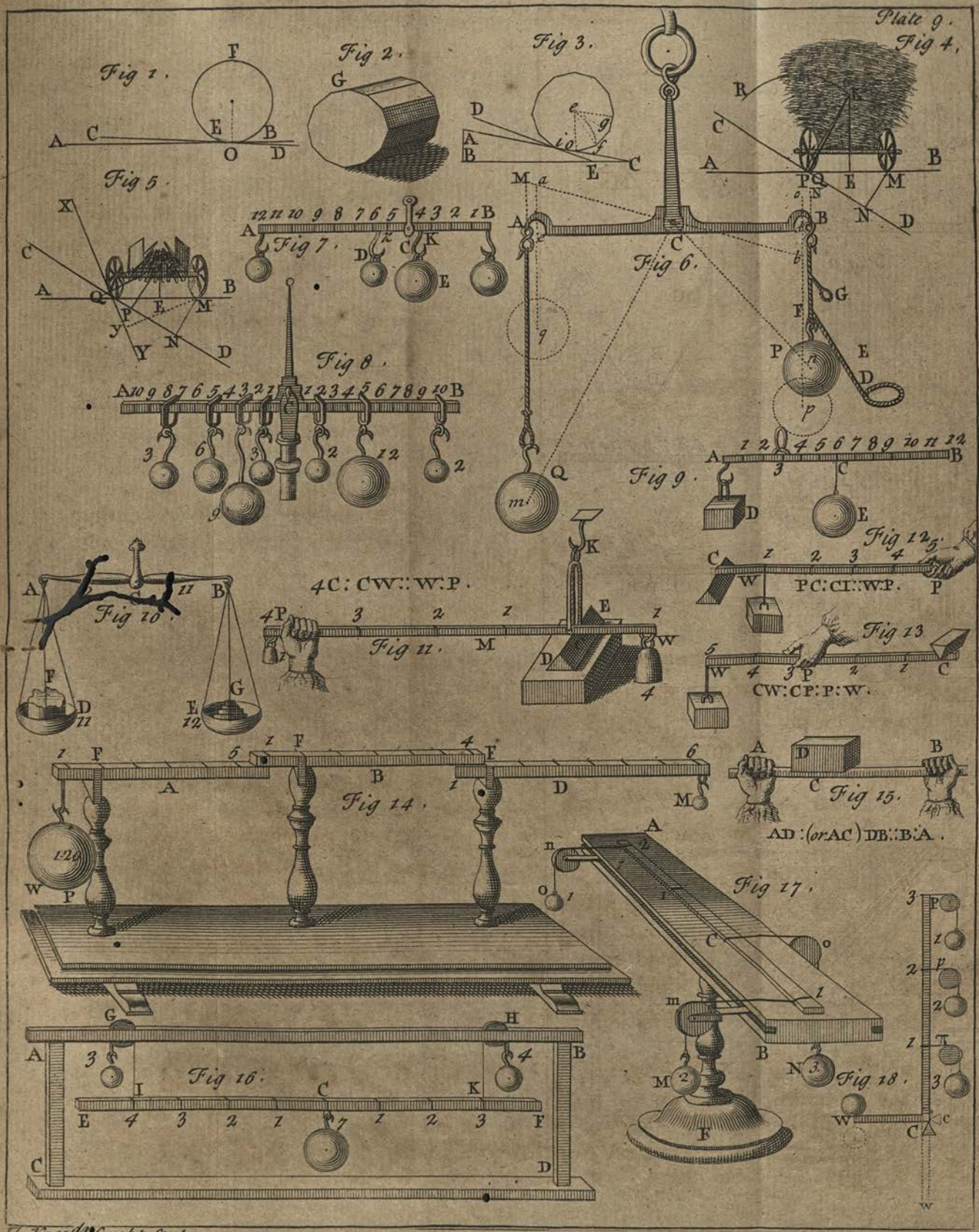
14. If the *Momentum* of a Power be greater than that of the Weight, or, (on the contrary) the *Momentum* of the Weight be greater; that Power or Weight is said to *præponderate*, or overpower.

15. When either the *Velocity* (the Mass being equal) or the *Mass* or *Intensity* of a Weight or Power (the Velocity being equal) or *both* Mass and Velocity together are greater in a Power or Weight than in the opposite (or counter-acting) Weight or Power; the *Momentum* of the former will be greater than that of the latter *.

* L. 2. 15. 17.

N. B. *This will happen, let the Difference be ever so small; though then the Resistance made by the Friction will hinder the Effect from being visible.*

Of



T. Van.dr Gucht Scul.

Of the BALANCE.

16. THE essential Parts of a Balance are (1.) the Beam, as A B *; (2.) the Axis of Motion, considered only as a Point or Center of Motion C, which divides the Beam into two Parts; (3.) those Parts called the Arms or *Brachia*, as A C and C B, which are either equal, as in *Fig.* 6, or unequal, as in *Fig.* 7. (4.) the Points of Suspension, as * A, B, in the 6th, and A, B, K, x, in the 7th Figure. * Plate 9. Fig 6, 7. * Plate 9. Fig. 6, 7.

17. WHEN Weights hang freely from the Points of Suspension, they gravitate neither more nor less for hanging nearer to or farther from the said Points.

EXPERIMENT I. *Pl.* 9. *Fig.* 6.

18. LET the Weight Q, with its Rope Q A hanging at the End A of the Balance A B, be equal to the Weight P, together with its Rope D B. Hang the Weight P at any of the Loops G, F, E, D, of its Rope, and it will at any of them make an *Æquilibrium* with the opposite and equidistant Weight Q.

19. WHEN the Beam of the Balance is equally divided by the Center of Motion (as in the 6th *Figure*) with Scales hanging freely from the Points A, B, instead of the Weights Q and P, it is the *Libra*, or common *Pair of Scales*.

20. THIS Instrument serves to compare together Bodies which have equal Quantities of Matter, though sometimes differing in Bulk; for when the Commodities to be bought or sold are placed in one Scale, so as to keep the Weights in the opposite Scale *in Æquilibrio*, the *Momenta* are equal; and since the Velocities are equal on account of the equal Distances A C and C B *, the Quantities of Matter must be likewise equal: and this is shewn by the horizontal Position of the Beam hanging freely on its Center of Motion, which is placed a little above its Center of Gravity. See the 4th Annotation. * 12. Ann. 4.

HENCE follows, that the Distance (that is, the acting Distance) of any Weight is not to be measured from the Center of Motion of the Balance to the Center of Gravity of the Weight;

Lect. III. Weight; and therefore the Lines C *n*, C *m*, do not expreſs the Diſtances of the Weights P and Q; but their Diſtances are properly C B and C A, the leaſt Diſtance from their Lines of Direction N *n* and M *m* to the Center of Motion C: And therefore when the Weights hang freely upon an Horizontal Balance, the Diſtance of their Points of Suſpenſion from C may be called their Diſtance, and meaſured upon the Beam. But if the Balance be in an inclined Poſition as *a b*, not *b* C and *a* C, but *d* C and *e* C will be the Diſtances of the Weights *p* and *q*, being Lines perpendicular to their Lines of Direction *p o*, *q a*, and going through the Center of Motion, conſequently the leaſt Diſtances from it to the ſaid Lines of Direction.

* Pl. 9. Fig. 7. 21. THE Balance, whoſe *Brachia* are unequal *, as A B (*Fig.* 7.) is the *Roman Statera*, or our Steel-yard; or the little Inſtrument which the *Chineſe* (who take all their Money by Weight) always carry about them, called the *Dotchins*. See Num. 13. of *Lect.* 2. *Plate* 4. *Fig.* 3. This Inſtrument ſerves to compare together, at one Operation, Bodies that have equal or unequal Quantities of Matter; but in weighing heavy Goods, it is not ſo exact as the Scales.

* Pl. 9. Fig. THE * Balance of *Fig.* 8. may ſerve as a Steel-yard or as a Pair of Scales, on account of the ſeveral Diviſions on each *Brachium*.

THEOREM.

22. *One or more Weights ſuſpended on one* Brachium *of a Balance, will be* in Æquilibrio *with one or more Weights ſuſpended on the other* Brachium; *provided that the Sum of the* Momenta *(or the whole Quantity of Motion) of the Weights on one Side of the Center of Motion, be equal to the* Momenta *on the other Side of it.*

EXPERIMENT II. *Pl.* 9. *Fig.* 8.

23. UPON the *Brachium* A C hang three ſingle Pounds at the Diviſion 8, a ſix Pound Weight at Number 5, a three Pound Weight at Number 1, and a nine Pound Weight at Number 3. Then on the *Brachium* C B, hang a two Pound Weight at

at 2, a twelve Pound Weight at 5, and a two Pound Weight at 10. Then the Balance will hang *in Æquilibrio.* Lect. III.

24. SINCE the Velocity of Weights hanging to a Balance depend upon their Distances from the Center of Motion, each Weight being multiplied by its Distance from that Center will give its *Momentum:* Therefore $8 \times 3 (=24) + 5 \times 6 (=30) + 3 \times 1 (=3) + 3 \times 9 (=27)$ make up the Sum of 84 for the *Momentum* or Quantity of Motion on the *Brachium* A C: And $2 \times 2 (=4) + 5 \times 12 (=60) + 10 \times 2 (=20)$ make the like Sum 84 for the *Momentum* on the *Brachium* C B: And consequently these equal *Momenta* acting in contrary Directions must produce an *Æquilibrium* *. If all the Weights on the *Brachium* A C were reduced into one, *viz.* a twenty one Pound Weight hanging at the 4th Division, it would keep *in Æquilibrio* a Weight equal to all the Weights on the *Brachium* C B hanging at a quarter of a Division beyond the 5th, because $4 \times 21 = 5\frac{1}{4} \times 16 = 84$, as has been shewn in the second Lecture *. * 9. * L. 2. 13.

PROBLEM.

25. *Equal or unequal Weights being suspended at the Ends of a Balance of known Length and Weight: How to find the fixed Point or Center of Motion about which the said Weights will be* in Æquilibrio.

EXPERIMENT III. *Pl.* 9. *Fig.* 7.

A B is a Balance weighing four Ounces, and of 12 Inches in Length, at whose Ends A and B are suspended Weights of four and eight Ounces. Find out the common Center of Gravity of the said Weights *, which will be directly under the Point K at Numb 4. Let K be made the Center of Motion, and then the Center of Gravity being at its lowest Place †, the Problem will be solved, if the Balance A B has no Gravity. But as the Balance weighs four Ounces, the *Brachium* A K will over-weigh the *Brachium* K B, and so destroy the *Æquilibrium.* Now a second Operation like the former, will perfectly solve the Problem, after the following Preparation. Suspend the Weight E equal to the two former Weights or 12 Ounces at the Hook under K, at the common Center of Gravity of the two Weights; * L. 2. 39. † L. 2. 26.

Lect. III. which reducing their Weights into the Center of Gravity, they will act as before *. Reduce also the Weight of the Balance into its Center of Gravity, by hanging the Weight D of four Ounces (or equal to the Weight of the Balance) at *x* under 6 the Center of Magnitude, which is also the Center of Gravity, because the Balance is regular and homogeneous. Then shall we have the short Balance 6 4 or *x* K without Weight, and the Point C or true Center of Motion will be found by this Analogy, E + D (16 Ounces) : D (4) :: *x* K (or a Length of two Inches) : K C (or half an Inch) This Point will be directly over the Center of Gravity of the Balance and all the Weights; and taking away the Weights D and E, the *Æquilibrium* will remain; the Alteration of the Center of Motion from K to C making Amends for the unequal Weight of the *Brachia* of the Balance.

* L. 2. 34.

PROBLEM.

26. *A given* Weight *hanging at one of the Ends of a* Balance *of known Weight, to find the fixed* Point *about which the* Balance *and* Weight *will be* in Æquilibrio.

Experiment IV. *Pl. 9. Fig. 9*

Having suspended the given Weight D equal to four Pounds (for Example) at the End A of the Balance A B, which weighs also four Pounds; since the Weight of the Balance is considered in this Operation, we must suppose the whole Weight of the Balance reduced into its Center of Gravity * as if the Weight E of four Pounds hung at C the Center of Gravity of the Balance, which is at its middle Division 6: then we shall have a new Balance (*viz.* A C) without Weight, at whose Ends hang the Weights D, and E, whose fixed Point will (by the last Prop.) be found at 3; or more generally by this Analogy:

As D + E (or the Weight of the Body and the Weight of the Balance):
Is to E (the Weight of the Balance) : :
So is C A (half the Length of the Balance):
To A 3 (the Distance of the fixed Point from the given heavy Body.)

PRO-

PROBLEM.

27. *To make a deceitful* Balance *or* Pair of Scales, *whose Beam will hang* in Æquilibrio *without the* Scales, *or with the* empty Scales; *and yet shall also be* in Æquilibrio *when unequal Weights are placed in the Scales; so as to* cheat in any Proportion *intended in making the Balance at first.*

THIS Problem is solved by making a *Statera* or *Steel-yard* with the Appearance of a common Beam, as in the following

EXPERIMENT V. *Pl. 9. Fig. 10.*

To the Beam A B (23 Inches long, whose *Brachium* C B, of 11 Inches in Length, keeps *in Æquilibrio* about the Point C, the *Brachium* C A, of 12 Inches in Length, by being made so much thicker, or having so much more Matter, as may make Amends for its being shorter) suspend the Scales D, E, in such Manner that D, which weighs one Part in twelve less than E, shall hang at the longest End of the Beam, and they will keep each other *in Æquilibrio* *. Then placing 12 Pound Weight at G in the Scale E, it will keep *in Æquilibrio* no more than 11 Pounds of F, the Commodity to be sold if plac'd in the Scale D; because then F will be to G, in a reciprocal Proportion of B C to A C *. * 9. 13. * 9. 13.

THO' such a Balance may be so nicely made as to deceive the Eye; the Cheat is immediately discovered by changing the Weights, and the Commodity F, from one Scale to another; for then the Owner of the Balance must either confess the Fraud, or add to the Commodity he sells, not only what was wanting, but also as much as he intended to cheat him of, and a Fraction of that added Weight proportional to the Inequality of the *Brachia* of the Balance. That is, in this Case, the Buyer instead of 11 Pounds offered him for 12 his Due, will have (by changing the Scales $13\frac{1}{11}$ Pounds. For whereas in the first Position of the Balance F (11) × A C (12 was equal to G (12) × B C (11), when G or 12 Pounds is placed in the Scale D, then 12 × 12 will be equal to no less than C B (11) × $13\frac{1}{11}$ G; or,

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As the *Brachium* C B, 11 Inches long:
Is to the *Brachium* C A, 12 Inches long::
So will be F, or the Weight 12, placed in the Scale D:
To G = $13\frac{1}{11}$, or the Weight of the Commodity keeping the Weights *in Æquilibrio*.

AND therefore as this Analogy gives a reciprocal Proportion between the Weights and their Velocities, the *Momenta* will be equal, which with contrary Directions destroy one another.

N. B. *In all these Cases we suppose the Weight to hang freely from those Ends of the Balance to which they are fastened.* See the other Cases in the Notes *. Ann. 5.

Of the LEAVER.

28. THE *Leaver* (a known Instrument commonly called a *Hand-spike*, when of Wood, and a *Crow*, when of Iron) is, in Theory, to be looked upon as an inflexible Line like the Beam of a Balance, and subject to the same Proportions, only that the Power applied to it is commonly an *animate* Power; and from the different ways of applying it, it is called a Leaver of the *first*, of the *second*, or of the *third Kind*.

EXPERIMENT VI. *Pl.* 9. *Fig.* 11.

29. LET the Steel-yard P W be taken off its Hook K, and let its Center of Gravity C be placed upon a *Fulcrum* or triangular Prism D E; and instead of the Weight 1 hanging at P to keep *in Æquilibrio* the Weight W of 4 Pounds, let an animate Power, as the Hand, be applied at P: The *Statera* or *Steel-yard* will then be turned into a *Leaver of the first Kind*, so called because the *Fulcrum* or fixed Point is between the Ends as at C, in which Case the Power may be four times less in Intensity than the Weight; but equal to it, if C or the *Fulcrum* was removed to M (the middle Point of P W) and four times greater, if C was removed to 3. N. B. *In all these Cases the Leaver is still said to be of the* first Kind *. * Ann. 6.

30. WHEN the *Fulcrum* is at one End, the Power at the other, and the Weight between them, the Leaver is of the *second Kind* *. * Plate 9. Fig. 12.

31. BUT

31. BUT it is of the *third Kind*, when the fixed Point is at one End, the Weight at the other, and the Power between them *. Lect. III.

N. B. *The Power and Weight are always supposed to act at right Angles with the Leaver, except it be otherwise expressed; for then the Cases will vary, as may be seen in the Notes † upon the Balance, which are equally applicable to the Leaver.*

* Plate 9. Fig. 13.

† Ann. 5.

THE Proportions, which the Powers and Weights bear reciprocally to their Distances, are set down at the 11th, 12th and 13th Figures.

EXPERIMENT VII. *Pl* 9. *Fig.* 14.

32. PLACE the three Leavers, A, B, D, in such Manner upon the *Fulcra* F, F, F, that A may have the Proportion of its *Brachia* as 5 to 1, B as 4 to 1, and D as 6 to 1; and let these Leavers act upon each other: then making use of a Power equal to one Pound at the End M of the Leaver D, it will keep *in Æquilibrio* W, or 120 Pounds at the End 1 of the Leaver A.

IN this compound Leaver, the *Proportion* or *Ratio* of the Weight W to the Power M is compounded of the several *Ratios* of the long *Brachium* of each Leaver to its short one; for $5 \times 4 \times 6 = 120$. And accordingly you will find by measuring the Ascent and Descent of the Ends of the first and last Leaver, that whilst the Weight W descends $\frac{1}{10}$ of an Inch, the Power or little Weight M will ascend $\frac{120}{10}$ or 12 Inches; the Force to be gained by such a compound Leaver being shewn by the reciprocal Proportion between the Masses and Velocities of W and M.

WHEN two Powers applied to the Ends of a Leaver, support a Weight resting upon a Leaver, they are to one another reciprocally as their Distances from the Weight. The Proportion is marked in the Figure *.

* Plate 9. Fig 15.

EXPERIMENT VIII. *Pl.* 9. *Fig.* 16.

33. IN the Frame A B C D hang the Leaver E F at the Points I, K, loaded with the Weight 7 at C; and the Weights whose Ropes go over the Pullies G H, being to one another reciprocally as the Distances I C, and C K, will keep *in Æquilibrio* the seven

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Lect. III. seven Pound Weight. N. B. *The Weight of the Leaver is not considered here; for unless the whole* Brachia E C *and* C F *be to one another as* 3 *to* 4, *the Center of Gravity* (*or Point* C) *must be removed a little farther towards* I, *as was said before concerning the Balance* *.

* 25.

It is upon this Principle that Horses of unequal Strength may draw equally in a Coach: For if the Spring-tree Bar be unequally divided, that Horse must employ more Strength which is applied to the short End of the said Bar. Two Men also who carry a Barrel hanging from a Staff are unequally pressed upon their Shoulders, if the Barrel does not hang in the Middle, the Man carrying most who is the nearest. This is farther illustrated by

Experiment IX. *Pl.* 9 *Fig.* 17.

34. The Foot F supports an horizontal Board A B, on which must be laid the Leaver 1, 2, divided in the Proportion of 2 to 1, (supposing it a Spring-tree Bar;) then placing the Pulley *n* over-against 2, and the Pulley *m* over-against 1, let the Weights O (1 lb) and M (2 lb) hang by Strings over the said Pullies, and they will keep *in Æquilibrio* the Weight N = 3 lb, drawing the Leaver in a contrary Direction over the Pulley *o* against C. N. B. *The Pullies slide in a Groove in the Edge of the Board, and remain in any Place where they are set.*

Experiment X. *Pl.* 9. *Fig.* 18.

35. If there be a Leaver whose *Brachia* make an Angle, as is represented in the Figure, with its fixed Point at the Angle as C; the Weight W, pressing perpendicularly upon the End W, will be kept *in æquilibrio* by a Weight of one Pound drawing the other End of the Leaver P perpendicularly (by means of a Pulley over which the Line of Direction of the Power is carried) by two Pounds at *p*, and three Pounds at *π*. It is in this manner that a Hammer is made use of to draw a Nail. Some call this a Leaver of the fourth Kind; but it is evidently a Leaver of the first Kind, because the Weight W is at one End, the Power P at the other, and the Center of Motion C between: And if the Arm C W be set straight in a Line with P C, so as to bring W to *w*, and *c* be made the

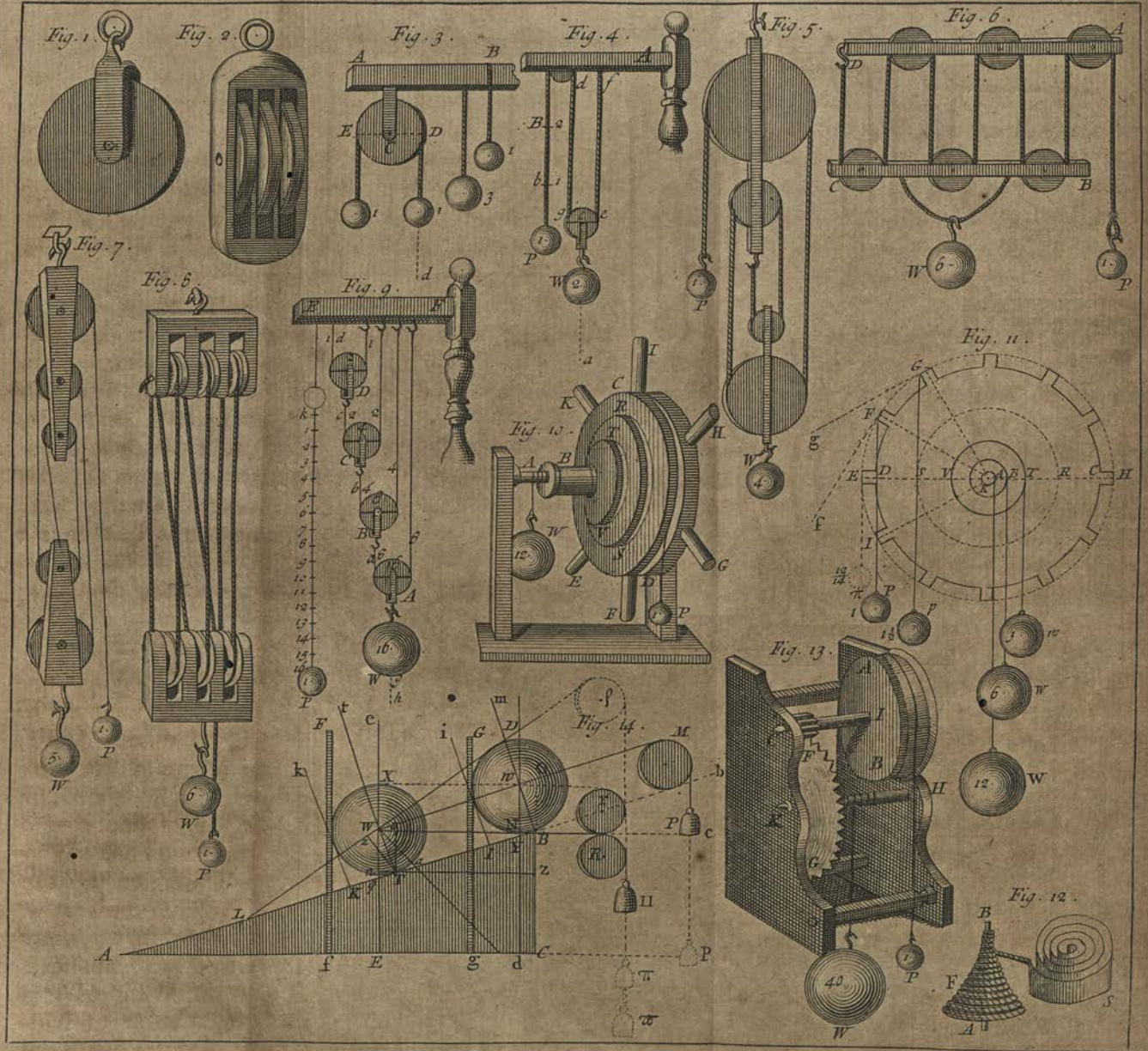
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.
Fig. 14.

Fulcrum, the Inſtrument will plainly appear to be a Leaver of the firſt Kind *. Lect. III.

* Ann. 7.

Of the PULLEY.

36. WHEN a little Wheel, commonly a *Sheeve* or *Sheever*, is ſo fixed in a Box or Block as to be moveable round a Center-Pin paſſing through it, ſuch an Inſtrument is called a Pulley *: And ſometimes, though improperly, a Box or Block with ſeveral Sheevers in it is alſo called a Pulley, as in the ſecond Figure †. The firſt of theſe is by Workmen called a *Snatch-block*.

* Pl. 10. Fig 1.

† Pl. 10. Fig. 2.

A Rope going round one or more Pullies to raiſe a Weight, is called the *Running-rope*.

WHEN a Block with its Sheevers is ſo fixed, that whilſt it remains immoveable, another Block and Sheevers riſes with the Weight hanging at it, ſuch a Machine is called a *Pair of Blocks*.

37. AN upper or fixed Pulley adds no Force to the Power, but only prevents the Friction by making the Rope run eaſily; and ſo much the more as the Sheever is bigger than the Center-Pin about which it turns *.

* Ann. 8.

EXPERIMENT XI. *Plate* 10. *Fig.* 3.

HAVING faſtened to the Ends of a pliable Rope the Weights 1 and 3, the 1ſt of one, and the other of three Pounds, if the Rope be thrown over the ſquare but ſmooth Beam A B, the Friction of the Rope on the Beam will be ſo great as to hinder the three Pound Weight from raiſing the one Pound, though its *Momentum* (without that Hindrance) is three times greater, becauſe its Velocity is the ſame and its Matter triple *. But if only one Pound be faſtened at the End of the ſaid Rope, and the Rope thrown over the Pulley E D, the ſingle Pounds will keep each other ſo exactly *in Æquilibrio*, that the leaſt Weight added to either of them will make it over-power. That theſe Weights ought to be *in Æquilibrio*, appears from a Sight of the Figure, ſince the Weight on the right Hand cannot deſcend to *d* without cauſing the Weight on the left to aſcend up to E preciſely with the ſame Velocity, 1 *d* being equal to 1 E. N. B. *This Pulley is alſo called a Roller.*

* L. 2, 3.

38. A

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38. A lower Pulley, that is, ſuch a one as is moveable with the Weight, takes off half the Weight; ſo that a Power of half its Intenſity will ſuſtain it.

EXPERIMENT XII. *Pl.* 10. *Fig.* 4.

To an Hook coming from the Center of the Pulley *g e* hang a Weight of 2 lb; then having made faſt the running Rope to the Hook *f* of the Arm A, bring it under the ſaid Pulley *g e* and over the Pulley *d*, and 1 lb at the End of the Rope will ſuſtain the 2 lb hanging from *c* the Center of the Pulley *g e*.

THAT the Power pulling down at 1 acts in the ſame Manner as if it pulled upwards at *d*, is evident, becauſe we have already ſhewn *, that an upper Pulley neither increaſes nor diminiſhes the Action of the Power.

WE may at one View ſee in Pullies how the Force of a Weight is diminiſhed, by conſidering how many Ropes (or Parts of the Rope) are employed to raiſe it, and which divide the Weight as they are applied to the lower Pullies to which the Weight hangs, whilſt the Power only draws by one Rope. For Example, in this Caſe the Ropes *f e* and *d g* ſuſtain the Weight; but *f e* is ſupported by the Hook *f*, whilſt the Power only draws up the Rope *d g*.

39. FROM this we may deduce this general Rule to know the Advantage to be gained by a Pair of Blocks, let their Number of Pullies or Sheevers be what it will, *(viz.) As One is to the Number of the Ropes (or of the Parts of the Rope) applied to the lower Pullies, ſo is the Power to the Weight.*

* Plate 10. Fig. 4, 5, 6, 7, 8.

THUS it is evident, only by a Sight of the Figures *, that one Pound will ſuſtain 4 Pounds, as in *Fig.* 5. Six Pounds as in *Fig.* 6. Five Pounds, as in *Fig.* 7. and ſix Pounds, as in *Fig.* 8. N. B *The Pullies and their Ropes repreſented by the 5th, 7th and 8th Figures, are called* Tackles of Four, Tackles of Five, *and* Tackles of Six.

40. THE Machine repreſented in *Fig.* 6, is the moſt inconvenient for raiſing the Weight 6; but the moſt convenient for bringing together the Ends of two Beams, without Danger of bend-

bending them, as if the Ends A and B were to be brought together gradually. N. B. *This Way of using Pullies is called* lacing. Lect. III.

41. We must observe that the Rule above-mentioned is only applicable to the Cases in which the lower Pullies rise all together in one Block with the Weight; but when they act upon one another, and the Weight is only fastened to the lowest of them, the Force of the Power is very much increased, each Pulley doubling it. As for Example *, a Power whose Intensity is equal to 8 Pound (applied at *a)* will, by means of the lower Pulley A, sustain 16 Pound †: A Power equal to 4 Pound (at *b*) will, by means of a lower Pulley B, sustain the Power of 8 Pound acting at *a:* A third Power equal to 2 Pound (at *c*) will, by means of the Pulley C, sustain the Power of 4 Pound at *b:* A fourth Power of 1 Pound (at *d)* will, by means of the Pulley D, sustain the Power of 2 Pound (at *c*); and this is not altered by having its Rope carried over the upper Pulley or Roller E *. N. B. *What Weight each Pulley and each Rope bears in this System of Pullies is set down in the Figure.* * Pl. 10. Fig 9. † 38. * 37.

EXPERIMENT XIII. *Plate* 10. *Fig.* 9.

A WEIGHT of 16 Pound hanging at the Pulley A, (of the Machine *Fig.* 9. made up of four singly moveable Pullies, a Roller at E, and four Hooks upon the Arm E F) let the little Weight 1, which keeps *in Æquilibrio* the Weight 16, be raised up to *k* 16 Inches high, and the Weight 16 will only descend in the Line *g h* an Inch long. This shews the reciprocal Proportion between the Weights and their Velocities to be applicable to this, as well as all other Cases of Pullies, as may be known by moving the Weight or Power in any Combination of Pullies, and measuring the Spaces which they go thro'. Thus in *Fig.* 4 †. while 2 goes down to *a*, 1 goes up to B, just twice as far, *&c* *. *This Proportion therefore will always produce an* Æquilibrium *in this as well as in all other Mechanical Engines.* † Pl. 10. Fig. 4. * Ann. 9.

N. B. *The Ropes ascending and descending, are always to be supposed parallel, except where it is otherwise expressed* †; *and in every Figure representing Pullies, the Power and Weight are marked with the Letters* P *and* W. † Ann. 10.

Of the A X L E in the Wheel.

42. When a Power by Help of a Rope, or any other Means, is so applied to the Circumference of a Wheel, as to cause the said

Lect. III. Wheel, together with its Axle, to turn round and raise a Weight applied to the Axle in any Manner, such a Machine is called an *Axle in the Wheel*, or *Axis in Peritrochio* *.

* Plate 10. Fig. 10, 11.

SINCE in this Instrument the Wheel and its Axis move together; it is evident, that in one Turn of the Wheel, when the Power * P descends a Length equal to the Circumference of the Wheel, the Weight W rises an Height equal to the Circumference of the Axis A by the winding of the Rope which carries the Weight upon the said Axis. And since, when there is an *Æquilibrium* between two Weights, as W and P, there must be a reciprocal Proportion between their Masses and Velocities; W will be to P, as the Circumference of the Wheel to that of the Axis (supposing the Rope of no Thickness) or as the Semidiameter of the Wheel to the Semidiameter of the Axis (that is, as † D K to K A) because the Semidiameters of different Circles are in the same Proportion to one another as the Circumferences.

* Plate 10. Fig. 10.

† Plate 10. Fig. 11.

HENCE it follows, that the less the Axle is in proportion to the Wheel, the greater may the Weight be, which is sustained or raised by the Power.

EXPERIMENT XIV. *Plate* 10. *Fig.* 10.

43. THE Machine represented by the Figure is a Model (made by a Scale of an Inch to a Foot) of such an Axle and Wheel as is often made use of to draw Water out of a Well, by means of a Power drawing by a Rope applied to the Circumference of one of the Wheels of the Machine, or by pressing down successively the Handles E, F, G, H, I, K, whilst another Rope or Chain is wound up upon the Axis A or B, a Bucket hanging at it instead of the Weight W. Here in the Experiment one Pound hanging at the Circumference of the biggest Wheel C D will keep *in Æquilibrio* 12 Pounds, hanging at the smallest Axis A, or 6 Pounds at the Axis B, and only 3 Pounds upon the Circumference T V. In the same manner, when the Weight hanging at the Axis continues in the same Place, and to be of the same Quantity, *viz.* 12 Pounds, then the Power which, at the Circumference of the Wheel C D, is equal to 1 Pound, must be equal to 1 $\frac{1}{2}$ Pound, if it be applied at S R; but if it be applied at any of the Handles at the Distance of $\frac{1}{4}$ of an Inch from the Circumference of the Wheel C D (which is the same as if a new Wheel was added of $\frac{1}{2}$ an Inch more in Diameter) then a

Power

Power equal to no more than $\frac{12}{144}$ of a Pound, will keep the Weight *in Æquilibrio*; and raise it, if its Intensity be increas'd ever so little. Lect. III.

This is more clearly represented in *Fig.* 11. * where the Weights are represented by the Letters W, w, *w*, and the Powers by the Letters P, *p*, *π*, and where you must observe, that unless the Power acts upon the Handles above-mentioned at right Angles, or in the Lines E *π*, F f, or G g, *&c.* the Effect cannot be the same as making use of a new Wheel E, F, G, H, to which the Lines E *π*, F f, and G g are Tangents. * Plate 10. Fig. 11.

44. For if a Power, as for Example P, acts obliquely upon any of the Handles, as in the acute Angle P F K, or its correspondently obtuse Angle P I K, the Line of Direction of the said Power * becomes the Tangent of the Circumference D C; and consequently the Power acts as if it drew by a Rope going round the Wheel C D: And if the Obliquity was greater, as when the Power *p* draws the Handle G in the Angle *p* G K, the Effect will be no greater than if the said Power should draw by a Rope going round the Circumference S R. The Powers in these Cases must be increased in the same Proportion as the Lines D K and S K are shorter than E K. *See the Numbers in the Figure, which express the* Intensities *of the Powers.* * Ann. 5.

N.B. *We have taken no Notice here of the Thickness of the Rope, to which a Regard is to be had in Practice, always adding half the Thickness of the Rope to the Semidiameter of the Axis: And if the Rope is wound up upon itself, for every such Turn we must still add half its Thickness, which is the reason why more Power must be applied when the Axis is thus thickned, as often happens in drawing Water from a deep and narrow Well, over which we cannot have a long Axis.*

45. If the Rope to which the Power is fastened be successively applied to different Wheels, whose Diameters are bigger and bigger, the Axis will continually be turned with more ease, unless the Intensity of the Power be diminished in the same Proportion; and if so, the Axis will always be drawn with the same Strength by a Power continually diminishing. This is practised in Spring-Clocks and Watches, where the spiral Spring S, which is strongest in its Action when first wound up, draws the Fuzee F, or continued Axis *in Peritrochio* by the smallest Wheels near B: and as it unbends and grows weak, draws at the larger Wheels near A, in such manner that the Watch-work is always carried round with the same Force *. * Plate 10. Fig. 12.

Lect. III.

THERE is another very curious Contrivance, whereby a Power, whoſe Intenſity is continually diminiſhing, does yet produce an Effect continually increaſing, which is a ſort of a mechanical Paradox; but as it is in common Uſe, we are apt to overlook it. I mean the Application of the main Spring in a Gun-Lock, to drive the Cock, which carries the Flint againſt the Hammer (or Steel-Plate, to ſtrike Fire) with an accelerated Motion, though the Spring is all the while unbending itſelf. F S P * is the Spring bent up to the higheſt Degree, when the Gun is cocked; but its natural Situation when wholly unbent, is F S *p*. A C T is the Tumbler, whoſe Axis of Motion is at C, which is the Middle of an Axle or Arbor, going ...gh the Plate of the Lock, on the End of which the Cock (being the Weight to be moved by the Power) is fixed upon a Square. Now, the Tumbler is in effect an *Axis in Peritrochio*, with ſeveral Wheels, where the Power is ſucceſſively applied from the leaſt to the greateſt, as in the Fuzee of a Watch. I have repreſented only three of them, according to the three moſt equal Poſitions of the Spring, by the pointed Circles $\alpha\beta$, ab, and AB. Now, though the greateſt part of theſe Wheels is cut away when the Tumbler is reduced to its proper Figure A a $\alpha\beta$ T t, there is enough left of each of them for the Hook of the Spring P to apply itſelf ſucceſſively to them all, as it preſſes on the Arm A α, with a ſliding Motion from α to A, which is the whole Range of the Spring in its Action. When the Gun is cocked, as in this *Figure*, the End of the Spring being applied at α, acts upon the Tumbler at the Diſtance α C, by the Wheel $\alpha\beta$, ſo as to preſs upon it at the greateſt Diſadvantage, at the ſame time, that the Spring is moſt bent, or has the greateſt Intenſity. This is felt by applying the Hand to the Cock, which then makes the leaſt Reſiſtance.

* Plate 22. Fig. 7.

IN the 8th *Figure* *, the Gun being at Half-Cock, the Spring acts upon the Tumbler with more Advantage, preſſing in the Tangent of the Circle, or Wheel ab, at the Diſtance aC, which gives the Cock more Force to deſcend, as may be felt by the Hand: For though the Spring is a little unbent, and ſo ſomewhat weaker; the Diminution of Intenſity in the Spring, is not ſo great as the mechanical Advantage gained by the Diſtance being increaſed from αC to aC.

* Plate 22. Fig. 8.

IN

In the 9th *Figure*, the Spring is still more unbent; but yet the Cock is pressed down with a much greater Force, because the End of the Spring P now acting in the Tangent of the Circle A B, has A C for its Distance of Power, about 3 times greater than α C (the Distance of Power when the Gun is cocked) whereas it is not unbent above $\frac{1}{3}$, as may be seen by comparing the 7th *Figure* with the 9th.

46. As Leavers and Pullies acting upon one another are sometimes joined together to increase the Action of the Power, whereby it may sustain or lift a greater Weight; so it is usual to make a compound Axis *in Peritrochio* by combining together two or more of these Machines. Because though by means of a long Axle capable of receiving a great deal of Rope, one may draw Weights from a great Depth; yet, as a very small Axis would be too weak for very great Weights, or a large Wheel would be very costly, if sufficiently strong, or make a cumbersome Engine which would take up too much room; it is more adviseable to combine Wheels and Axles, by means of Pinions or small Wheels upon the Axles, whose Leaves (or Teeth) take hold of Teeth made in the large Wheels, as we see in Clocks which have several Axles and Wheels, or some sort of Cranes which have only two of them combined together *. * 82, 83.

Experiment XV. *Plate* 10. *Fig.* 13.

47. This Machine consists of two Wheels with their Axles, the first of which A B C (on whose Circumference A B is coiled the Rope which carries the Power P, which is a Weight of one Pound) has a Pinion of eight Teeth on its Axis at C, which takes the Teeth of the Wheel F G of the other Axis *in Peritrochio*. The Wheel F G has forty Teeth, and its Axis H K is in Diameter the eighth Part of the Wheel A B. Hang the Weight W of 40 Pound upon the Axis H K, and it will be kept *in Æquilibrio* by the Power P, which is equal but to 1 Pound.

If we suppose the Axis C I of the same Diameter as the Axis K H, it is evident that the Power P would have sustained but 8 Pound hanging on the said Axis; nor would it have sustained more

Lect. III. more on the Axis K H if this laſt Axis had gone round as often as the firſt Wheel A B (which would have happened if the Wheel F G had had no more Teeth than the Pinion C) but ſince its Wheel has five times more Teeth than the Pinion on the Axis C I, it muſt go five times ſlower than the ſaid Axis; and conſequently the Weight W goes up five times ſlower than it would have done on the Axis C I; therefore it has forty times leſs Velocity than the Power P, riſing only one Inch whilſt P deſcends forty.

Plate 10. Fig. 13.

HENCE it follows that the Ratio of the Power to the Weight is compounded of the Ratio of the Diameter of the Axis of the laſt Wheel (where the Weight hangs) to the Diameter of the firſt Wheel (where the Power is applied) and the Ratio of the Number of Revolutions of the laſt Wheel, to the Number of Revolutions of the firſt Wheel in the ſame time. As for example, here the firſt Ratio is of 1 to 8, and the laſt Ratio is of 1 to 5; therefore the Ratio of the Power to the Weight is as 1 to 40, *viz.* the Compoſition or Multiplication of thoſe two Ratios, becauſe $5 \times 8 = 40$. *And this holds good, let the Machine conſiſt of ever ſo many Wheels.*

Of the Inclined P L A N E.

48. FOR the better underſtanding the Uſe of the inclined Plane in Mechanicks, we muſt remember what has been ſaid before concerning the Velocity of a Weight *, *viz.* that *whatever Line the Weight deſcribes in its Aſcent by the Action of the Power, we are to call its* Velocity only that Line which repreſents its *perpendicular Aſcent or Deſcent.*

* Ann. 8. Lect. 2.

IF it ſhould be required to lift up a very heavy Body as W or *w* *, the Height C B, it would be impracticable to raiſe it up in the Line C B without a Power whoſe Intenſity is equal to that of the Weight; and even in that Caſe very inconvenient to do it, eſpecially in building. But if an inclined Plane A B, be laid riſing from the horizontal Line A C, from whence the Weight is to be raiſed, a leſs Power than the Weight will ſerve for that Purpoſe, unleſs it puſhes the Body directly againſt the Plane, (as in the Direction W T) or draws the Body away from that Plane (as from W towards *e*, *t*, or L, or) in any Direction on that Side of the Line E *e* *.

* Plate 10. Fig. 14.

* Ann. 11.

49. THE

49. THE Direction in which the Body can moſt eaſily be drawn or puſhed up the Plane (as in driving a Wheel-barrow) is in the Line W *w* M, parallel to the Plane and paſſing through the Center of the Weight; for whether the Power drives a Plane *k* K (in a Direction perpendicular to it) along the Line W M, or the Power P (by its Deſcent to *p*) draws it in the ſame Line, the Velocity of the Power will be equal to the Line W *w*, the Space deſcribed by the Center of Gravity of the Weight, whilſt the ſaid Weight riſes only the perpendicular Height Z B (= *n* W) or has the ſaid Line properly to expreſs its Velocity. If the Body was a Cylinder, as a Rolling-Stone, and the Plane T *t* were to paſs through the Gudgeons or Axis of the ſaid Stone; it is evident that the Caſe would be the ſame; and as the Weight P has its Rope running over the Roller (or upper Pulley) M, the Line P *p* will be the Velocity of the Power. Therefore in this Caſe the Weight (if kept *in Æquilibrio*) will be to the Power as W *w* (=T B) to *w* Y (= B Z) or as the Hypotenuſe A B is to the Perpendicular B C, which (by *Eucl.* 4. 6.) are in the ſame Proportion, and conſequently if the Power be ever ſo little increaſed, it will draw the Weight up the Plane. Plate 10. Fig. 14.

N. B. *In Practice the Power muſt be pretty much increaſed, if the Body is not ſmooth and ſpherical or cylindrick, and the Plane very ſmooth. But as all Sorts of Bodies are to be drawn up to reduce them as near as we can to a Sphere or Cylinder, we muſt faſten Wheels to them, or (which is the ſame thing) lay them on a Wheel-Carriage.*

50. THAT the Power acts with the greateſt Advantage, whilſt it draws in the Line of Direction W *w* (parallel to the Plane) is evident; becauſe if one End of the ſaid Line of Direction remaining fixed at W, the other ſhould move towards B, or beyond it, then the Body would be partly drawn againſt the Plane, and therefore the Power muſt be increaſed in proportion to the greater Difficulty of Traction: And if the End *w* of the Line abovementioned ſhould be carried to D, or beyond it, the Power muſt be alſo increaſed, in as much as it endeavours to lift the Body off from the Plane. *How much the Power muſt be increaſed in proportion to the Angle, which its Line of Direction makes with the Plane, will be ſhewn in the Notes* *. * Ann. 12.

51. If

Lect. III.

51. If the Power draws in a Line of Direction W B *, parallel to the Base of the Plane; then in order to keep the Weight W *in Æquilibrio* by the Power Π, the said Power must be to the Weight, as Z B to Z T, or as the Perpendicular B C to the Base A C of the Triangle A C B. For if we suppose the Pulley R at so great a Distance from W, that the Line of Direction W R may not sensibly alter its horizontal Position, then will the Power Π descend to ϖ whilst W rises the Height B Z, in such manner that Π ϖ (= W Y, and not W w) will be the Velocity of the Power. So that the Velocity of the Power to that of the Weight will not be as the Hypotenuse to the Perpendicular, as in the former Case, but as the Base to the Perpendicular in the Triangle A C B.

* Plate 10. Fig. 14.

If the Power be increased just enough to overcome the Friction of the Plane and draw up the Body W, let the Pulley R be lifted up gradually to r, so as to keep the Line W R parallel to itself till it comes to w r, and the Power will be descended to π when the Weight is come to w B. But Π π together with the Distance R r, is equal to Π ϖ, or W Y, *&c.* And this Traction being constantly made in the Angle W B T, is the Case.

Of the W E D G E.

52. The Wedge is a short triangular Prism, whose two opposite and parallel Planes are rectangular Triangles, such as the Section A B C *, the rest being rectangular Parellograms; the Edge or entring Part of the Wedge is made by the meeting of two Planes in whose Section is the Point A, and the Back of it is the Plane opposite to the Edge, on which the Hammer or Mallet strikes to drive the Wedge forwards. The Representation of it is seen in the Figure A B C D E *.

* Plate 10. Fig. 14.

* Plate 11. Fig. 1.

If upon the horizontal Line A C † produced towards p, be laid the Weight w at the Point d, and a Plane as G g to stop it from going towards A, whilst the Wedge A B C is pushed under it from d towards A; as the Wedge is driven from C to A, the whole Length of its Base A C, the Weight w rises just the

† Plate 10. Fig. 14.

Height CB, or the Thickness of the Wedge: Therefore the Power will be to the Weight, as B C to A C. Lect. III.

53. THE Proportion of Force used would be just the same, if a Plane as F *f* were to move parallel to itself, and perpendicular to A C, and push up the Weight, as W, from A to B along the Wedge supposed then immoveable. Nay, it would be the same, if the Weight was only pushed from one Part of the Wedge to another, the Plane moving but from F *f* to G *g*, or (which is the same) from E *e* to D *d*; for then *w* Y would express the Rise of the Center of Gravity (or the Velocity) of the Weight, and W Y the Velocity of the Power, which are still in the Ratio of B C to C A.

THIS is the second Case we mentioned concerning the inclined Plane, and may be confirmed by the following

EXPERIMENT XVI. *Plate* II. *Fig.* 2.

54. TAKE the Machine described in the Figure *, in which the smooth Plane or Board B A H I, moveable on Hinges at B and I, may be raised so as to make an Angle with the horizontal Board N L B I, by help of the Quadrant Q, and fixed in its Position by means of the Screw at T; and having made fast the Head D E in the Groove S *s* by a Nut under the horizontal Board, raise the Pulley G of the Arm D G to C, so that a Line going over the said Pulley in the Direction *c* M, may be parallel to the Plane A B. Then take a little wooden Cylinder M, the Ends of whose Axis or Pivots go through a brass Frame as at O, so that it may be drawn easily by a String fixed at M; and lay on the said Cylinder upon the inclined Plane, having fastened to M a String which goes over the Pulley *c*, and is joined to the Ball P, which being made use of as a Power, will sustain the Cylindric Weight M, when P is to M as A C the Height of the Plane, to A B its Length. *A little more Weight added to* P *makes it draw up the Cylinder.* * Plate II. Fig. 2.

BUT if the Arm E *c* be lowered, so as to come into the Position E G, *p* must be to M (in order to keep it *in Æquilibrio*) as A C the Height of the Plane, to C B its Base. N. B. *Here it is to be observed, that though the Addition of a little Weight to* p *will make it begin to draw the Cylinder* M *up the Plane; that the Cylinder*

Lect. III. *will not be drawn quite up by the Deſcent of* p, *becauſe the Angle which the Line of Direction of the Power makes with the Plane will be increaſed as the Cylinder riſes up; but if the Pulley* G *be gradually lifted up to* K, *whilſt the Cylinder is drawn up to* m, *the Power, which then will be at* π, *will have the ſame Effect on the Weight at* m, *as it had before at* M, *&c.* and the Weight (when kept *in Æquilibrio* in the Line of *Direction* M G *or* m K) *will ſtill be to the Power as* M n *to* n m, *or* B C *to* C A.

THIS Machine alſo experimentally ſhews the Effect of the Power, whatever Angle its Line of Direction makes with the Plane.

55. THE Wedge, which we have hitherto conſidered, is a Wedge acting in the moſt ſimple manner, only with one of its Surfaces; for when it ſlides upon the Plane C A * to lift the Weight W, it does only act with its Surface A B, the Surface A C only applying itſelf to the Line A *p*, without removing it from its Place. Thus if a Moulding was to be ſeparated from a Wainſcot, as M *m* * from W *w* by means of the Wedge A C B, it is plain that *a* A the Velocity of the Wedge (ſo far as it is driven in, when from the Poſition *a b c* it comes into the Poſition A B C) would be to *a m* the Velocity of the Moulding as A C to B C. Therefore, *&c.* So it would be if a Pillar ſtanding upon a Floor was to be raiſed up without moving the Floor.

* Plate 10. Fig. 14.

* Plate 11. Fig. 3.

56. BUT in the common way of uſing the Wedge, both Sides of it act, as in cleaving Wood: Then the Proportion of the Power and Weight are different from the former; for then the Power will be to the Weight as half the Wedge's Thickneſs to its Length. But then we may eaſily reduce this to what has been ſaid before; becauſe here we make uſe of a double Wedge. For let us ſuppoſe C *a* * an immoveable Plane, and on each Side of it a Wedge, as B C A and *b* C A, made uſe of to remove a Weight as *c* or *d* from the ſaid Plane; the Power puſhing the Wedge will be to the Weight but as *c* C (= A *c* = B C) or *d* D (= A D = C *b*) to the Length of the double Wedge C B A *b*, which is now got to A *c a d*. Some mechanical Writers have miſtaken this Caſe by adding together the ſingle Velocities of *c* and *d*, *viz.* C *c* + D *d*, and calling that Sum the Velocity of the Weight, which they compared with A *a* the Velocity of the Power; but we muſt conſider,

* Plate 11. Fig. 4.

sider, that if the two Bodies to be removed from each other (as the Parts of the Wood to be cloven) were laid upon one another on either Side, as for Example at *d*, their Velocity would only be *d* D, and they would be removed as easily then by one Wedge as *b* C A, from *d* to D, as by the two Wedges in the former Case. *For though in considering the* Momentum *of Bodies, the Sum of the* Momenta *of all the Parts is to be taken for the* Momentum *of the Whole; we are not to take the Sum of the Velocity of the Parts for the Velocity of the Whole, but only the Velocity of the Center of Gravity of* the Body, when we consider *how far it is removed out of its Place. If a Leaver or Balance was made like a Fork*, and the Part* C A *was just double of either of the Parts of the Fork as* C B *or* C D, *and the Center of Motion fixed at* C; *it is evident that one Pound at* A *will keep* in Æquilibrio *two Pounds at the forked End, though the Pounds were to hang single, a two Pound Weight being always equivalent to two single Pounds: but if the Sum of the Velocities of* B *and* D *were to be taken, there must then be two Pounds at A to sustain the Weights at* B *and* D, *the Radius* C A *giving no more Velocity to the Weight at* A, *than the Radius* C B *or* C D *of half its Length, to the Weights at its Ends, which is absurd,* &c. Lect. III.

* Plate 11. Fig. 5.

57. In both Cases of the Wedge, it is not only necessary to apply a Force something greater than either in proportion of the half Thickness or of the whole Thickness of the Wedge to its Length, that the Power may overcome the Weight; but because the Surfaces even of the most polished Wedge are very rough (in comparison to the mathematical Smoothness which we have supposed) and the Bodies to be separated likewise very far from having their Surfaces truly plane; we must make use of an additional Force, to overcome the Stickage arising from that Roughness.

This Friction or Stickage, which is not great in other Engines, is very considerable in the Wedge, Experience shewing that a Wedge laden with a vast Weight, has hardly any Effect especially in cleaving Wood; because not only the Surfaces of the Wedge, as we said before, but the Parts of the Wood to be cloven are always rough, and so close, that their Friction very much hinders the Motion; which Obstacle we endeavour to remove by Percussion, which here is of wonderful Use: for Experience shews, that a Blow upon the Head of a Wedge, makes it enter easily into a hard Body; the Reason of which seems to be, that a Blow, by

Lect. III. putting all the Parts of the Wood in motion, makes them tremble and be diſunited, ſo as to leſſen the Stickage, and facilitate the Motion of the Wedge. The Effect of Percuſſion will be greater in proportion as the percutient Body is heavier and moves ſwifter.

Plate 11. *Fig.* 6.

58. To prove by Experiments what has been ſaid of the Wedge, we muſt make uſe of the Machine repreſented in the 6th Figure of *Plate* 11.

A B C D is a Braſs Frame, conſiſting of two horizontal Pieces A B and C D, and two vertical ones A D and B C, ſtanding upon and fixed to the former; each of theſe laſt Pieces has on the Inſide about the Middle, two little Pullies N O, P Q not exactly in the ſame Plane, leſt the String that goes over the one, ſhould fall upon the String which goes over the other. E F, G H are two Cylinders with Steel Axes, which are brought together (their Axes rolling on the vertical Pieces) by the Deſcent of the Weights R S, each of which is divided into two Parts by means of its Pulley T or V, ſo as to draw the Cylinders equally towards one another, by the Strings and Braſs Loops T N H, and *t* Y *y*, and two ſuch other Loops at the other End of the Axis of the Cylinders. That the Cylinders may come cloſe together, without touching the Pulleys N, O, P, Q, the Plates at their Baſis (which are a pretty deal larger than the Cylinders) are made convex towards the Ends of the Axis. The two Plates Z M, Z M, juſt wide enough to apply themſelves to the Cylinders, without rubbing againſt the Places at their Ends, are joined at M in the manner of an Hinge, ſo as to make a Wedge, and be opened to any Angle meaſured by the graduated Arc I K L, which paſſing through the Plates, holds them faſt, by means of the two little Screws Z, Z. X is one of two bended Wires, whoſe Ends being ſlipped into two Holes, keep the Cylinder E F from coming out of its Place, and only allow it to turn upon its Axis, when the Loop F (and its oppoſite under E) is carried over the Pulley Y (and the contrary Way) to the Axis of the other Cylinder at *f*, when only this laſt Cylinder H G is to be puſhed away by the Deſcent of the Wedge, which by its own Weight, or the Addition of the Weight W, ſeparates one Cylinder from the other, which is fixed (when the pricked Line repreſents the String) or removes them from each other (when Y *y* repreſents one String, and H N the other.)

Ex-

EXPERIMENT XVII.

59. EVERY thing being in the Position represented in the Figure *, open the Wedge to any Angle at pleasure; as for example, to an Angle of 20 Degrees, and hang on such a Weight W, as may together with the Weight of the Wedge, draw down the Wedge, and by separating the Cylinders, raise the Weights R and S. Open the Wedge to an Angle of 40 Degrees, and twice the Weight will be required to bring down the Wedge; but if you slip on the Wires, such as X, to fix the Cylinder E F, and hang the four Loops and Strings upon the Cylinder H G, the Wedge acting only with one Surface to remove the Cylinder H G, will require twice the Weight to bring it down, that it would if it acted with both Surfaces; that is, being opened to an Angle of 20 Degrees, as much Force will be required to bring it down, when only one Cylinder is moveable (though the other can freely turn about its Axis) as if it was opened to an Angle of 40 Degrees, and both Cylinders moveable. * Plate 11. Fig. 6.

Of the SCREW.

60. A Screw is a Cylinder cut into several concave Surfaces, or rather a Channel or Groove made in a Cylinder, by carrying on two spiral Planes the whole Length of the Screw, in such manner that they may be always equally inclined to the Axis of the Cylinder in their whole Progress, and also always inclined to the Base of it in the same Angle.

61. THE Screw may be also considered as a Wedge carried round a Cylinder, which in that Case is called the Arbor of the Screw; the Wedge so carried on making what is called the Thread of the Screw, as may be seen in the 7th, 8th, 9th, 10th, and 11th Figures *: The Arbor of the Screw being A B in *Fig.* 7. and *a c b d* in *Fig.* 8. as if the Cylinder A C B D was inscribed within the Screw. * Plate 11. Fig. 7, 8, 9, 10.

62. IN the 8th Figure we may see the manner how a Screw is made, if it be cut out of the Cylinder P H I Q, then H K L M N O P is a spiral Line going about the Cylinder, marking the prominent Part to be left of the said Cylinder, and *b k l m n o*, the Line

Lect. III. Line marking the Depth to which the Screw is to be cut (ſuppoſing the ſame Line to go round the inner Cylinder or Arbor A B C D *, though not expreſſed here, to avoid Confuſion) and then *b* L *l*, *l* N *n*, &c. will repreſent the prominent Part or Thread of the Screw. Now, if inſtead of cutting the Hollows H *b* L, L *l* N, N *n* P, &c. into the Cylinder P H I Q, a continued Wedge be fixed to a ſmaller Cylinder as A C B D, or rather *a c b d*, the ſame kind of Screw will be made, and *a b c d* will be the Arbor of that Screw. Sometimes the moſt prominent Part of the Thread, as L, N, &c. is not ſharp but flat, and then the Thread is called a ſquare Thread, as in *Fig.* 11. which repreſents the Section of ſuch a Screw. This Sort of Thread is not uſed in Wood, but in Iron and other Metals it is of good Service, being commonly more durable, and raiſing the Weight with more Eaſe than the ſharp Thread, as will be more fully ſhewn in the Notes *.

* Plate 11. Fig. 8.

* Ann. 13.

Of the Force of the S C R E W.

To make an Eſtimate of the Force of the Screw (which may be compared either to an inclined Plane, as we have conſidered it among the mechanical Powers) or to a Wedge according as its Arbor does or does not advance in a progreſſive Motion, whilſt it turns round its Axis to raiſe or ſtop a Weight, or to preſs Bodies together, which are the ſeveral Uſes of a Screw) let us take a flexible Wedge, as for example one of Paper, and coil it round a Cylinder *, as is repreſented in the Figure, where A B is the Arbor, C I D one Thread or Helix, D H E another, and E F G Part of the Wedge left to ſhew the Proportion between the Power that turns the Screw and the Weight W.

* Plate 11. Fig. 7.

63. If the Weight puſhed up the Wedge (or which is the ſame, raiſed perpendicularly by the Wedge ſlipping under it) from F to H in the Direction W *w*, then will H G be the Velocity of the Weight, and G F the Velocity of the Power, which is the Caſe of the inclined Plane becoming a Wedge; and this will be the Analogy for the Screw * thus acting.

* 52.

As a Circle whoſe Diameter is H *b* :
To H I the Diſtance of two Threads : : (or as the Baſe F G : to the Perpendicular H G : :)

So

Plate. 11

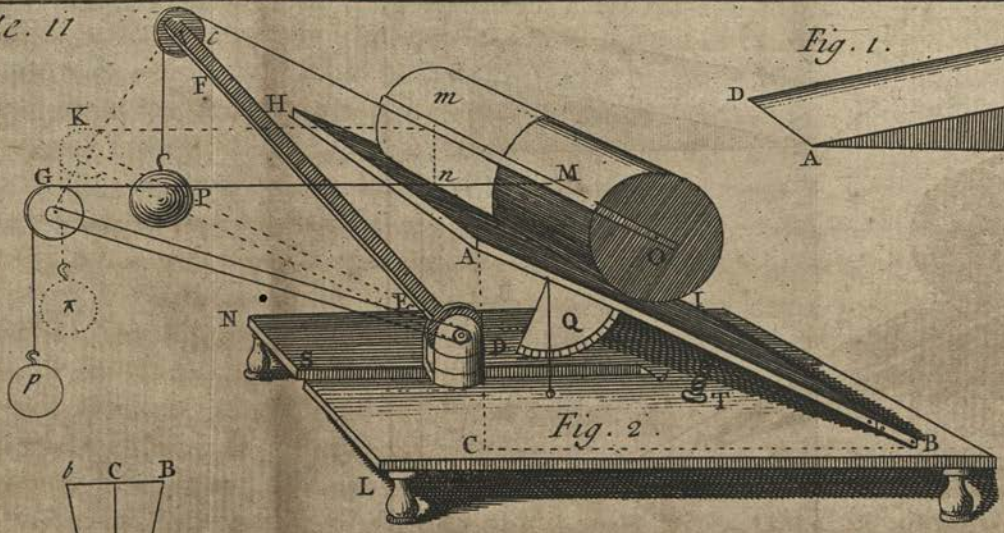

Fig. 2.

Fig. 1.

Fig. 3

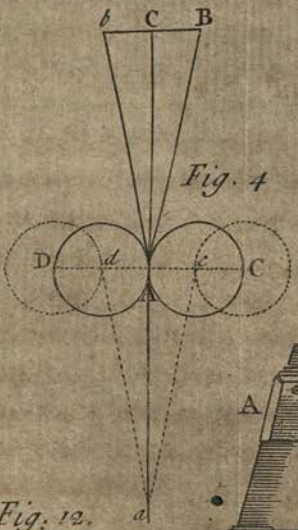

Fig. 4

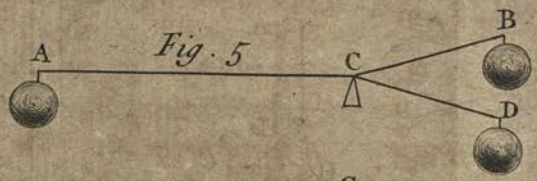

Fig. 5

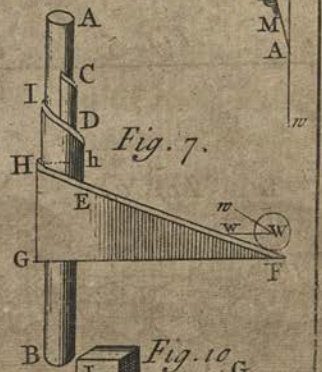

Fig. 7.

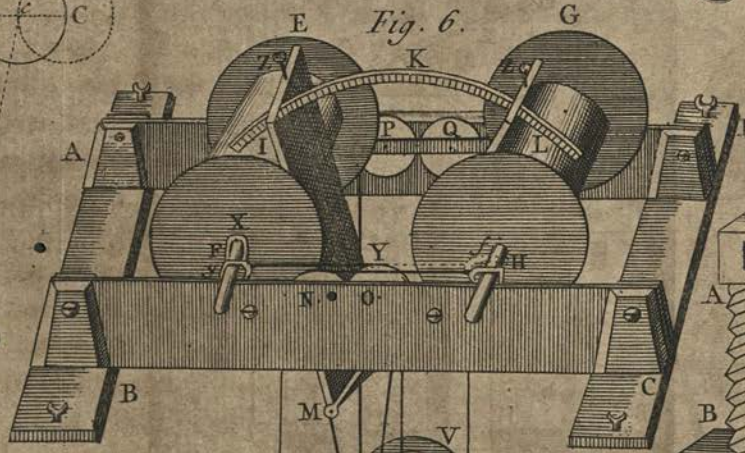

Fig. 6.

Fig. 12.

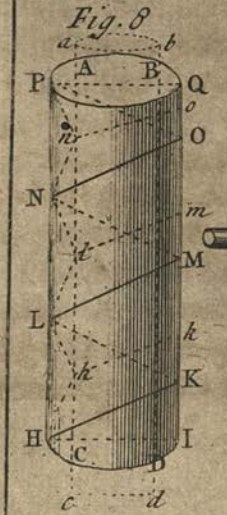

Fig. 8

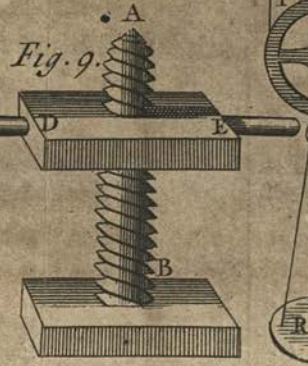

Fig. 9.

Fig. 11

Fig. 14.

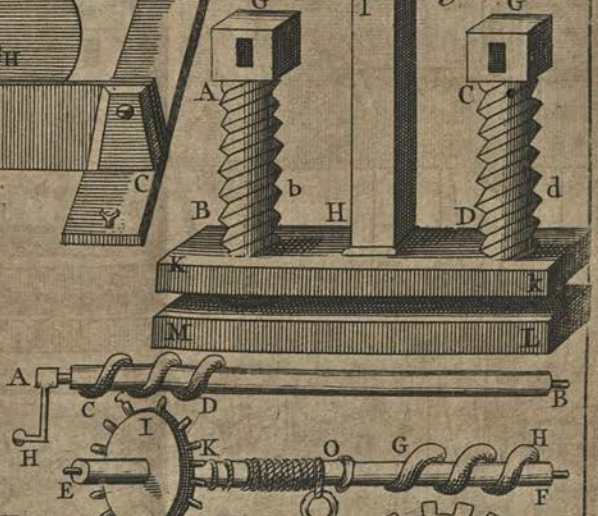

Fig. 10

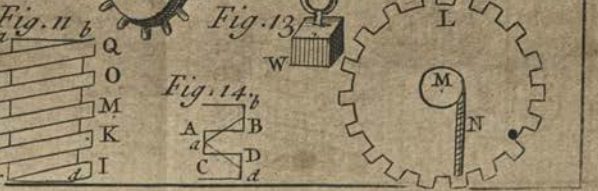

Fig. 13

Lect. III.

So is the Weight :

To the Power applied to the Arbor at A to raiſe a Weight up the Thread H I D C.

N.B. *We ſuppoſe the Diameter of the Arbor at* A, *and of the Screw at* H, *nearly equal.*

THIS is the Caſe of the 10th *Fig.* where the moveable Plank D K is carried down by turning round the Heads G, G, of the Screws A B and C D, in order to preſs ſtrongly the Bodies placed between the Planks D K and M L, whilſt the Piece H I fixed on the upper Plank is either guided through an Hole, or being only looked at, ſerves to ſhew whether the Plank K D be brought down horizontally as the Screws are turned. When long Leavers are thruſt into the ſquare Holes at the Heads of the Screws, the Force of the Screw is much increaſed, and then the Weight : will be to the Power : : as the Circumference of the Circle deſcribed by that Part of the Leaver to which the Hand is applied : to the Diſtance between two Threads. So in *Fig.* 13. as the Circumference of the Circle whoſe Radius is A H : to C *c* the Diſtance of two Threads of the endleſs Screw C D : : ſo is the Reſiſtance of the Teeth of the Wheel I : to the Power applied at H. Plate 11. Fig. 10. Plate 11. Fig. 13.

64. BUT if the Weight W * be driven along on the Wedge H F G in the Direction W *w* parallel to the Surface of the Wedge, then will this Caſe be the ſame as that of the inclined Plane †; and therefore the Analogy for the Screw in the Progreſſion of it will be, * Plate 11. Fig. 7. † 49.

As the Spiral Line H D I :

To H I the Diſtance of two Threads : : (or as the Hypothenuſe F H : to the Perpendicular H G) :

So is the Weight :

To the Power applied to the Arbor at A in a ſpiral Direction parallel to H D I C. * Plate 11. Fig. 7.

THE 9th Figure repreſents the Practice of this, whether the Female Screw D E be brought down on the Male Screw towards Plate 11. Fig. 9, 10, 12.

Lect. III. wards the Plank B, or the Plank B and its Screw be brought up towards D E by ſcrewing its Screw up higher into the Box or Nut D E. Therefore when Handles as D and E are uſed to increaſe the Force, the Weight : will be to the Power : : as the ſpiral Circumference deſcribed in one Revolution by their Ends to which the Power is applied : to the Diſtance between two Threads of the Screw. The Thread of the Female Screw within the Nut or Piece of Timber with the Handles, is repreſented by the pricked Lines in *Fig.* 12. compared with *Fig.* 8.

THIS Caſe is applicable in Practice to the Screw at the End of Gimblets, to facilitate the boring of Wood; to the bringing down the Nuts in large cold Preſſes, and to the common Uſe of Screw-Pins for holding together the Parts of Machines.

THE greateſt Part of mechanical Writers have only taken notice of this laſt Proportion of the Force of the Screw.

65. THOUGH in the Theory, if the Product of the Intenſity of the Power into its Velocity does ever ſo little exceed the Product of the Weight into its Velocity, the Power muſt raiſe the Weight; yet here the Power, or its Velocity, muſt be ſenſibly increaſed to produce this Effect in Practice, by reaſon of the great Friction in the Screw, which is the ſame that we have taken notice of in the Wedge; only we are to obſerve, that there is more Friction in the ſharp than the ſquare Thread, which may appear by a Sight of the 14th Figure *, where B A C D repreſents the Section of a ſquare Thread, and *b a d* the Section of a ſharp Thread. As the Part A C does not touch the Female Screw, which only bears with its Force and Weight upon the Flat A B, the Line A B in the ſquare Thread compared with *a b* in the ſharp Thread (on which Part the Female Screw bears) will ſhew the Friction to be leſs on the ſquare Thread in proportion to thoſe Lines, beſides the Obliquity to be conſidered elſewhere *.

* Plate 11. Fig. 14.

* Ann. 13.

66. THE Diſadvantage to the Force of the Power occaſioned by the great Friction in the Screw is fully recompenſed in the Uſe of that Machine; becauſe, on that account, the Screw continues to ſuſtain the Weight, even after the Power is removed, or ceaſes to act, or to preſs the Bodies againſt which the Power had driven it; whereas in the Balance and Leaver, and other mechanical Powers, (except the Wedge of which the Screw is made)

the Weight ceases to be sustained, and goes back when the Power ceases to act. The Reason of this is, that Weights when they are raised by Screws endeavour to descend perpendicularly, whereas the Screw has been pushed against them very obliquely, so that it can only be pushed back again in that oblique Direction, which cannot be given to them by the Gravity of the Weight tending downwards, if there be the least Friction against the Thread of the Screw: So likewise when a Body is press'd by a Screw, its Surface re-acts in Lines perpendicular to it, that is, in the Direction of the Arbor of the Screw; whereas the Screw cannot be driven back unless it be moved in the Direction of its Thread, which makes a great Angle with the Axis of the Arbor. Thus the * Plank K *k*, *Fig.* 10. cannot fall down in the Direction of its Gravity towards M L, unless it could cause the Screws to move in the Direction B *b* or D *d*. Neither can the Teeth of the Wheel I *, *Fig.* 13. pressing against the Screw C *c* D in the Direction C B move the Screw, or cause the Handle H to move in the Direction of the Circle whose Radius is A H, though the Power which went round in that Circle be removed, and the great Weight W does very strongly endeavour by the Axis E F to turn the Wheel I. Lect. III. * Plate 11. Fig. 10. * Plate 11. Fig. 13.

HENCE appears the great Use of the Machine of *Fig.* 10. which by means of the strong Piece I, may support the Side of an House, and several of them applied in different Parts can sustain a whole Building whilst the Foundation is mended or renewed.

67. WHETHER the Screw be considered as a Wedge or an inclined Plane, it follows from what has been said, that the closer the Thread of the Screw is, the greater is its Force, and the more advantageous the Friction; the Power bringing forward the Screw with most Ease, and the Difficulty to push back the Screw becoming the greater.

68. As Percussion is useful in the Wedge to lessen Friction; so in some Cases there is a kind of Percussion used to drive on a Screw, as in *Printing*, *Coining*, and *Sealing* Instruments with large Seals, where a great Pressure is required; and that is done by a *Fly*, which is a Balance going through the Arbor of the Screw, and loaded with Weights at its Ends, or sometimes only one *Brachium* of it, as in the Printing-Press.

Lect. III. HERE inſtead of the Hammer or Mallet, which comes on upon the Back of the Wedge with an accelerated Motion in a circular Direction to drive it into the Wood; the Weight of the Fly comes down alſo with an accelerated Motion, but on an inclined ſpiral Plane, and thereby having overcome the Friction of the Threads of the Screw as it deſcends, puſhes the End of it with great Force againſt the Bodies to be preſſed. *But this will be farther explained hereafter in another Lecture.*

69. THE Friction of a Screw joined by a Fly is often made uſe of to regulate Motion by retarding a Weight which would deſcend too faſt, reducing the accelerated Motion of an heavy Body in its Deſcent to one that is uniform, by deſtroying juſt as much Force as Gravity would ſuper-add to the deſcending Body in Motion. But we ſhall explain this more fully when we ſpeak of the Fall of Bodies.

THOUGH Balances, Leavers, Pullies, Axes *in Peritrochio*, and even Wedges can be made to act upon one another to increaſe the Force of the Power (as has been ſhewn in the Conſideration of the Leaver, Pulley and Axis *in Peritrochio*;) yet Screws cannot be applied to each other directly, without the Intervention of ſome other mechanical Power: but in Compoſition with other mechanical Powers or ſimple Machines, the Screw will ſerve to make an Engine of vaſt Force. Which leads me to the Conſideration of

Compound ENGINES.

70. THE Combination of two or more of the ſimple Machines, for the Uſes of Life (whether they be of the ſame or of different Kinds) by means of a Frame of Wood or any Metal, makes what we call a *compound Engine*. As it would be endleſs to recount the ſeveral Sorts that are made uſe of, we ſhall only give an Account of ſome few, whereby we may judge of what may be performed by any Engine which has been made, or may be hereafter put in Execution, upon ſight of an exact Draught of the intended compound Machine; that we may have a juſt Value for what is or may be uſeful, and not be deceived by Pretenders to Perpetual Motions, and thoſe who promiſe greater Effects by Machinery than is conformable to the reciprocal Proportion between

tween the Intensities of the Powers and Weights and their Velocities *. Lect. III.

* Ann. 14.

71. The two Machines represented in *Fig.* 1 and 2, of * *Plate* 6. represent the Scorpions which the Ancients used in War for throwing Stones, as I have described them already †; I only take notice here, that they are compounded of a forked Leaver, two Axes *in Peritrochio*, and two Pullies, which last in *Fig.* 1, only direct the Rope as they are Rollers, but double the Force of Power in *Fig.* 2.

* Plate 6. Fig. 1 and 2.

† L. 2. Ann. 7.

72. The Water-Engine of * *Plate* 7. *Fig.* 14 and 15, consists of two Leavers E D and I K, and its Use is described where it is first mentioned †.

* Plate 7. Fig. 14 and 15.

† L. 2. Ann 9. at the End.

73. The 14th *Figure* of *Plate* 9. represents a Machine of great Force, compounded of three Leavers acting upon one another as described above *.

* Plate 9. Fig. 14.

74. Though in the tenth *Plate*, *Fig.* 4, 5, 6, 7 and 8, the Machines represented are made up of several single Pullies or Sheevers (Wheels moving round a Center-pin) like that of *Fig.* 1. yet as the same Rope runs over or under them all, they must be considered only as one Machine; for the Workmen call them by the single Name of a *Pulley* or a *Pair of Blocks*; and the Ancients considered a single or many Pulleys joined in the manner represented in those Figures only as one Engine, which they called by various Names, according to its Number of Sheevers; as *Monopastum* when it had but one, *Dispastum* when it had two, *Trispastum* when three, *Tetraspastum* when four, *Pentaspastum* when five, and usually *Polyspastum* when it had many Sheevers.

75. But the Machine of *Fig.* 9 *, is a compound Engine; because in it the Pullies act upon each other, and increase the Force of the Power in a greater Proportion than according to the Number of Sheevers in the others above-mentioned, as has been shewn *.

* Plate 10. Fig. 9.

* 41.

Lect. III.

76. THE 13th *Fig.* repreſents an Engine compounded of two Axes *in Peritrochio*, whoſe Operation has been deſcribed and Force calculated *. * 47.

77. THE 9th and 10th *Figures* of *Plate* 11 *, are compound Engines, becauſe ſuch great Screws cannot be turned without Leavers applied to them. N.B. *It is to be obſerved, that when a Leaver or Leavers carry an Axis round, in which Caſe they are called* Hand-ſpikes, *or* Bars, *they perform the Office of a Wheel turning an Axis, and may therefore together with the Axis they turn be conſidered as an Axis* in Peritrochio. * Plate 11. Fig. 10.

THIS is more evident in obſerving the Motion of the Handle A H of *Fig.* 13 *, made uſe of to carry round the Screw and Axis A B. * Plate 11. Fig. 13.

78. BECAUSE a Screw, which greatly adds to the Force of the Power, by reaſon of its Shortneſs, raiſes the Weight but to a ſmall Height; and the Axis *in Peritrochio* (for the Reaſons given in the Deſcription of it *) though it may raiſe the Weight to a very great Height, or from a great Depth, does not much increaſe the Force of the Power; the Combination of theſe two Machines preſerves the Advantages of each of them, and takes off their Defects: And this is done by cauſing the Threads of the Screw C D to take hold of the oblique or ſkew Teeth of the Wheel as *c*, and by continually turning the Wheel round to draw up a great Weight as W by means of the Rope which is wound on the Axis E F. If on the Axis E F another Screw be added, as at G H, to turn a ſecond Wheel L, whoſe Axis M being of ſufficient Length receives the Rope N inſtead of the Axis K O, a much greater Weight may be raiſed; or an Engine of vaſt Force may be made for boring hard Metals by means of cutting Inſtruments fixed on the End of the Axis M, or performing Operations where much Strength is required. 46.

79. A Crane is an Inſtrument of ſuch general Uſe, that we cannot avoid giving its Deſcription here. It is of two Kinds: in the firſt, only the Gibbet moves upon its Axis, and in the ſecond Kind called the *Rat-tailed Crane*, the whole Crane, with its Load, turns upon a ſtrong Axis.

80. THE

80. THE firſt ſort of Crane is repreſented by the 1ſt *Fig.* of *Plate* 12.* ſeen in Profil. L B E D is a Section of that Part of a Wharf on which it is fixed, L B being the horizontal Line. A C is a ſtrong horizontal Piece of Timber making the upper Part of the Crane, into which are framed the three upright Pieces X, Y, Z, (of which, the laſt called the main Piece, is ſtronger than the others) with its Cill I E, and its Braces H I and *b* E longer and ſtronger than the Braces and Cills M N and D S of the other two upright Pieces, and pinned with Iron where the others are only pinned with Wood. When the Wharf is not of Stone-Work where the Crane is fixed (as is here repreſented) the three Cills muſt be all in one Piece, reaching from D to E. Four Braces, ſuch as K, join the Upright to the horizontal Pieces. To the abovementioned horizontal Piece is faſtened with ſtrong Iron Pins, a ſhort Piece P P, having a Bell-metal Collar to receive the Iron Pevet or Axis of the upright Shaft R F, which is an Axis *in Peritrochio*, whoſe lower End of the Axis is alſo of Iron, turning in another Bell-metal Collar let into the firm Piece of Wood F. The Axis *in Peritrochio*, inſtead of a Wheel, has four Bars *e*, *f*, *d*, and another behind *d*, going through its thicker Part, which is eight ſquare, the upper Part being round to receive the Rope. When this Piece is hooped with Iron above and below *d*, it is beſt to uſe but two Bars inſtead of four, puſhing them quite through, as *e b*; and Men at each end of them go round in the Direction *b* O *e*, to wind up the Rope and raiſe the Weight at the End of it. This upright wooden Axle, with its Bars, is called the Capſtane of the Crane (every ſuch Axle being called a Capſtane when it turns in a perpendicular Situation, like that of the Capſtanes in Ships, and a Windleſs when it runs in an horizontal Poſition, though it ſhould be for the ſame Purpoſes as the other) and the Rope R r *r*, which goes firſt over the Pulley or Roller T, then between the Pulleys P and Q; and laſtly over the Pulley r, has at its Ends a double Iron-Hook, called a Ram's-Head, to which the Goods to be craned up are faſtened. The Gibbet G V B is moveable upon its Axis C B, by means of the Iron Center-Pins or Pevets at its Ends B and C; ſo that when the Weight is raiſed up ſufficiently high, by a ſmall Rope faſtened to it, or to the End of the Gibbet at g, it may be eaſily brought from over the Ship or Barge, ſuppoſed on the Water at W, to a Cart, or any other Carriage on the Wharf

to-

* Plate 12. Fig. 1.

Lect. III. towards *w* on the right or left of the Piece Z. There is a Roof or ſmall wooden Shed A *a* Q to ſhelter the Rope from Rain, when the Crane is not in uſe, the Gibbet being brought under it towards Y.

* Plate 12. Fig. 2. THE ſecond Figure * ſhews the Plane of the upper Part of the Crane, or as it would be ſeen from above, where we are to obſerve the Poſition of the Pulleys P and Q, and of the Place of the Center of the Gibbet which muſt be at C, in a Line touching the Circumference of both the Pulleys; for if the ſaid Center of Motion of the Gibbet were in a Line with the Center of the Pulleys, the Gibbet when loaded would require a Force to bring its End *g* over the Wharf on either Side; and that Force ceaſing to act, the Weight and Gibbet would run back, and reſt over W. See this more particularly explained in the Notes, where the 3d Figure, being Part of the Crane and various Situations of the Gibbet and Rope drawn by a larger Scale, is conſidered *.

Ann. 15.

* Plate 12. Fig. 3. THIS Crane is very expeditious with many Hands, it being always requiſite that ſome ſhould ſtand at the Bars to keep the Weight from running down again, which might be of dangerous Conſequence. But if inſtead of the Capſtane at O, there was an Iron endleſs Screw, and horizontal Wheel and Axle (commonly called a Worm and Wheel) fixed to the Piece X made very ſtrong for that Purpoſe, or a Machine in the manner of *Fig.* 13. of *Plate* 11.* only with a Pinion inſtead of a Screw at G H, making the Axis E F ſhort and the Axis M long, to receive the Rope; then two Men, nay ſometimes one Man, may eaſily draw up Goods from a Barge; becauſe by the Reſiſtance of the Screw againſt the Teeth of the Wheel the Machine will hold the Weight at any Height, whilſt the Man quits the Handle to bring the Weight by the Guide-rope faſtened at *g* * over a Cart on the Wharf to receive it; and yet the Weight will go down gently of itſelf, if a Man with a ſudden Jerk ſets the Handles a going the contrary way to that whereby he raiſed the Weight, which is convenient for loading the Cart. N. B. *This ſudden Impulſe on the Handles does to the Screw what a Blow does to the Wedge to ſet it a going* *. As this laſt ſort of Crane with a Worm and Wheel does very much increaſe the Force of the Power, I need not put the Reader in mind that more Time muſt be ſpent in raiſing the Weight; becauſe the Velocity of the Weight, compared with that of the Man's Hand who turns the Handle, muſt

* Plate 11. Fig 13.

* Plate 12. Fig. 1.

* N° 57.

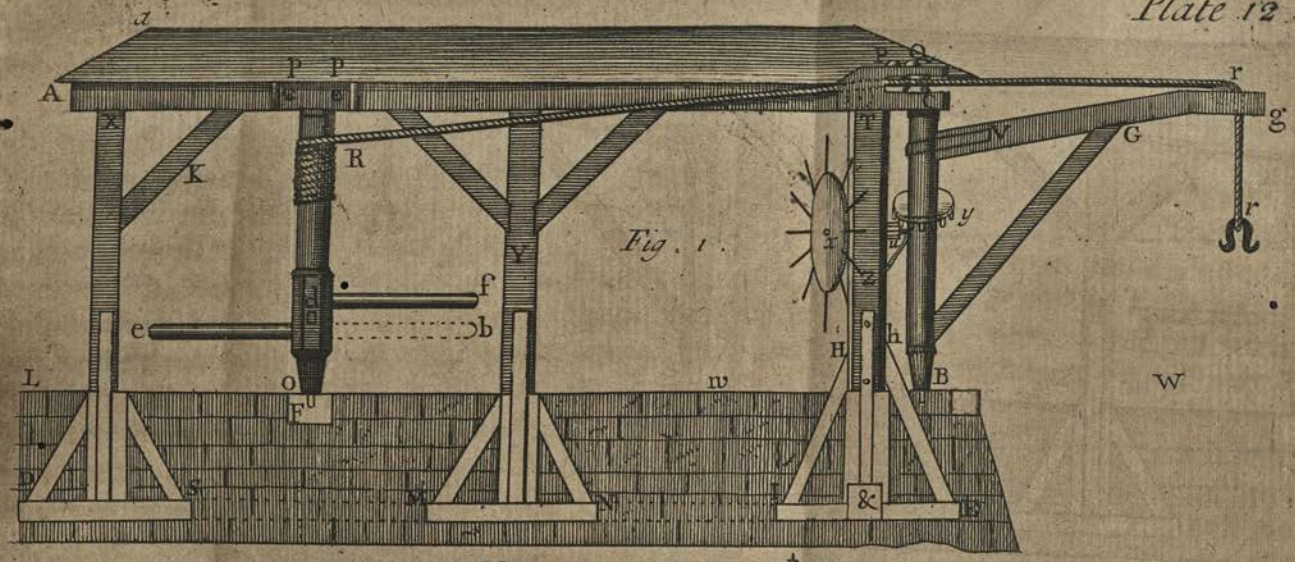

Fig. 1.

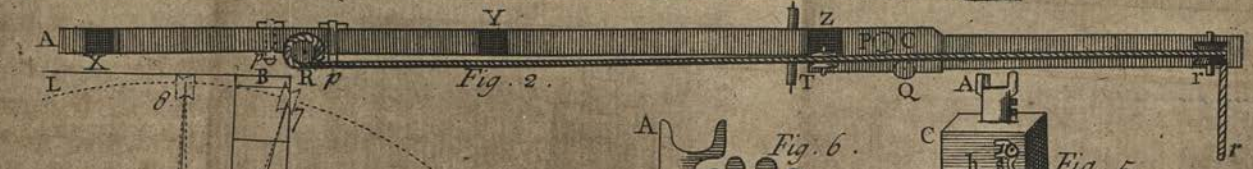

Fig. 2.

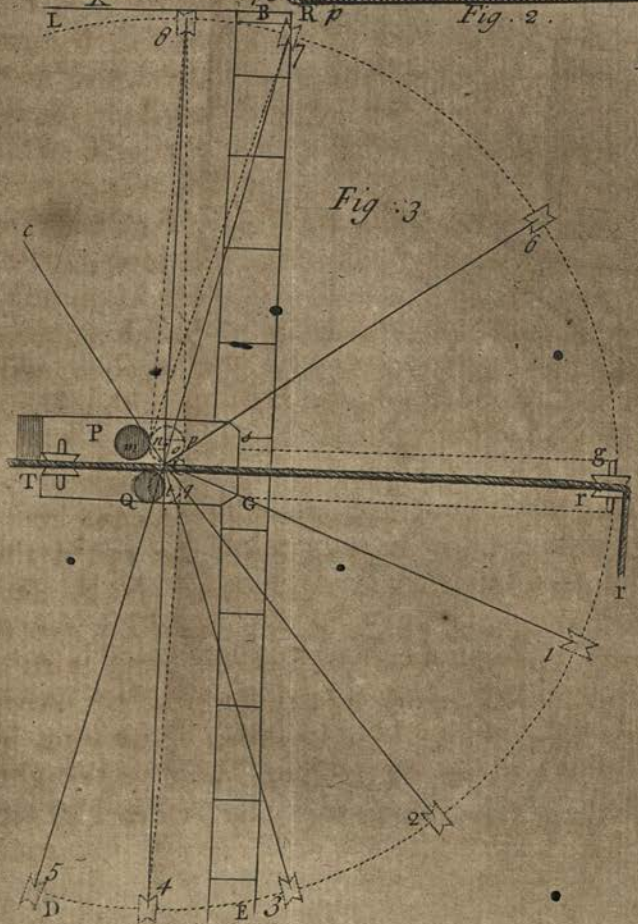

Fig. 3.

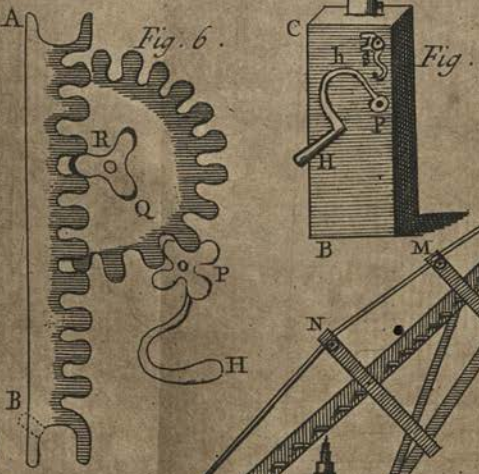

Fig. 6.

Fig. 5.

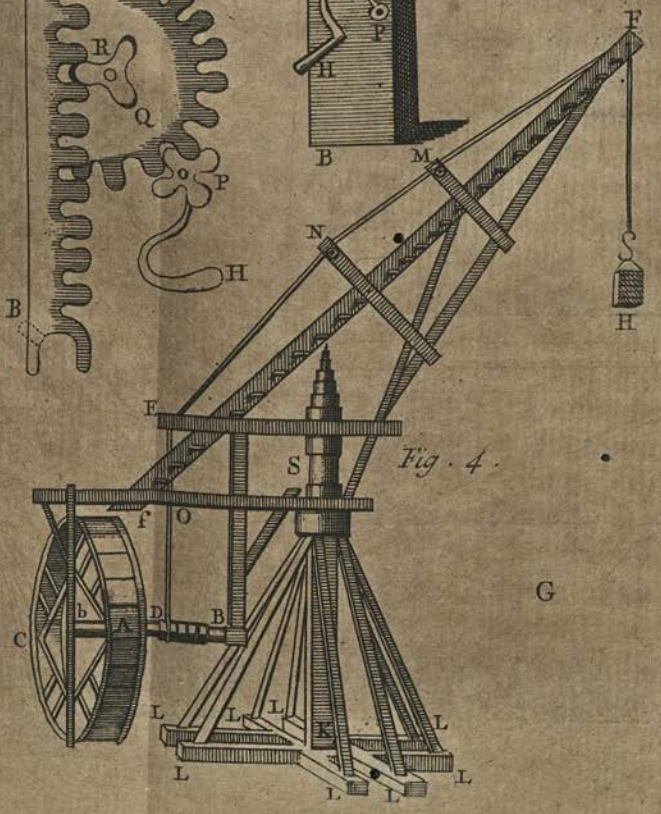

Fig. 4.

must be diminished in a reciprocal Proportion of the Intensity of the Weight to the Intensity of the Man's Force *: This being true in all compound as well as simple Engines. Lect. III.

* L. 2, 9, 12, 17.

81. THE fourth Figure * represents the *Rat's-tail Crane*, not only useful on a Wharf to crane up heavy Goods, but also of great Service in Building to raise great Stones and bring them round to any designed Place. It consists of the following Parts. On the Cross-ground Cills L L L L L L is supported and fixed by oblique Braces the strong upright Piece K, called the Gudgeon of the Crane, on whose upper Part or Spindle S, covered with Iron (and sometimes made wholly of Iron) the whole Machine turns, being easily moved from C to G, when it is charged with its Load H. C A is the Counter-wheel with its Axis D B, bearing only on the Iron Ends of the said Axis in two hanging perpendicular Pieces at B and *b*; *f* F is the Brace and Ladder, whose Top F carries the Pulley above the Weight, the other Pulleys being in the Ends of the Pieces M, N, E; the remaining Parts are too plain in the Figure to need farther Explanation. The Power is sometimes applied by means of a Rope on the outer Circumference of the Wheel A, but most commonly Men, or an Horse, or an Ass, turn the Wheel round by walking in it. Sometimes also Strength is gained by having the Counter-Wheel made with Teeth, and giving Motion to its Circumference by means of a Pinion.

* Plate 12. Fig 4.

82. THE 5th Figure * represents a Jack, which is an Instrument of common Use, for raising heavy Timber or very great Weights of any kind; but as the Wheel-work of it is shut up in the strong Piece of Timber C B, I thought it not improper to represent the Inside of it in *Fig.* 6. * where you must only suppose the Rack A B at least four times as long in proportion to the Wheel Q (the Figure of the Rack being here contracted for want of Room) and the Teeth, which will then be four times more in Number, to be contained about three in an Inch. Then if the Handle be seven Inches long, five Turns of it, that is, five times 22 Inches, or 110 Inches, will be the Velocity of the Power; whilst the Weight raised by the Claw A, or depressed by the Claw B, moves one Inch: for as the Pinion has but four Leaves, and the Wheel Q twenty Teeth, there must be five Revolutions

* Plate 12. Fig. 5.

* Plate 12. Fig. 6.

of

Lect. III. of the Handle fixed to it to turn the ſaid Wheel once round, whoſe three-leaved Pinion R will in that Revolution juſt move the Rack three Teeth, or one Inch. This might have been alſo known without ſeeing, or even knowing the Numbers of the Teeth of the Wheel and Pinions, by meaſuring a Revolution of the Handle in *Fig.* 5. and comparing the Space gone through by it with the Space gone through by the End A or B. So in any other compound Engine, we may judge of its Force by comparing the Velocity of the Part to which the Power is applied with the Velocity that moves the Weight, as has been before ſhewn *. Sometimes this Machine is open behind from the Bottom almoſt up to the Wheel Q, to let the lower Claw (which in that Caſe is turned up as at B) draw up any Weight. When the Weight is drawn or puſhed ſufficiently high, it is kept from going back by hanging the End of the Hook S fixed to a Staple over the curved Part of the Handle at *b* *. All the Work muſt be made very ſtrong in this Machine, but chiefly thoſe Parts which immediately ſuſtain the Weight; and this muſt alſo be obſerved in all other compound Engines.

* 32, 47.

* Plate 12. Fig. 5.

83. ALL the Mechanical Powers are united together in the Machine repreſented by the firſt Figure of *Plate* 13 *, which reſembles our common Chimney-Jacks for roaſting Meat. In a Frame ABCD faſtened by the Nut *o* upon the Stand *o* O, and held together by the Pillars V W and B *q*, is adapted firſt the Piece E F, whoſe Fans or Flies may be put in motion by the Wind, or drawn by an Hair faſtened at F, which repreſents the Leaver and Balance: At right Angles to this Piece is joined the perpendicular Spindle G H, having upon it the endleſs Screw H, which may be alſo conſidered as a Wedge *. This endleſs Screw or Worm takes the ſkew Teeth of the Wheel K, which is the *Axis in Peritrochio*, and, in turning round, winds up the String L M upon its Axis, which paſſing round the Pulleys at M and N (or drawing by a *Tackle of Five*) raiſes the Weight P. But as the Screw has no progreſſive Motion on its Axis, it cannot here be ſaid to take in the inclined Plane; therefore to make this Engine take in all the mechanical Organs or Powers abovementioned, we may add the inclined Plane *r q* Q R, by making it reſt on the Ground at Q R, and on the Pillar *q* B at *q r*, and thereby the Force of the Power drawing at F will be farther increaſed in the Ratio of Q T to T S *. The whole Force gained by

* Plate 13. Fig. 1.

* 63.

* 49.

by this Machine is found by comparing the Space gone through by the Point F, with the Height that the Weight is raised in any determinate Number of Revolutions of F *. An hundred Pounds Weight at P will be easily raised by the Hair of a Man's Head drawing at F. Lect. III. * 32, 47, 82.

84. In the Machines above mentioned, both simple and compound, the Power according to its Intensity is so applied to one Part of the Machine, as immediately to act upon the Weight whose Resistance destroys all the Force of the Power, when an *Æquilibrium* is made by giving the Body moved and the Mover a Velocity reciprocally proportionable to their Intensities; and when the Product of the Power into its Velocity exceeds that of the Weight into its Velocity, there is no more *Momentum* left to the Power, but so much as it has more than the Weight *. But there are other mechanical Organs, in which the Force of the Power is accumulated, and, as it were, *condensed*, before the Weight is acted upon at all; such as an Hammer, the Cock of a Fire-lock which carries the Flint to strike upon the Steel, the Battering-Ram (already mentioned by the by in another Lecture) the Rammer to drive Piles into the Ground, the Fly applied to printing or making a Stamp on Medals or current Coin (cursorily taken notice of in this Lecture *) and the Pendulum; and in a word, whatever is made use of, to give a sudden Blow, or strong Impression instantaneously; which though all reducible to geometrical Calculation, whereby one may be certain of their Effects, yet cannot be explained from the Principles already given; and therefore the Consideration of them must be deferred, till we have explained those Laws of Motion on which they depend. In the mean time I think it not improper to give Sir *Isaac Newton*'s Account of all the Mechanical Powers by one Scheme, as he has explained them in the second Corollary to his Laws of Motion; for though that Explanation depends upon such Laws as are to be hereafter considered, so much of them as is contained in his first Corollary being taken for granted, will serve our present Purpose. * 14. * 68.

65. *If upon a Body* * *at* A *two Forces act at once, whose Intensities are as the Length of the Lines* A B *and* A C, *and Directions in those Lines; the Body so acted upon will describe the Line* A D, *the Diagonal of the Parellogram compleated by drawing the two Lines* * Plate 13. Fig. 2.

Lect. III. C D *and* D B *respectively equal and parallel to the two former; and that Diagonal will be described by the joint Forces in the same Time that either of the Forces singly would have caused the Body to go through the Line* A B *or* A C. *And as by Composition of Forces a Body will move in the Line* A D; *so likewise a Body moving in the Line* A D, *though by the Action of one single Force, may be considered as if it had been acted upon by two Forces, namely, resolving the single Force into two, as* A C *and* A B.

* Plate 13. Fig. 3.

86. If the unequal *Radii* O M * and O N drawn from the Center O of any Wheel should sustain the Weights A and P by the Cords M A and N P; and we would know the Forces of those Weights to move the Wheel: Through the Center O draw the right Line K O L, meeting the Cords perpendicularly in K and L; and from the Center O with O L, the greater of the Distances O K and O L, describe a Circle, meeting the Cord M A in D; and drawing O D, make A C parallel, and D C perpendicular thereto. Now, it being indifferent whether the Points K, L, D, of the Cords be fixed to the Plane of the Wheel or not, the Weights will have the same Effect, whether they are suspended from the Points K and L, or from D and L. Let the whole Force of the Weight A be represented by the Line A D, and let it be resolved into the Forces A C and C D; of which the Force A C, drawing the Radius O D directly from the Center, will have no Effect to move the Wheel: But the other Force D C, drawing the Radius D O perpendicularly will have the same Effect, as if it drew perpendicularly the Radius O L equal to O D; that is, it will have the same Effect as the Weight P, if that Weight is to the Weight A as the Force D C is to the Force D A; that is, (because of the similar Triangles A D C, D O K) as O K to O D or O L. Therefore the Weights A and P, which are reciprocally as the *Radius* O K and O L that lie in the same Line, will be equipollent, and so remain *in Æquilibrio*, which is the well-known Property of the Balance, the Leaver, and the Wheel. If either Weight is greater than in this Ratio, its Force to move the Wheel will be so much the greater. If the Weight *p* equal to the Weight P, is partly suspended by the Cord N *p*, partly sustained by the oblique Plane *p* G; draw *p* H, N H, the former perpendicular to the Horizon, the latter to the Plane *p* G; and if the Force of the Weight *p* tending downwards is represented by the Line *p* H, it may be resolved into the Forces

p N,

p N, H N. If there was any Plane perpendicular to the Cord *p* N, cutting the other Plane *p* G in a Line parallel to the Horizon; and the Weight *p* was supported only by those Planes *p* Q, *p* G; it would press those Planes perpendicularly with the Forces *p* N, H N; to wit, the Plane *p* Q with the Force *p* N, and the Plane *p* G with the Force H N. And therefore if the Plane *p* Q was taken away, so that the Weight might stretch the Cord, because the Cord now sustaining the Weight, supplies the Place of the Plane that was removed, the Force of that Weight will be obtained by considering it as the same Force *p* N which pressed upon the Plane before. Therefore the Tension of this oblique Cord *p* N will be to that of the other perpendicular Cord *p* H as *p* N to *p* H. And therefore if the Weight *p* is to the Weight A in a Ratio compounded of the reciprocal Ratio of the least Distances of the Cords *p* N, A M, from the Center of the Wheel, and of the direct Ratio of *p* H to *p* N; the Weights will have the same Effect towards moving the Wheel, and will therefore sustain each other, as any one may find by Experiment. Lect. III.

But the Weight *p* pressing upon those two oblique Planes, may be considered as a Wedge between the two internal Surfaces of a Body split by it; and hence the Forces of the Wedge and the Mallet may be determined: for because the Force with which the Weight *p* presses the Plane *p* Q, is to the Force with which the same, whether by its own Gravity, or by the Blow of a Mallet, is impelled in the Direction of the Line *p* H towards both the Planes, as *p* N to *p* H; and to the Force with which it presses the other Plane *p* G, as *p* N to N H. And thus the Force of the Screw may be deduced from the like Resolution of Forces; it being no other than a Wedge impelled by the Force of a Leaver. Therefore the Use of this Corollary spreads far and wide, and by that diffusive Extent the Truth is farther confirmed. For on what has been said depends the whole Doctrine of Mechanics, variously demonstrated by Authors. For from hence are easily deduced the Forces of Machines, which are compounded of Wheels, Pulleys, Leavers, Cords and Weights ascending directly or obliquely, and other mechanical Powers; as also the Force of the Tendons to move the Bones of Animals.

End of the THIRD LECTURE.

ANNOTATIONS upon the Third LECTURE.

1. [1. *Simple Machines, or Organs, call'd by some* Mechanical Faculties, *or* Mechanical Powers.]

Annotat. Lect. III. *L. 2. N°. 19.

THE Word *Power* here is to be taken in a very different Sense from what it bears in the Second Lecture, where the Word *Power* * (signifying whatsoever is made use of to raise a Weight) is defined at large; for here it only signifies the Organ or Instrument, whereby a Power of known Intensity is made to act upon a Weight: and therefore we must take care not to attribute any real Force to any simple or compound Machines, as a great many People are apt to do merely because the Name *Power* has been given to Mechanical Organs, not from their Effect, but from the Effect which the Power produces by their Means. For how much soever the Force of a Power is thereby increased in order to sustain or raise a Weight far superior to it in Intensity; yet this cannot be done without losing in Space and Time what is gained in Force; contrary to what some have vainly imagined, because the Vulgar commonly speak of a Machine as they do of an Animal, and attribute that Effect to the Machine, which is the Effect of the Power by Means of the Machine, as it is usual to say, *Such a Machine raises such a Quantity of Water*, or *performs such and such Work*; when we should say, if we would speak in the proper and philosophical Sense, *Such a running Stream, such a Fall of Water, the Wind*, or *so many Men, Horses*, or *Oxen*, &c. *raise so much Water in such a Time*, &c. *by such or such a Machine*; as we have observed in the Notes of the last Lecture *. It were therefore to be wished, that the Word *Power* were confined to its proper Sense, and not used to signify a Mechanical Organ; but as it has been customary to use it in that Sense, I thought proper to make use of it; but withal to give this Caution.

* Ann. 6. L. 2.

2. [3.——*Reducing all the Mechanical Powers to the Leaver, or explaining all their Operations by that of*, &c.] Though one may easily shew (as we shall at the End of this Note) that every other Mechanical Organ does potentially contain a Leaver; that is, that if so much of every other simple Engine was cut away as to leave only a Leaver, the same reciprocal

Proportion between the Velocities of the Power and Weight and their Intensities; would as plainly appear on such a Leaver as on the Engine, before it is reduced to it: Yet we are not from thence to conclude, that the Leaver alone might be made to serve for the Purposes of all other Mechanical Organs; for they are contrived of different Forms to answer to different Ways of working, required in Mechanical Operations for the Uses of Life, it being sometimes impossible to use one Mechanical Organ instead of another; and always more convenient to apply one than another, the Choice of which depends upon the Skill of the Artist. Annotat. Lect. III.

Thus in the Balance (whether a Pair of Scales or a Steel yard) the Commodity to be bought, or heavy Body whose Quantity of Matter is to be estimated, is not to be raised with any Velocity, but only supposed to make an *Æquilibrium* with a known Weight, which in this Machine serves for a *Power:* But a Weight is seldom or never made use of as a Power in any other Mechanical Organs, except in some few, and those Engines which the Ancients made use of in War, which are now out of Use.

The Force of one or more Men is the Power applied to the Leaver; there the Power must always overcome the Weight, by adding a little more Velocity or a little more Intensity to the Power, over and above the reciprocal Proportion required in the Balance. With this Instrument heavy Bodies are removed a little way at a time, as great Stones in Building, large wooden, leaden, or iron Pipes in Water-works, and large Pieces of Timber; but Leavers will only serve to raise those Bodies high enough to lay them on a Carriage, *&c.*

But if we would raise up a Stone to a considerable Height to lay it in its proper Place in a Building, or any other Body to any Height above three or four Foot, the Leaver becomes ineffectual, and then we must use Pulleys in some of the Ways mentioned in this Lecture*. Tackles of Pulleys are very convenient where there is no room for a Capstane, and where Bodies are to be raised in different Places, because they are easily moveable; but the Weight must not be too great, by reason that many Men cannot pull at once, and equally, by one running Rope; and if the Power was to be very much increased by the Number of Sheevers or Wheels in the Pulleys or Tackles, the Rope must be of a prodigious, and therefore inconvenient Length. * 37, 38, 39, 40, 41.

An Axis *in Peritrochio*, Capstane or Windless, which are all the same Organ differently posited, is of use where Pulleys are deficient. For example, if Water is to be drawn out of a deep Well, a Wheel with Arms or Spokes serves to turn the Axle on which the Rope winds to bring up the Bucket or Buckets. In Building, a Capstane whose Make gives the Power no more Advantage than a Tackle or Pulley of many Sheevers, is yet more useful, because it will admit of eight, ten, or twelve Men to work at it, and push its Bars, when only three or four could pull at the running Rope of the Pulley. If the four Bars of the Capstane are so long, that three Men applying their Strength to each of them, the Man in the middle of the three is at the Distance of three Foot from the Axis of Motion,

Annotat. Lect. III.

Motion, and the Axle on which the Rope winds is of ſix Inches and a half Diameter, thoſe twelve Men will do the Work of 72, but in ſix times more Time. The ſame would be done, and in the ſame Time, by two Men walking in a vertical Wheel of 24 Foot in Diameter (ſuch as is called by ſome a Counter-Wheel) whoſe horizontal Axle might be of eight Inches Diameter; and about the Weight of one Ton and a half, or two Tons, might thus be raiſed by either of theſe Machines. *But if at a Wharf,* where only two Men ſhould be employed to work at the Crane, it was required to draw up heavy Blocks of Marble of three or four times the Weight above mentioned, (for a Block of Marble ſix Foot long, four Foot wide, and four Foot deep, will weigh between ſeven and eight Ton) the Counter-Wheel muſt be 72 Foot in Diameter for an Axle of eight Inches, which is impracticable, on account of the Bulk and Coſt; or the Axle muſt be made three times leſs in Diameter, and then it cannot be ſtrong enough to bear the Weight. In ſuch a Caſe therefore there muſt be a compound Axis *in Peritrochio*, ſuch as is deſcribed in this Lecture *, but with one more Pinion: for Example, if an iron Wheel with Teeth of four Foot in Diameter be fixed to the above-mentioned Axis of eight Inches Diameter, and that Wheel be led by an iron Pinion of ſix Inches Diameter, whoſe Wheel is of three Foot Diameter, and led by a Pinion of eight Inches Diameter, whoſe Handles are a Foot long, the ſame Operation will be performed by two Men, but they muſt employ three times more Time. N.B. *If the Teeth of the Wheel be of Braſs, and the Teeth or Leaves of the Pinions of Iron, the Machine will move more ſmoothly, and the Teeth wear equally.*

* N°. 47.

The Numbers of Teeth in the Wheels and Pinions may be as follows;

The firſt Pinion, to which the Handles are fixed, 28.
The firſt Wheel, led by the ſaid Pinion, 112.
The ſecond Pinion, 19.
The ſecond Wheel, 171.
And the Axis on which the Rope winds muſt be of about eight Inches, becauſe it immediately bears the Weight by the Rope, which runs over ſome upper Pulleys or Rollers that add no Force to the Power.

If the Capſtane with Bars above-mentioned be fixed to do the Work for which it is proper, and unexpectedly there ſhould be occaſion to lift very heavy Weights, there is no Neceſſity for taking it down to ſet up ſuch a Combination of Wheel-work as we have juſt deſcribed, becauſe a Pair of Blocks to help at ſuch a time may be fixed in any Part of the Building over the Place where the Weight is to come up and receive the Rope from the Capſtane, ſo as to increaſe the Force of the Power, according as the Blocks are a *Tackle* of *Two*, of *Three*, of *Four*, or *Five*, &c. and then the Blocks may be taken off again, and the Capſtane worked as before. In the Uſe of the Counter-Wheel or Rat's-Tail Crane * the Power may be alſo thus occaſionally increaſed.

* N°. 82.

Where

Where there is no Room for the compound Axis *in Peritrochio* of two large Wheels and Pinions, the ſame may be effected by making a Worm-ſcrew of one Thread turned by two Handles of a Foot long each, to lead a Wheel of two Foot Diameter, having 72 Skew-Teeth, and a wooden Barrel or Axle of 8 Inches Diameter, the Advantage of which Machine is ſhewn in this Lecture *. Annotat. Lect. III.

* N°. 79, 81.

If Hogſheads or Pipes of Wine, or other Liquors, are to be let down into a Cellar, or brought up out of it, a Plank is laid along the Stairs, which in that Caſe is an *inclined Plane*, the only mechanical Organ fit for that Purpoſe: So likewiſe in making Reſervoirs for Water, in Gardening, and in building Fortifications, where Carts cannot come; inclined Planes made of Wood ſerve effectually for Wheel-barrows to run on in removing the Earth from a lower to an higher Place.

In cleaving Wood, the Wedge only is of Uſe; for an Hatchet, which may cleave ſmall Wood, is only a Wedge with an Handle. Another conſiderable Uſe of a Wedge, is the raiſing up a Beam to underprop it when a Floor begins to give way, by reaſon of too great a Burthen laid upon it, as in a Warehouſe; and ſo much Force may be applied this way, that ſome thouſand Tons may be raiſed up together with the Floor, and all ſecured by means of this ſmall Machine. For though Screws turned by long Leavers might ſerve in a great meaſure for the ſame Purpoſe; there muſt be Space to go round with the Hand-ſpikes, which cannot be had when the lower Part of the Warehouſe is full of Goods, without removing them with great Trouble and Coſt. See the manner of performing this in the 4th *Figure* of *Plate* 13.

B A *b* D E C is a Beam, which in the horizontal Situation, marked by the pricked Lines B *b* and C D, ſupports a Floor. Now when very great Weights being laid upon the ſaid Floor, make it give way and bend into the Curve B A *b*, or, which is the ſame, C E D, an upright Poſt P *p* is framed into a ſtout [illegible] horizontal Piece E (here ſeen end-ways) is ſlipped under the Beam, and on the Top of the Poſt; then the two wooden Wedges W, *w*, as broad as the Poſt, are driven in with heavy Hammers, in ſtriking at once in contrary Directions, ſo as to ſet the Beam ſtreight, and reduce the Floor to its place, without removing any of the Goods that lie on the ſaid Floor, or moving any of thoſe below, but juſt to make room for the Poſt and Plank. Plate 13. Fig. 4.

The Uſes of the Screw for raiſing, ſinking, drawing, puſhing, preſſing, or joining together Bodies, are commonly known; and it is evident by Inſpection, that no other of the mechanical Organs will anſwer the ſame Purpoſes.

Now we will ſhew how all the mechanical Organs *or Inſtruments may be reduced to the Leaver.*

This has been already ſhewn concerning the *Roman* Balance (and is applicable to the common Pair of Scales) by the 6th *Experiment* of this *Lecture* *.

* N°. 29. Pl. 9. F. 11.

Secondly,

Annotat. Lect. III.

Secondly, Pulleys are thus reduced to the Leaver *. An upper Pulley as E D (*Plate* 10. *Fig.* 3.) appears plainly to be a Leaver of the first Kind, by cutting away all the Sheever, except the pricked Line E D, which about the Center C keeps *in Æquilibrio* two equal Weights. In the 4th *Figure* † the lower Pulley *g e*, by leaving only the pricked Line *g e*, and Center-Pin *c*, appears to be a Leaver of the second Kind, whereby a Power applied at *g*, does in the Direction *g d* raise a Weight W suspended at the Center-Pin *c*. It is here evident, that *e* is the fixed Point or *Fulcrum* of the Leaver, *g e* the Distance of the Power, and *c e* the Distance of the Weight; and accordingly, in the Experiment, the Power P : is to the Weight W : : as *c e* : to *g e* : : 1 : 2. And by comparing the compounded Leavers of *Plate* 9. *Fig.* 14. with the System of Pulleys of *Plate* 10. *Fig.* 9. one may see that the four Pulleys reduced to their horizontal Diameters, act upon one another as four Leavers of the second Kind, in every one of which the Distance of the Power is Two, and that of the Weight is One, and therefore the Ratio compounded of them all *, is that of the Weight to the Power, or 16 to 1; for 2x2x2x2=16. Or comparing this System of lower Pulleys with *Fig.* 5. *Plate* 13, which is just such a System or Assemblage of Leavers of the second Kind, where the Leavers are marked with the same Letters, as well as the Forces pulling down each Leaver *.

* Plate 10. Fig. 3.

† Plate 10. Fig. 4.

Pl. 9. Fig. 14.

Pl. 10. Fig. 9.

N°. 32.

* Pl. 13. F. 5.

One may, by the 11th *Figure* of *Plate* 10, easily reduce an Axis *in Peritrochio* to a Leaver of the first Kind, represented by the pricked Line E T; the fixed Point being at K, the Power being applied at E, and different Weights successively at the Points A, B and T, cutting away the rest of the Machine. But as the Rope sustaining the Weight does not move in the same Plane as the Rope drawn by the Power, it is better to consider it as a Leaver of the first Kind twice bended, and an Axis of Motion going through one of the bended Parts, as in the 6th *Figure* *, where the bended Leaver A C *c* B moves on the Axis I I, fixed in the Frame I K L I. B *c* represents the Radius of the Axle, and A C the Radius of the Wheel, supposing B *c* and A C in the same Plane, and at right Angles to the Axis; otherwise, if oblique, they must be reduced to right ones, by calling the Lengths only the perpendicular Distances of B and A from the Axis I I: Then P being the Power, and W the Weight, the reciprocal Proportion will be thus, A C : B *c* : : W : P.

* Plate 13. Fig. 6.

To reduce the inclined Plane to a Leaver, we must look for a bended Leaver in the Weight rolling up the Plane, whose Arms shall be as the Length of the Plane to its Height.

Since the Triangle A B C * is similar to *w* Y B (by 4. 6. *Eucl.*) and *w* Y B similar to *w* B N (by 8. 6.) in the bended Leaver *w* B N, *w* B : B N : : A B : B C. Now, since *w* N is the Line of Direction of the Weight *w*, that Weight may be considered as pressing on N B the short Arm of the Leaver at the Point N, the Center of Motion being at B, where the spherical Weight touches the Plane, and the Power applied at right Angles at the End *w* of *w* B the longer Arm of the Leaver; therefore, calling P the Power, and *w* the Weight, P : *w* : : N B : B *w* : : A B : B C.

* Plate 10. Fig. 14.

N. B.

N. B. *Here the Power by a Rope over the Pulley* M, *draws in a Direction parallel to the Plane; but if it drew in any other Direction, one might calculate the oblique Force of the Power by means of a bended Leaver. But we must refer this to our particular Observation of the inclined Plane, and statical Considerations, in other Notes; except the Direction parallel to the Base, which reduces the Wedge to the Leaver.* Annotat. Lect. III.

Now the Triangle A B C must represent the Wedge, which being driven under the Weight, makes it rise up the perpendicular Height C B, while the Power drives the Wedge the Length of its Base A C, or, which is the same Thing, the Power Π draws in the Line *w r* parallel to the Base A C. Here the bended Leaver is O B N, with the short Arm N B supporting the Weight at N, while the Power fastened to O draws the Arm O B at right Angles.

Authors have shewn other Ways of reducing the Wedge to a Leaver: For Example, the Wedge B F C * of *Plate* 13. *Fig.* 7. is considered as a Leaver of the second Kind, whose *Fulcrum* is at F, and the Weight at W; which moving round the Center of Motion F, by a Power carried on at the End of the Leaver from A to L, raises up the Weight W. Or else, the Leaver being kept by the Prop B, so as always to make the same Angle with the Horizon, is carried on from the Position C F B into the Position *c f b*, the *Fulcrum* advancing forwards along with it to raise the Weight to *w*. In the 8th *Figure* *, where two Bodies are separated to represent the cleaving of Wood, the Wedge is reduced to two Leavers of the first Kind, having the *Fulcra* at F, *f*, the Weights at W, *w*, and the Powers at L, *l*. Or in the 9th *Figure* †, two Leavers of the second Kind are supposed to be very thin, and thrust in between the two Weights, so as to press against each other's Ends, and make a common *Fulcrum* at F, the Powers moving from L, *l*, to *a* and *b*, while the Weights W, *w*, are separated: or else (what comes to the same thing) the said two Leavers being joined in a fixed Angle by the Prop L, *l*, (*Fig.* 10*.) are considered as thrust in between the Weights, while the common *Fulcrum* advances in the Line F G. But because in all these Ways of explaining, the Distance of the Power (and consequently its Force) is continually changing, which is not true in the Wedge; I would rather propose another Method, which will agree with the Case of the single Wedge or double Wedge (distinguished thus, as the Instrument acts with one or both Surfaces *) always making the Distance of the Power and Weight to keep the Proportion agreeable to the Angle of the Wedge. As for Example, in the Wedge L F W, *Fig.* 11. * L F W is a bended Leaver, whose short *Brachium* always remaining the same, lifts up the Weight *x*, and brings it to *w*, whilst the Leaver turns round the Center F, the Power at L describing the Arc L *l*: So in the double Wedge *Fig.* 12. * two bended Leavers, moving round the Center F, by their short Arms F W, F *w*, separate the Weights W, *w*; and when they have brought them round to X and *x*, the long Arms L F, *l* F turning round F in the Arcs L M, *l* M, one may see that

* Plate 13. Fig. 7.

* Plate 13. Fig. 8.

† Plate 13. Fig. 9.

* Plate 13. Fig. 10.

* Nº. 55, 56.

* Plate 13. Fig. 11.

* Plate 13. Fig. 12.

Annotat. Lect. II.

the ſame thing is performed as if the whole Wedge F M had been driven between them in the Direction N M.

Having ſhewn that the Screw acts either as an inclined Plane, or as a Wedge, it is evident that what has juſt been ſaid reduces it to a Leaver.

3. [— *The Quantity of Friction in Engines will be conſidered*, &c.] That there is a Loſs of Force in the working of Engines on account of the Rubbing or Friction of their Parts, has been obſerved by moſt Writers of Mechanics; but that Friction has not been enough conſidered by them: Upon which account, ſeveral Perſons (who having applied themſelves to the Study of Mechanics, are not yet much acquainted with the Practice) imagine by comparing the Effects which are performed by means of ſuch Engines as they examine with the Powers applied to them, that the Machine muſt be very ill contriv'd, becauſe the Effect does ſo much differ from the Calculation which they make excluſive of Friction; ſuppoſing indeed, that ſomething is to be allowed on that account, but nothing near what they find to be loſt in Force. Poſſeſſed with this Notion, Projectors contrive new Machines (new to them, though perhaps deſcribed in old Books, formerly practiſed and then diſuſed, and forgot) which they ſuppoſe will perform much more than they have ſeen done with the ſame Power; becauſe they allow too little for Friction. Full of this they go to the Charge of 70 or 80 *l.* for a Patent for their new Invention; then divide it into Shares, and draw in Perſons more ignorant than themſelves to contribute towards this (ſuppoſed advantageous) Undertaking; till, after a great deal of Time and Money waſted, they find their own Engine worſe than others which they hoped by many degrees to excel. This has been very much the Practice for theſe laſt twenty Years: For though ſome Projectors have been altogether Knaves, yet the greateſt Part have firſt deceived themſelves; and thoſe who are really deceived, by their Eagerneſs and Earneſtneſs moſt eaſily deceive and draw in others. For this Reaſon I thought it would be of Uſe to the Publick, to give as full an Account of Friction, as I poſſibly could gather from the Experiments made by others (eſpecially the Members of the Royal Academy at *Paris*) and my own Experiments and Obſervations.

We ſhall not here conſider, that Friction or rather Reſiſtance which ariſes from an ill Contrivance of the Parts of an Engine which are to act upon one another by Application; making thoſe Parts act obliquely which ought to act at right Angles, or at leaſt more obliquely than they ſhould do; that being owing to an incomplete Theory or a bad Workman. But only that Friction, which is unavoidable from the Nature of the Materials, however poliſhed, at the firſt making uſe of an Engine; and that which Time brings on, as the Parts wear unequally, or grow ruſty or rotten for want of Oil, Greaſe, or by conſtant Uſe: So that the touching Surfaces, which were as ſmooth as the Hand of a ſkilful Workman could make them, become very rough and uneven by this means, and add much Friction to that which, on account of the Nature of the Materials, could not be avoided

at

at first. So an old Jack for roasting Meat requires more Weight to make it go when the Pivots and their Holes are pretty much worn; and a well made Lock grown rusty for want of Oil to guard it from the Action of the acid Salts in the Air will not without Difficulty be opened with a rusty Key; though the Figure both of the Lock and Key be as perfect as at first. Wood will grow rotten, and swell beyond, or shrink from its first Dimensions by the Weather; and Cords run round Pulleys, or wind about Rollers with more Difficulty according as they stiffen by wet, or any way become more twisted. Annotat. Lect. III.

To proceed methodically, we will consider the unavoidable Friction of the simple Machines or mechanical Organs severally.

The Leaver, in respect to the Work done with it, is subject to very little Friction, moving on a small Surface crossing the Instrument like a Line where it is applied to the *Prop* or *Axis of Motion*; which in Theory is considered only as a Point, and called the *Center of Motion.*

When the Balance (whose Make is nearest to that of the Leaver) has no more Friction in Proportion to its Length than the Leaver; yet it has a great deal too much for a nice Balance, as may be found by Experience. Few of the Scalemakers know wherein the Nicety of a Balance consists; but generally follow a Fashion, or, when they would excel, endeavour to out-do one another in Ornaments or a fine Polish, consulting Beauty more than Use; and thinking the Business is done if they have brought the Balance to turn with a small Part of a Grain.

For the Benefit of those who would make, and those who would use very exact Scales, I shall here mention the Faults I have found in Scales esteemed extraordinary good ones; and shew in what Manner I think such Faults may be avoided; as also the utmost that may be expected from a well-made Balance.

Mr. *George Graham* and I, last Summer, in order to try some Experiments with Brigadier *Armstrong*, Surveyor of his Majesty's Ordnance, examined a Pair of Scales made by a very nice Workman, and kept in a Glass Case, that the Air might have no Power over them. These Scales were thought extremely nice, because they turned with the 256th part of a Grain; but upon Examination they appeared to differ from that Nicety, sometimes three of the Parts above-mentioned, which made us imagine one *Brachium* longer than the other; but at last we found that this Difference was owing to the Situation of the Axis of Motion; for if in the Circle A *a* B *b* *, (on the lower Part of which the Axis or sharp Edge of C ought to bear at A) the said Edge should not rest at A, but at *a* or *b*, the *Brachium* on the other Side of A will preponderate; which it did in our Trials, so as to make a Difference three Times greater than what turned the Balance in its true Situation. Now since the Friction increases in Proportion to the Weights bearing on the Axis of Motion, (as we shall hereafter shew) this Error will increase in the same Manner and become considerable, as heavier Bodies are weighed; so that when we think we have an *Æquilibrium*, by only lifting up the Beam E L with the Weight hanging at its

* Plate 13. Fig. 13.

Annotat. Lect. III.

Ends in the Scale, we ſhall upon letting it down again find the *Æquilibrium* to be loſt; ſo difficult it is to lay the Edge or Axis of Motion upon the ſame Place which it had before, or to give it a true Bearing at firſt over the lower Part of the Ring at A. There is alſo another Fault which Scale-makers are ſometimes guilty of, which is, not to make the *Brachia* nicely of a Length; and then to hide that Error, they adjuſt the Balance by filing away ſome of the Thickneſs of the longeſt Part of the Beam, and ſometimes by the Scales. Others again, by making Ornaments in a nice Balance weaken a ſlender Beam juſt under the Axis, where it ought to be ſtrongeſt.

To make a very exact Pair of Scales, the following Directions ſhould be obſerved.

1. The Axis C muſt be made of good Steel hardened and well poliſhed, but the Edge of it muſt not be ſo ſharp as to cut.

2. The two Rings on which the Axis is to bear, ſuch as A *a*, B *b*, muſt alſo be of hard Steel and well poliſhed, but the lower Part of them ſhould be the narrow End of an Oval, the Hole being of the Figure K. The Planes of theſe Rings with the Pieces that carry them (here repreſented by *m n* as broken, with the Examen *o* between them alſo broken) muſt be exactly parallel to one another, and one and the ſame Line muſt be their common Axis.

N. B. *If theſe Rings were made of Agate, or any harder Stone well poliſhed, they would do better than Steel.*

3. The Points of Suſpenſion of the Scales, ſuch as S, muſt be exactly equidiſtant from C the Middle of the Beam, and the lower Part of the Hole S, muſt be a ſharp Edge of hard and poliſhed Steel.

4. Each Scale being fitted with its Hook and Strings, muſt be weighed ſingly in another Pair of Scales, obſerving to weigh each of them in the ſame Scale againſt the ſame Counterpoiſe in the other Scale, without ſhaking the Beam to alter the Poſition of the Axis in the Manner above-mentioned.

5. When the Beam being ſuſpended, does by its *Examen* (that is, the ſlender perpendicular Piece over the Axis) appear to have its two *Brachia* exactly *in Æquilibrio*; to try whether the Points of Suſpenſion are exactly equidiſtant from the Axis at A, hang on the Scales (prepared as before directed) and if they are *in Æquilibrio*, and continue ſo upon changing them for one another, then you may be ſure that both Scales and Beam are well adjuſted.

6. But if by the preponderating of one Scale, the Beam appears to be unequally divided; then with a Pair of Pliers bend the Part L of the Hook, in order to bring S nearer to, or to remove it farther from the Point A, and if that alters the *Æquilibrium* of the Beam without the Scales, hang a Thread or ſome ſmall Weight on that End of the Beam which has been made ſhorter, to reſtore that *Æquilibrium*; then try with the Scale, and if the *Æquilibrium* continues, file off ſo much of the heavieſt *Brachium*, as the Weight of the Thread amounts to. If the Beam be made

made of Steel, after the Hook L S has been hardened, one must with a Blow-Pipe bring down the Temper of the Part L, that it may be soft enough to bend without breaking. Annotat. Lect. III.

7. The Points of Suspension for the Scales, as S, must be in the same horizontal Line as the Axis A, the Scales must hang very freely on their Hooks, and the Center of Gravity of the Beam must be a very little under A.

8. When the Beam of a Pair of Scales weighs from three or four Ounces to about one or two Pounds, sometimes the Points of Suspension of the Scales are shut up within Boxes at the Ends of the Beam such as B *, through which a square Piece of Steel C passes, whose upper Part in the Middle has an Edge like the Axis of the Beam, but only with the Edge upwards, to sustain the Eye E *e*, and the Hook of the Scale C *c* †, represents one of those Pieces out of the Box, and its middle Section is marked D, whose upper Edge supports the Eye E *e* ‖. In order to adjust such a Balance nicely (all other things being fixed as before-mentioned) there should be a long Hole like H *h* ‡, for the suspending Pieces as C *c*, to be moved in, so as to be brought nearer to, or farther from the Axis of the Balance, by means of a Screw-Pin P; that when they come to be exactly equidistant from the said Axis, (which can only be known by hanging on Scales or Weights exactly equal, having due Regard to the Effect of Pins lengthned or shortned without the Box at P by screwing) the Pieces C *c*, may be fixed, the Screw-Pins filed off, and the rest of the Hole filled up on each Side the supporting Piece C *c*.

* Plate 13. Fig. 14.

† Plate 13. Fig. 15.

‖ Plate 13. Fig. 16.

‡ Plate 13. Fig. 17.

N. B. *It is not to be expected that a large Pair of Scales should be so exact as a small Pair; because the Friction increases according as it is heavier: So that if a Balance, whose Beam and Scales weigh six Ounces Troy, be turned with* $\frac{1}{16}$ *of a Grain, it may be said to be as nicely adjusted as Brigadier* Armstrong's *Balance above-mentioned, which weighing* 16 *Times less, turns with* $\frac{1}{16}$ *of a Grain. So likewise when a Balance turns with a small Part of a Grain, we must not expect it to turn as easily when the Scales are loaded; for they will always become less nice, according as they increase in Weight.*

The Pulley is liable to a great deal of Friction on account of the Stiffness of the Ropes, the Smalness of the Wheels or Sheevers in Proportion to their Center-Pins, and their rubbing against the Sides of the Block or Frame in which they move.

Great Care should be taken, that Pulleys which are to be used in Building, or any where at Land, should receive no Wet; for by so doing, the Ropes twist and thicken, so as often to require a great Force to draw them through the Blocks in that Condition, even when no Weight is raised; but when one cannot avoid wetting them, tarred Ropes must be used as at Sea.

To prevent the Wheels rubbing too much against the Blocks that contain them, there ought to be thin Collars of Brass or Iron of a Diameter much less than the Wheels on the Pins on each Side of the Wheel.

To

Annotat. Lect. III.

To lessen the Friction on account of large Pins when great Weights are to be raised, we should considerably increase the Diameter of the Wheels, though in some Cases the Machine would be too cumbersome, and therefore not so easy to be managed. But as People do not commonly imagine so much Difference as there is between small and large Pullies, provided their Number and Combination be the same; I shall explain that Case in considering the way of finding what the Quantity of Friction is, and only mention here that according to *Monsr. Amonton*'s Experiments and Calculations *, there is so much Friction in Pulleys on account of the Force required to bend the Ropes and overcome the Friction of the Pins, when the Wheels or Sheeves are small, that if over an upper Pulley of three Inches Diameter with an Inch Pin, there was a Rope of $1\frac{2}{3}$ Diameter, having 800 Pound Weight at each End, which Weights must keep each other *in Æquilibrio* in this Case, (because here the Pulley only does the Office of a Roller) in order to make one of the Weights preponderate and overcome all the Frictions, so as to bring the other Weight over, one must add $436\frac{2}{3}$ to that Weight called the *Power* in such a Case: But if the Pulley had been of 24 Inches Diameter, the Diameters of the Rope and Pin being as in the small one, 45 Pound would have been a sufficient Addition to the Power, to enable it to give Motion to the Weight by overcoming the Friction. It is therefore well worth considering the Quantity of Friction in order to direct us in Practice; since in this one Instance, by only using a Pulley-Wheel of 24 Inches Diameter instead of one of 3, the Force to overcome the Friction is less by the Quantity of $391\frac{3}{4}$ Pound; or less than $\frac{1}{16}$ of the Power added to it will bring up the Weight, when in the other Case there must be more added to the Power than one half of it.

* *Memoirs of the Royal Academy for the Year* 1699.

The Axis *in Peritrochio* has but little Friction if the Wheel be large and the Axle small, except what arises from the folding of the Rope round the Axle, if a large Rope be made use of to raise a great Weight. But yet we shall shew how to find the Friction of the Axle, whatever it be, after we have taken Notice of the Friction of the other Machines in general.

The inclined Plane is not liable to much Friction, if the Weight which is rolled up be spherical or cylindrick; for then the whole Friction arises only from what the Plane wants of perfect Hardness, so as to suffer the Body rolling up to sink a little, which alters the Inclination of the Plane in that Place by making it steeper, and renders the Line of Direction a little inclined to the Plane: So likewise, if the Plane being hard, the Body yields and alters its Figure a little; then it must be lifted up at every Pull, or go up by jumps. But if the Body to be drawn up be flat, or of any Figure not spherical or cylindrical, as a Piece of Timber, or at least a Sledge loaded, then the inclined Plane will have a great deal of Friction, which we shall shew how to estimate.

The Wedge has a great deal of Friction; for besides all this last mentioned Friction of the inclined Plane when flat Bodies slide against one another, there must be added the third Part of the Pressure which the same Body gives more to the Wedge than the inclined Plane, by reason of the

Ob-

Obliquity of the Draught, a Wedge being only an inclined Plane with the Line of Direction parallel to the Base, instead of being parallel to the Hypotenuse of a Triangle, whose Height is the Thickness of the Wedge. Annotat. Lect. III.

N. B. *We do not here take notice of the Cohesion of the Wood or Bodies to be cloven, because the Resistance there to be surmounted is to be considered as a Weight, and therefore attributed to the Weight, and not the Power.*

The Screw has a Friction of the same Nature as that of the Wedge, because it is compounded of a Wedge; but greater, because it touches in all its Parts at once, which a Wedge does not ‡. The flat or square-threaded Screw * representing only a single Wedge rising in the Direction H K, or L M, *&c.* has less Friction than the sharp-threaded Screw †; because in this last, the Surface of the Thread of the Screw is inclined to the Base as well as to the Axis or Arbor of the Screw. But the endless Screw * has yet more Friction than the sharp-threaded common Screw, because it is obliged to take obliquely the Tooth of the Wheel which it drives. * Pl. 11. F. 11. † Pl. 11. F. 14. * Pl. 11. F. 13.

For this Reason in Clock or Watch-work, those that would change Wheels and Pinions into endless Screws, should be aware of the great Friction that is in them, and not make use of them unless the Nature of the Movement requires it, and there be more gained by the Alteration of the Direction of Motion than is lost by Friction.

To settle such a Theory of Friction, as may serve to direct our Practice (that we may not only make a just Estimate of its Quantity in every simple Engine, but also find out the Friction of the several Parts which make up a compound Engine, so as to enable us to know what to allow for the Friction of the whole complex Machine) depends upon so many Experiments and Observations, that I rather choose to consider that Subject in a *Lecture* on purpose, (*viz.* the 4th *Lecture*) than go on with it here, which would swell these Notes to too great a Bulk; so much remaining to be considered in relation to the several Particulars of the 3d *Lecture*, that they will far exceed the Length of the *Lecture*; unless I were to omit some Things very material, which yet are too difficult to be mentioned in the *Lectures* themselves, which I have made so easy, as to require only the Reader's Attention, without any previous Knowledge of Mathematics.

4. [20. —— *The Beam hanging freely on its Center of Motion, which is placed a little above its Center of Gravity.*] As no Position of the Beam of the Balance but the horizontal can make us judge of the Weight of the Bodies to be compared by that Instrument, we must be careful that the Center of Motion or Point of Suspension of the Beam be not in the Center of Gravity, because then not only the Beam would remain in any given Position * (as well inclined as horizontal) but also continue in that Position * L. 2. 26.

‡ Though, strictly speaking, the Friction of Bodies of the same Weight does not increase in Proportion to the Number of Parts that rub; yet it so happens here, because in this Case the rubbing Surfaces apply less exactly to one another than in the Wedge.

Annotat. Lect. III.

sition when equal Weights are suspended at its Ends. To illustrate this, let us examine the 1st *Figure* of *Plate* 14 †.

† Pl. 14. F. 1.

H C *b* D, represents the Section of a Beam of some Thickness, which when suspended by its Center of Gravity K, will as well remain in the inclined Position A E B F as in the horizontal Position: Now if the heavy Bodies P, W, (equal, if the Beam be equally divided in its Length by the Point K, or reciprocally proportionable to the *Brachia* if the Beam be unequally divided) be suspended at the Ends A, B, they will hang *in Æquilibrio* in any Inclination of the Beam, let the Beam be of any Size whatever. *First*, Let the Beam be supposed so slender as to have little or no Weight in Comparison of the Bodies, as the Line A B, with its Center of Gravity at K: It is evident, that the common Center of Gravity of P and W, which is at G, will not be sensibly removed from the Point G by the Addition of the Beam supposed of little or no Weight, nor by the Removal of the Bodies to *p*, *w*, when the Beam is inclined into the Position *a b* *. *Secondly*, If we consider the Beam with all its Weight; when in the horizontal Situation C D, the heavy Part C B pressing upon A B being equal and equally distributed over the Beam, and the lower Part A D equal to C B, hanging under in the same Manner as C B presses above, the Center of the Gravity of the Beam will not on that Account be removed from the Point K; but the common Center of Gravity of the Bodies P, W, and the Beam will be removed from G to *g* under the Center of Motion K; therefore the Balance and Weights will remain in that Position, because the Point K of the Line of Direction is supported ‖. Neither will the Weights (which in a Balance always hang freely) be able to alter this horizontal Position by their Suspension, because their Distances A K, B K from their Lines of Direction to the Center of Motion K, (upon which their Velocities depend) are equal, or reciprocally proportionable to their Masses *. Now if the Beam be inclined in the Position E A F B, we may still consider in it the slender Beam A B, loaded above and below with the two equal Prismatical-Wedges A F B and A E B, whose Centers of Gravity bein at *m* and *n*, their Lines of Direction will go through the Points *r* and *s* equally distant from the Center of Motion K, therefore they will balance each other †, and consequently not alter the inclined Position of the Beam. Then if we consider the Weights *p* and *w* suspended from the Points *a* and *b*, the common Center of Gravity of the Beam and Weights will still be at *g*, and the Distances of the Lines of Direction of the Weights now becoming *t* K and *u* K, decrease exactly in a reciprocal Proportion to the Weights; so that there can be no Motion occasioned by this Position of the Beam and Weights, because there is no Alteration of Place in the common Center of Gravity of the whole loaded Balance, or the respective Velocity of the Weights.

* L. II. 31, 32, 33.

‖ L. II. 47.

* L. II. 13. and L. III. 13, and 22.

† 18.

But if we remove the Center of Motion or Point of Suspension of the Balance to *k*, a little above the Center of Gravity of the Beam K, the Line of Direction, which in the horizontal Position of the Beam is *k g*, will in the inclined Position of the Beam be out of the Point of Suspen-

sion, which will then be removed to *c*, and the Center of Gravity must move from *g* to *q*, which it can do describing a small Arc round the Point *c*; which will reduce the Beam to an horizontal Position, in which the Line of Direction will be *c q* going through the Point of Suspension *c*, and consequently *Weights that are* in Æquilibrio *upon a Balance whose Center of Motion is above the Gravity of the Beam will reduce the said Balance from an inclined to an horizontal Position.* Annotat. Lect. III.

Now in fixing the Center of Motion of the Beam above the Center of Gravity, Care must be taken to fix it above the Points of Suspension, as some Authors have taught, and as the Practice is in making the common Scale-Beams, which may be considered as made up of two Leavers making an obtuse Angle at *k* the Center of Motion, whilst the Points of Suspension A, B, are under its Level in the Line A B. Such a Balance is used for common Purposes, because it comes sooner to an *Æquilibrium*, than if A *k* B was one Line; *but it is a false Balance, and a skilful Person may cheat with it in proportion as the Angle* A *k* B *is more acute, especially when there is no perpendicular Piece or Examen as* C *r* *, *to shew when the Balance is truly horizontal; for unequal Weights may make an* Æquilibrium *on such a Balance, and not be discovered by changing the Scales,* which presently discovers the Cheat in a Balance whose Beam has its *Brachia* unequal †. For Example, let the Balance A C B *c* (whose Center of Gravity is at *c*, and Center of Motion at C, with the equal Weight P, P, hanging at its Extremities A B) be placed in the inclined Position *a b*; I say, that as the Line of Direction D d of the Weight P is brought nearer to the Center of Motion (*viz.* to *q π*) the said Weight may be increased in proportion as its Distance C *u* is diminished by being reduced to C *q*, and act with the very same Force on its Point of Suspension: Whilst the Line of Direction of the opposite Weight *P* being removed from *D d* to *b ϖ*, its Distance C *o* becomes C *b*, and therefore *P* may be diminished in proportion as its Distance is increased; therefore in that Situation of the Balance the Weights P and *P* will keep each other *in Æquilibrio*, when they differ in the reciprocal Proportion of their Distances *q* C and *b* C, or b C and C *α*, if the Balance be inclined the other Way in the Line *α β*; the Discovery being only made by the visible Inclination of the *Examen* towards *s* or *t*. *Q. E. D.*

* Pl. 14. F. 2.

† 27.

The nearer the Center of Gravity of the Beam is to the Center of Motion, the nicer will be the Balance, because the Beam will be the more apt to vibrate quick from Side to Side. As for Example, if *α c b* C * be the Beam, and C the Center or Axis of Motion, the Difference between the Effect of having the Center of Gravity at K, or *c*, will be the same as if we compared the Vibrations of two Pendulums of the Length C K and C *c*, whose Velocities in their Vibrations reciprocally are in a subduplicate Ratio of their Lengths (as I shall farther shew when I come to treat of Pendulums) for the Beam is really a Pendulum.

* Pl. 14. F. 2.

Annotat. Lect. III.

EXPERIMENT. *Plate* 14. *Fig.* 3.

Pl. 14. F. 3. To the Beam A B, whoſe Axis of Motion is C, fix a ſcrewed Wire K *c*, and a Ball W ſo contrived, that it may be ſcrewed on towards, or ſcrewed fromwards the Axis C. When by bringing the Ball to W the Center of Gravity is at K, the Vibrations of the Beam will be quicker than when the Center of Gravity is brought to *k* by lowering the Ball to *w*. *This may be uſed in Practice for ſome nice Experiments, becauſe by ſuch a Contrivance the Center of Gravity may be brought as near as you pleaſe to the Center of Motion.*

N. B. That the Center of Gravity ought never to be above the Center of Motion appears from what has been already ſaid in the ſecond *Lecture*, N°. 26. L. 2. 26.

5. [27. *In all theſe Caſes we ſuppoſe the Weights to hang freely from the Ends of the Balance to which they are faſtened.*] Though in the common Uſe of the Balance the counterpoiſing Weights, or the Scales, generally hang freely; yet there are ſome Caſes where they do not; and in compound Engines, where Balances are often a Part of a complex Machine; inſtead of Weights, Powers are applied to their Ends in all Manner of Directions, and then they become Leavers of the firſt Kind, ſuch as the Regulators in ſeveral Water-Engines, Beams to blow Bellows, *&c.* Therefore I ſhall conſider the Effects of Powers applied obliquely to a *Mathematical Balance* or ſtraight inflexible Line, which will alſo ſolve all Caſes of the Leaver, and (with proper Allowances) may be applied to all the Mechanical Organs.

*Pl. 14. F. 4. Let the Balance A B *, 12 Inches long and equally divided by its Center of Motion C, ſuſtain at its Ends the two equal Weights W, P, which laſt we ſhall conſider as the Power. Whilſt the Power draws in the Line †L. 2. 20. B P, it acts according to its whole Intenſity †, its Diſtance being then C B = C A the Diſtance of the Weight, both Diſtances being meaſured upon the Beam: But if the Power be removed to *P*, and (its String B *x* P running over the Pulley *x*) it draws obliquely in the Line B *x*, which makes with the Beam the acute Angle C B F, or (what comes to the ſame) the obtuſe Angle C B E (becauſe here we ſuppoſe it as much greater than a right Angle as C B F is leſs, C B F being = C B ϕ) the Force of the Power will be diminiſhed in proportion as C F or C ϕ, the Diſtance from the Line of Direction of the Power acting obliquely, is leſs than C B the Diſtance of the Line of Direction of the Power acting directly or at right Angles to the *Brachium* of the Balance C B *. Knowing therefore the Intenſity of the Power, which acting at right Angles at B (or which hanging freely from B, if an heavy Body, as repreſented here, by the Power) keeps the oppoſite Weight W *in Æquilibrio*, one may eaſily find how much the Power muſt be increaſed to keep the ſaid Weight *in Æquilibrio* when it draws obliquely

/* 20.

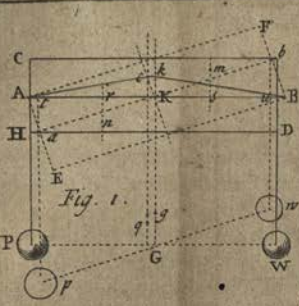

Fig. 1.

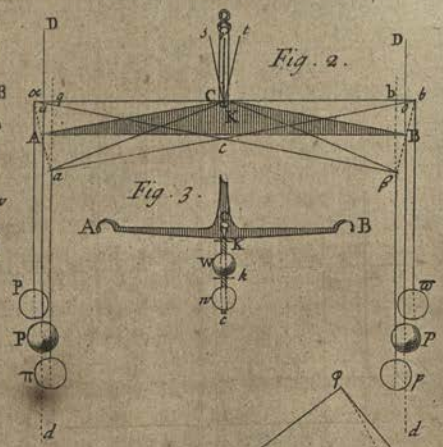

Fig. 2.

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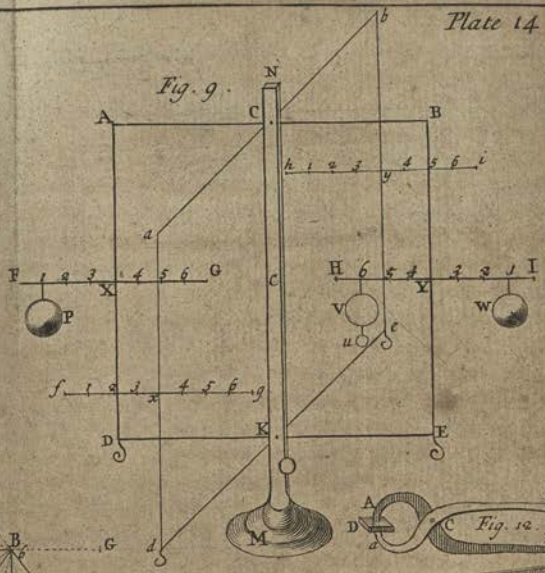

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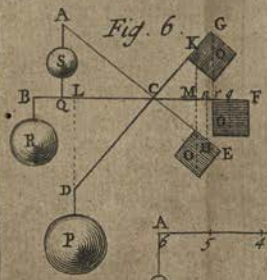

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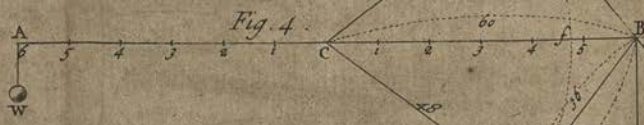

Fig. 4.

Fig. 12.

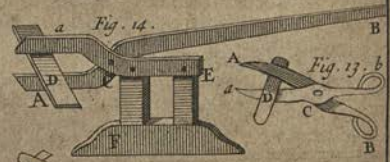

Fig. 14.

Fig. 13.

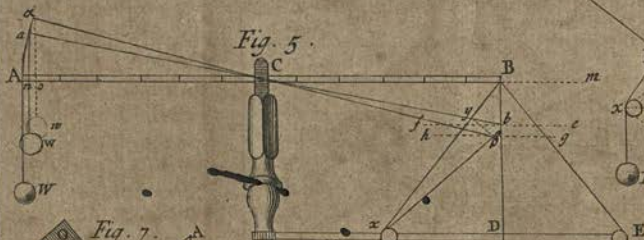

Fig. 5.

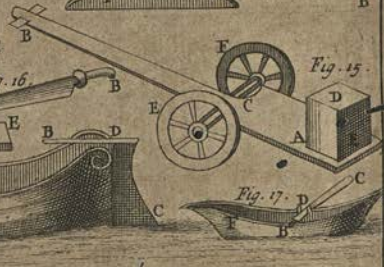

Fig. 15.

Fig. 16.

Fig. 17.

Fig. 18.

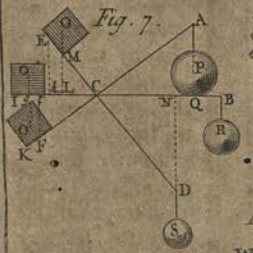

Fig. 7.

Fig. 8.

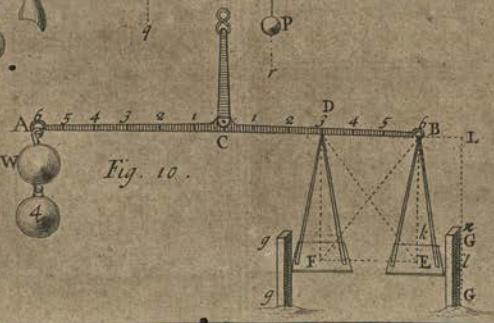

Fig. 10.

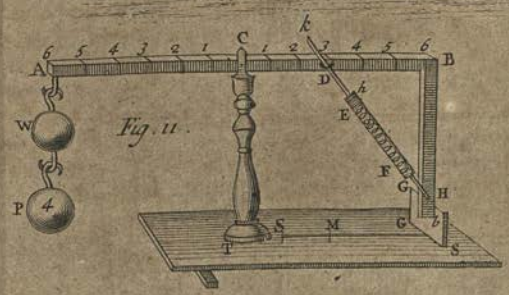

Fig. 11.

obliquely in any known Direction, as for Example in the Direction repre- Annotat.
sented in the Figure; or (which is all one) how much the Weight *P*, Lect. III.
whose String fastened to B runs over the Pulley *x*, must be greater than
the Weight P, which hangs freely, to have the same Effect. With the
Distance C F or C ϕ draw the Arc F *f* ϕ, which cuts C B at *f*; then you
will have the Quantity of *P* by this Analogy:

As the Length C *f upon the Beam, whose Length is* 4, 87 *Inches*:
Is to C B *the whole* Brachium *of the Beam, here* 6 *Inches long*::
So is the Intensity of the Power, or the Weight P, *here supposed* 40 *Pounds*:
To the new Power, or Weight P, 50 *Pounds.*

Hence it appears that the Weight *P* thus found, would keep the counterpoising Weight W *in Æquilibrio* by hanging freely at the Point *f*, as well as it does it by drawing obliquely over the Pulley *x*; because the Momentum of W (or W × A C) being divided by C *f*, will give the Quantity of the Weight *P*; or, in other Words, there will be a reciprocal Proportion between the Weights W and *P*, and their Distances C A and C *f* *. Therefore C *f* = C ϕ = C F the Distance of the Line of Direction * 13. and L. II. 9, 12, 13, *and* 15.
from the Center of Motion (*always found by the Length of a Perpendicular from that Center to the said Line of Direction*) may in all such Cases be properly called *the acting Distance of the Power*. See N°. 20.

Another Way.

Through the Point *x* taken in the Circumference of the Pulley, over which the String B *x P* runs, draw *x* E parallel to the Balance A B, which will cut the perpendicular Line of Direction of the Power (or freely hanging Weight) P, at right Angles at D, D *x* being = D E, and the Angle *x* B D = D B E by Supposition. In the rectangular Triangle B D *x*, as much as the Hypotenuse B *x* is longer than the Perpendicular B D, so much must the Quantity or Intensity of the Weight P (which hanging freely, keeps W *in Æquilibrio*) be increased when it draws obliquely in a Line B *x*; that is, so much must *P* be greater than P, to keep W *in Æquilibrio* drawing obliquely. The same would be true if *x* was removed to E; and therefore this will always be the Rule to estimate the Force of Powers drawing obliquely;

As the Sine of the Angle of Traction, viz. *the Angle which the Line of Direction of the Power makes with the Beam*:
Is to the Radius::
So is the Intensity of the Power drawing at right Angles with the Beam:
To the Intensity of the Power drawing obliquely.

N. B. The Angle of *Traction* here is C B *x* or C B E, whose common Sine is B D; then in this Case B D : B *x* :: P : *P*.

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That this Method will always have the same Consequence as the former, may be seen by comparing together the two Triangles B *x* D and C B F; for since C F B is a right Angle by Supposition, and C B *x* = B *x* D because of the Parallels C B and *x* D, B C F must be equal to *x* B D, and consequently the Triangles be similar, which will shew that C F (48) : C B (60) : : B D (40) : B *x* (50) : : P : *P*. *Q. E. D.*

This might also be explained a third Way, by resolving the Force which draws obliquely in the Line B *x*, into the Forces, the one drawing along the Leaver in the Line B C, and the other in the Line B D, after Sir *Isaac Newton*'s Manner *; but we shall consider it in the Case of Trusion or pushing obliquely against any Point of the Beam, which is the same as drawing the contrary Way.

85, 86.

All these Methods will appear to agree in the following

EXPERIMENT. *Pl.* 14. *Fig.* 5.

The Balance or Leaver A B, 12 Inches long, is moveable upon the Center C of the Stand S, which has a long Piece so propped at S, as to remain in an horizontal Position, so as to carry the Pullies *x* and E, each 3 Inches distant from the Point D, which is placed perpendicularly under B. When the Power P of four Ounces equal to W hangs freely, it keeps W *in Æquilibrio*; but if the String P X B be thrown over the Pulley *x* or E, then will the said Power P be overpois'd by W till it be changed for *P* or *p* a Five-ounce Weight, which drawing obliquely over *x* or E, will keep W *in Æquilibrio*. Now one may observe, that when P descends one Inch, or from P to *r*, it brings down the End of the Beam B only to the Point *b* in the horizontal Line *f e*, which raises the opposite End A to *a*, just as high above the Line A; but when the Power at *P* descends one Inch, namely to *q*, it brings down the End B to β in the horizontal Line *b g*, and consequently raises the opposite End A so much higher, so as to give more Velocity to the Weight W, bringing it to *w* instead of w. Now since the Powers P and *P* with the same Velocity (or descending equally) give W different Degrees of Velocity, their Intensities must be different in that Proportion, because *Causes are always proportionable to their Effects*; therefore *P* must be greater than P as much as the Arcs A α and B β are greater than A *a* and B *b*, or rather as much as the Sine α *o* is greater than *a n*. This also appears by observing (since B X = β *x* (= *y x*) and B *y*) that the Strings B P, B *x P*, and β *x P q* are all equal.

N. B. *This is strictly true only in the Beginning of the Motion of the Beam; but that is sufficient for our Purpose.*

The very Sight of the Machine makes it plain that a Power acting at right Angles is the most effectual. For as in removing the Power from the Perpendicular it draws more weakly at E, and at *m* it only pulls the Center C in the Direction B *m*, so as to do nothing towards raising A, the

oppoſite End of the Beam; and going the contrary Way, the Power weakened at *x*, becomes wholly ineffectual when brought to the Beam to draw in the Direction B C, becauſe then it only acts againſt the Center C, no way moving the End A: Therefore a mean Situation of the Line of Direction between thoſe two ineffectual Extremes, muſt be the moſt effectual; and that is in the Line B P perpendicular to the Beam. Annotat. Lect. III.

It is upon this Principle that moſt of the Feats of the pretended ſtrong Men, or modern Sampſons, *are performed; the Machines on which they ſit or ſtand being ſo contrived, that the drawing Horſes or hanging Weights pull thoſe Limbs of the Man reſiſting (which perform the Office of a Leaver or Balance) in ſuch a Manner, that they are drawn directly againſt the Center of* Motion. But this I ſhall explain more particularly when I come to ſpeak of thoſe Feats of Strength.

If at one of the Ends of the Leaver or Balance be fixed a Weight which (moving with the ſaid End of the Leaver) does not hang freely, whilſt the Power acting at the other End is either a heavy Body hanging freely, or an animate Power preſſing perpendicularly towards the Earth; I ſay, that ſuch a fixed Weight will vary in Force according to the Poſition of the Beam, and that Force will vary in a contrary Manner according as the Center of Gravity of the Weight is above or below the Beam: namely, when the Center of Gravity of the fixed Weight is below the Beam (as in Fig. 6.) *the Weight will become heavieſt (or act moſt ſtrongly) when raiſed above the horizontal Line (as at* G) *in the inclined Situation of the Leaver* D C G; *and it will become lighteſt when depreſſed below the horizontal Line (as at* E) *in the inclined Situation of the Leaver* A C E; *on the contrary, if the Center of Gravity of the fixed Weight be above the Leaver (as in Fig.* 7.) *it will become heavieſt when depreſſed below the horizontal Line (as at* K) *in the inclined Situation of the Leaver* A C K; *and become lighteſt when raiſed above the horizontal Line (as at* E) *in the Situation of the Leaver* D C E. *But the ſaid fixed Weight will act in the ſame Manner as if it hung freely when the Leaver is in the horizontal Situation, as at* B F (*Fig.* 6.) and B I (*Fig.* 7.) Pl. 14. F. 6. Pl. 14. F. 7.

The firſt Caſe (*Fig.* 6.) may be thus explained. In the Poſition of the Leaver B F the Line of Direction *q* O going thro' the Point of Suſpenſion *q*, and being at right Angles with the Leaver, C *q* is the *acting Diſtance* *, of the Weight as well as the Diſtance of the Point of Suſpenſion; therefore as B C : to C *q* :: ſo is the Weight F : to the Power R; in the ſame Manner as if the Weight F hung freely from *q*. But when the Weight is raiſed up to G, as the Center of Gravity O cannot get under the Point K (the ſame as *q* the Point of Suſpenſion in the horizontal Leaver) O *r* becomes the Line of Direction inſtead of K M, which would have been the Line of Direction if the Body had hung freely from K; therefore C *r* is the acting Diſtance of the Weight inſtead of C M, when the acting Diſtance of the Power is become L C; and conſequently the Weight has more Force, and can only be balanced by a greater Power as P. For whereas C L : C M :: F (or G) : R, now C L : C *r* :: F (or G) : P, a ſtronger Pl. 14. F. 6. * *See Page* 147.

Annotat. Lect. III. a stronger Power or heavier Counterpoise. But if the Weight be brought to E, its Line of Direction instead of *n* H becomes M O, and consequently its acting Distance is less than it ought to be in Proportion to C Q the acting Distance of the Power, which therefore may be diminished in Intensity and become S instead of R.

*Pl. 14. F. 7. From what has been said, and a View of *Fig.* 7, * one may see that the contrary must happen, when the fixed Weight has its Center of Gravity above the Leaver or Beam. For in the Situation of the Leaver E D, the Line of Direction of the Weight coming forward too fast (becoming O L instead of E *b*) C L the acting Distance of the Weight at E bears a less Proportion to C N the acting Distance of the Power at S, than C *g* to C B, the Proportion in the horizontal Situation; and consequently the Power S of less Intensity serves as a Counterpoise instead of R. But when the Weight is depressed to K, we must make use of P, a greater Counterpoise; because C *g* is become the acting Distance instead of C *b*. And in the horizontal Situation of the Beam the Body O weighs the same as if it hung freely, because O *g* is its Line of Direction, as it would be if the said Body hung down from *g*. This may be observed in Practice. Suppose the Man
Pl. 14. F. 8. M (*Fig.* 8.) is lifting Hay, Sheaves of Corn, or a large Faggot A, by means of the Prong or Fork A B resting upon his Knee *c* as a *Fulcrum*, and pressing down the End B of the Prong or Leaver; if the Fork A goes un-
Pl. 14. F. 7. der the Faggot, it will be the Case of the inclined Leaver A K (*Fig.* 7.) and the Burthen will grow lighter as it rises. But if the Fork had been thrust through the Binding above the Faggot, it would be the Case of
Pl. 14. F. 6. the inclined Leaver A E (*Fig.* 6) at whose End the Burthen becomes heavier as it is raised up; and then if the Man was just able to begin to lift it, he must let it go back again, or find a new *Fulcrum* at C in order to raise that Weight. Suppose again, that the Beam of a large Pair of Scales, was inclined in the Position A C E; a Man in the Scale suspended at H may, by thrusting up his Hand hard against the Beam, put himself in the Condition of the heavy Body H E, and consequently appear to weigh less than his true Weight, being counterpoised by such a Weight as S. But if the Scale which he gets into be raised up, its Point of Suspension being at K, in the Position of the Leaver D C K; the Man, by thrusting hard against the Beam above his Head, may throw his Body into a Position perpendicular to the Leaver, and together with the Scale be in the Condition of the Body K G; so that if there is more Weight in the opposite Scale, the Weight need not be lessened till its Intensity be equal to that of the Man's Weight, but it will begin to be lifted up whilst it is still greater; so that in such a Case, a Man's Weight will appear greater than it is, by as much as the Counterpoise P is greater than R. Just the Reverse would happen, if a Man was to sit upon the Beam; for then he would weigh least, when raised above the horizon-
Pl. 14. F. 7. tal Line going through the Center, as at E M (*Fig.* 7.) and most when below the said Line, as at F K.

Now

Now though the Action of heavy Bodies on one another may thus in the Balance and Leaver, and some other of the Mechanical Organs, be estimated by the Distance of their Line of Direction from the Center of Motion; yet this is only so far true, as the perpendicular Ascent and Descent is agreeable to that Distance; for there are Cases especially in the working of Compound Engines, where the Distance of the Line of Direction of a rising or falling Body from the Center of Motion is not proportionable to the perpendicular Ascent or Descent of the said Body. Therefore the Velocity of a Power, when it is an heavy Body, must be considered in the same Manner as that of a Weight; as has been fully explained in the 8th Note of the second Lecture *; and may be farther proved by the following Annotat. Lect. III. *L. 2. Ann. 8.

EXPERIMENT.

A C B E K D * is a Balance in the Form of a Parallelogram passing through a Slit in the upright Piece N O standing on the Pedestal M, so as to be moveable upon the Center-Pins C and K. To the upright Pieces A D and B E of this Balance are fixed at right Angles the horizontal Pieces F G and H I. That the equal Weights P, W, must keep each other *in Æquilibrio* is evident; but it does not at first appear so plainly, that if W be removed to V, being suspended at 6, yet it shall keep P *in Æquilibrio*; though the Experiment shews it. Nay, if W be successively moved to any of the Points 1, 2, 3 E, 4, 5, or 6, the *Æquilibrium* will be continued; or if, W hanging at any of those Points, P be successively moved to D or any of the Points of Suspension on the Cross-Piece F G, P will at any of those Places make an *Æquilibrium* with W. Now, when the Weights are at P and V, if the least Weight that is capable to overcome the Friction at the Points of Suspension C and K be added to V as *u*, the Weight V will overpower, and that as much at V as if it was at W. * Pl. 14. F. 9.

From what we have said above, the Reason of this Experiment will be very plain. As the Lines A C and K D, C B and K E always continue of the same Length in any Position of the Machine, the Pieces A D and B E will always continue parallel to one another and perpendicular to the Horizon, however the whole Machine turns upon the Points C and K; as appears by bringing the Balance to any other Position, as *a b e d*; and therefore as the Weights applied to any Part of the Pieces F G and H I can only bring down the Pieces A D and B E perpendicularly, in the same Manner as if they were applied to the Hooks D and E, or to X and Y, the Centers of Gravity of A D and B E; the Force of the Weights (if their Quantity of Matter is equal) will be equal; because their Velocities will be their perpendicular Ascent or Descent, which will always be as the equal Lines 5 *x* and 5 *y*, whatever Part of the Pieces F G and H I the Weights are applied to. But if to the Weight at V be added the little

Annotat. Lect. III. little Weight *u*, those two Weights will overpower, because in this Case the *Momentum* is made up of the Sum of V and *u* multiplied by the common Velocity 5 *y*.

Pl. 14. F. 9. Hence follows, that it is not the Distance *c* 6 multiplied into the Weight V, which makes its *Momentum*; but its perpendicular Velocity 5 *y* multiplied into its Mass *.. *Q. E. D.*

This is still farther evident by taking out the Pin at K; for then the Weight P will overbalance the other Weight at V, because then their perpendicular Ascent or Descent will not be equal.

To conclude all that relates to Forces applied in different Directions to Leavers and Balances, I shall explain the Action of oblique Forces by the *Composition* and *Resolution* of Motion after Sir *Isaac Newton*'s Manner; applying the Solution to a Proposition of the Balance, which has not been taken notice of by mechanical Writers, though often talked of by handicraft Workmen.

Pl 14. F. 10. THEOREM. *Fig.* 10.

A B *is a Balance, on which is supposed to hang at one End* B *the Scale* E *with a Man in it, who is counterpoised by the Weight* W *hanging at* A *the other End of the Balance.* I say, *that if such a Man, with a Cane or any rigid straight Body, pushes upwards against the Beam any where between the Points* C *and* B *(provided he does not push directly against* B*) he will thereby make himself heavier or overpoise the Weight* W, *though the Stop* G G *hinders the Scale* E *from being thrust out fromwards* C *towards* G G. I say likewise, *that if the Scale and Man should hang from* D, *the Man by pushing upwards against B, or any where between* B *and* D, *(provided he does not push directly against* D*) will make himself lighter, or be overpoised by the Weight* W, *which did before only counterpoise the Weight of his Body and the Scale.*

If the common Center of Gravity of the Scale E and the Man supposed to stand in it be at *k*, and the Man by thrusting against any Part of the Beam cause the Scale to move outwards so as to carry the said common Center of Gravity to *x*; then instead of B E, L *l* will become the Line of Direction of the compound Weight, whose Action will be increased in the Ratio of L C to B C. This is what has been explained by several Writers of Mechanicks; but no one, that I know of, has considered the Case when the Scale is kept from flying out, as here by the Post G G, which keeps in its Place, as if the Strings of the Scale were become inflexible. Now, to explain this Case, let us suppose the Length B D of half of the *Brachium* B C to be equal to three Feet, the Line B E to four Feet, and the Line E D (of five Feet) to be the Direction in which the Man pushes, D F and F E to be respectively equal and parallel to B E and B D, and the whole or absolute Force with which the Man pushes, equal to (or able to raise) 10 Stone. Let the oblique Force E D (= 10 Stone) be

be resolved into the two E F and E B (or its equal F D) whose Directions are at right Angles to each other, and whose respective Quantities (or Intensities) are as 6 and 8, because E F and B E are in that Proportion to each other and to E D. Now since E F is parallel to B D C A the Beam, that Force does no way affect the Beam to move it upwards; and therefore there is only the Force represented by F D, or 8 Stone to push the Beam upwards at D. For the same reason, and because Action and Reaction are equal, the Scale will be pushed down at E with the Force of 8 Stone also. Now, since the Force at E pulls the Beam perpendicularly downwards from the Point B distant from C the whole Length of the *Brachium* B C, its Action downwards will not be diminished, but may be expressed by $\overline{8 \times BC}$: Whereas the Action upwards against D will be half lost, by reason of the diminished Distance from the Center, and is only to be expressed by $8 \times \frac{BC}{2}$; and when the Action upwards to raise the Beam is subtracted from the Action downwards to depress it, there will still remain 4 Stone to push down the Scale; because $\overline{8 \times BC} - \overline{8 \times \frac{BC}{2}} = 4\,BC$: Consequently a Weight of 4 Stone must be added at the End A to restore the *Æquilibrium*. *Therefore a Man*, &c. *pushing upwards under the Beam between* B *and* D, *becomes heavier*. *Q. E. D.* Annotat. Lect. III.

On the contrary, if the Scale should hang at F from the Point D only three Feet from the Center of Motion C, and a Post *g g* hinders the Scale from being pushed inwards towards C; then if a Man in this Scale F pushes obliquely against B with the absolute Force above-mentioned; the whole Force, for the Reasons before given (in resolving the oblique Force into two others acting in Lines perpendicular to each other) will be reduced to 8 Stone, which pushes the Beam directly upwards at B, while the same Force of 8 Stone draws it directly down at D towards F. But as C D is only equal to half of C B, the Force at D compared with that at B loses half its Action, and therefore can only take off the Force of 4 Stone from the Push upwards at B; and consequently the Weight W at A will preponderate, unless an additional Weight of 4 Stone be hanged at B. *Therefore a Man*, &c. *pushing upwards under the Beam between* B *and* D, *becomes lighter*. Which was also to be demonstrated.

Scholium I.

Hence, knowing the absolute Force of the Man that pushes upwards (that is, the whole oblique Force) the Place of the Point of Trusion D, and the Angle made by the Direction of the Force with the Beam at the said Point, we may have a general Rule to know what Force is added to the End of the Beam B in any Inclination of the Direction of the Force or Place of the Point D.

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Rule for the firſt Caſe.

Firſt find the perpendicular Force by the following Analogy, whoſe Demonſtration is known to all that underſtand the Application of oblique Forces.

As the Radius:
To the right Sine of the Angle of Inclination of the Force to the Beam::
So is the oblique Force:
To the perpendicular Force.

Then the perpendicular Force multiplied into the Length of the *Brachium* BC, *minus* the ſaid Force multiplied into the Diſtance DC, will give the Value of the additional Force at B, or of the Weight required to reſtore the *Æquilibrium* at A.

Or to expreſs it in the algebraical Way: Let of expreſs the oblique Force, pf the perpendicular Force, and x the Force required, or Value of the additional Weight at A to reſtore the *Æquilibrium*.

$$\begin{array}{l} DE : DF\ (=BE) :: of : pf. \\ \overline{pf \times BC} - \overline{pf \times DC} = x. \end{array}$$

The ſame Rule will ſerve for the ſecond Caſe, if the Quantity found be made negative, and the additional Weight ſuſpended at B. Or having found the Value of the perpendicular Force, the Equation will ſtand thus: $\overline{-pf \times BC} + pf \times DC = -x$; and conſequently the additional Weight muſt be hanged at B; becauſe $-x$ at A is the ſame as $+x$ at B.

Scholium 2.

Hence it follows alſo, that if, in the firſt Caſe, the Point of Truſion be taken at C, the Force at B (or Force whoſe Value is required) will be the whole perpendicular Force; becauſe CD is equal to nothing. And if the Point D be taken beyond C towards A; the perpendicular Force puſhing upwards at that Point, multiplied into DC, muſt be added to the ſame Force multiplied into BC; that is, $\overline{pf \times BC} + \overline{pf \times BC} = x$.

Plate 14. Fig. 11.

The Machine I made uſe of to prove this experimentally, was as follows, (*Fig.* 11.) The Braſs Balance AB is 12 Inches long, moveable upon the Center C, with a perpendicular Piece B*b* hanging at the End B, and moveable about a Pin at B, and ſtopped at its lower End *b* (by the upright Plate GG) from being thruſt out of the Perpendicular by the puſhing Pipe FE, whoſe lower Point being put into a little Hole between the Letters G and H, the upper Wire or Point (when put into another little Hole under

under the Beam at D) is by means of the Worm-Spring E F pressing against the Plug E, to drive forward the said Wire *b* D, made to push the Beam upwards with the Force of the Spring E F. T S S is a Stand to which is fixed the Pillar T C, which sustains the Balance; and it has also a Slit S S to receive a Shank of the moveable Plate G G, to be fixed in any Part of the Slit by a Screw underneath. Annotat. Lect. III.

EXPERIMENT. *Plate* 14. *Fig.* 11.

Hang on B *b*, as in the Figure. Then let E F be so applied to the Hole H, that its upper Wire *b* D *k*, may go through a little Loop at D, so as not to thrust the Beam upwards, but be in the same Position as if it did, that by hanging on the Weight W the *Brachium* B C with B *b* and F E may be counterpoised, that the Action against D and H may be estimated without the Weight of the Pushing-pipe.

Then drawing down the End of the Wire *k*, thrust it into the little Hole under D, and B will be so pulled downwards as to require the additional Weight P of 4 Ounces to be hung on at A, to restore the *Æquilibrium*; when B H is 4 Inches, B D 3 Inches, and the whole Force of the Spring equal to 10 Ounces.

I need not here say, that for explaining the second Case, B *b* is to be suspended at D, with the Plate G G fixed to stop it at the Place M, to keep it from being pushed towards T, and that the upper End of G F E D *k* must push into a Hole made under B, in which Case the Weight P must be hanged at B, to restore the *Æquilibrium.*

N. B. To shew experimentally that the Force which the Spring exerts in this oblique Trusion is equal to 10 Ounces; take the Beam A B, which weighs 4 Ounces, from its Pedestal C T, and having suspended at each End A and B, 3 Ounces, support it under its Center of Gravity by the Pushing-pipe E H set upright under it, and you will find that the Beam with the two Weights will thrust in the Wire H *k* as far as D, the Place which the oblique Trusion drives it to.

6. [29. — *In all these Cases the Leaver is still said to be of the first Kind.*] There is another way of distinguishing the Leavers, according to *Aristotle* and the Mechanical Writers among the Ancients; and that is, as the Weight does, or does not, rise in the same Direction as the Power. As for example, in the Leaver of the first Kind; as it has the *Fulcrum* (Center of Motion, or *Hypomochlion*) between the Power and the Weight, the Power must move downwards whilst the Weight moves upwards; and that Leaver is by those Authors called the *Heterodromus* Leaver; that is, *working or moving different Ways.* But the Leaver of the second, and likewise the Leaver of the third Kind, are both called *Homodromous* Leavers, because the Power and Weight being on the same Side of the *Hypomochlion* or Propping point, they both go the same Way, though in the one the Power always gains, and in the other always loses.

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If we examine such Instruments as we have in common Use, we may plainly see that they are Leavers of one of the three Kinds. As for example, a Pair of Pincers * is made up of two Leavers of the first Kind, whose common Center of Motion is at the Rivet C, the Power being applied at the Handles B *b* to press them together, and thereby pinch the Body D as a Weight at the opposite End A *a*; in which Case, the Power acts six times more strongly than if applied directly to the Body D at A, *a*; since in the two Leavers A B and *a b* the Distance of the Power B C and *b* C is triple the Distance of the Weight C A and C *a*. So is a Pair of Sciffors † made up of two such Leavers, whose common Center of Motion is C, the Power being applied at B *b*, and the Body to be cut as a Weight at D; where it is evident that the nearer D is to the Points A, *a*, the greater will be the Difficulty in cutting it; and the less, the nearer it is brought to C. The Force of a Leaver in this Way is remarkable in the Brasier's or Tin-man's Sheers *, whereby one Man pressing upon the Handle B raising the lower Sheer A C, moveable about the Center C, is able to cut a Plate of Brass or Copper D a quarter of an Inch thick, the other Sheer *a* C E being riveted to a couple of strong Standards fixed into the Block F. Nippers, Forceps, Snuffers, and other such Instruments may in the same manner be found to be Leavers of the first Kind. The little Cart or Carr B C A *, very useful in Building, is also a Leaver of the first Kind: It is made to lift great Stones, and carry them to the Builders; upon the farther End of whose Plank or Bottom A the Stone D being laid, and with a little Pains moved towards C by wrigling the Plank; the Force of one Man, by taking hold of the long Handle at B, and weighing it down as it turns about the Axle-tree of the Wheels E F as the *Fulcrum*, will be able to raise that Stone; and being raised, by Help of the Wheels, be able to convey it to the intended Place.

* Plate 14. Fig. 12.

† Plate 14. Fig. 13.

* Plate 14. Fig. 14.

* Plate 14. Fig. 15.

The Cutting-knife * used by Druggists and Patten-makers, to cut Drugs, or the Wood they use, is moveable on the Joint or Center of Motion C, whereby it is fastened to the Plank C E; the Power is applied at the Handle B, and the Weight is D, the Wood or Drug to be cut: This shews the Instrument to be a Leaver of the second Kind †. So is a Door, whose Hinges are the Center or Axis of Motion, the Hand or Power being applied at the Part towards the Lock, whilst the Body of the Door is the Weight. A Pair of Bellows are two Leavers of the second Kind, whose common Center of Motion is at the End of the Boards where the Nose begins, the Power being applied at the Handles, whilst the Air to be pressed out between the Boards, by its Resistance acts against the Middle of the Boards as a Weight. Thus one may easily perceive that the Latch of a Door, when drawn up by a String, is a Leaver of the second Kind; as are also jointed Nut-crackers, Horse-barnacles, *&c.* The Oars of a Boat or Galley, and the Rudder, are also Leavers of the second Kind, though *Aristotle* supposed them Leavers of the first; but the Error lies in this, that he considered the Water as the Weight to be moved; whereas

* Plate 14. Fig. 16.

† 30.

the Boat or Galley is the Weight to be moved. For the Water makes Resistance at C * as a *Fulcrum*, the Man or Power acting at B, whilst the Vessel F D is moved by that Part of the Leaver which presses on it at the Point D, as the Oar in *Fig.* 17. and the Rudder in *Fig.* 18. But *Aristotle*, in his *Mechanics*, rightly reckons the Masts of Ships among Leavers of the second Kind, assigning the *Fulcrum* or *Hypomochlion* to be in the Bottom of the Ship B †, considering the Ship itself as the Weight or Burthen, resting with its Upper-deck C upon the Mast, as upon a Leaver, and so to be moved forwards. Lastly he affirms, the moving Force to be the Wind gathered in the Sail, which, by the Help of the Sail-yard, is applied to A; and then he gives a Reason from the Principles hitherto delivered, why, the higher the Yard D A E is, so much the swifter the Ship will be carried with the same Wind and Sail; *viz.* because the farther the *Fulcrum* is from the moving Force (all other Things being at the same time considered) so much the easier will the same Power or Force move the Weight.

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* Plate 14. Fig. 17 and 18.

† Plate 15. Fig. 1.

The same Instrument, according to its different Application may be a Leaver of the first or second Kind: As for example, when a Man who has carried a Stone upon the Carr * goes to deliver it at the Place intended, so as to lay it upon its Side E, he lets the End of the Plank rest upon the Ground at E, and lifting up the End B, turns the Stone over; which Operation changes the Leaver from one of the first Kind into one of the second, where E is the *Fulcrum*, B the Power, and the Weight is at A.

* Plate 14. Fig. 15.

The Sheep-sheers † are two Leavers of the third Kind; the common Center of Motion being at the springing Bow at C, whilst the Power or Hand is applied at P *p*, and the Wool to be cut is the Weight at W. Thus are the two Legs of a Pair of Tongs visibly Leavers of the third Kind. A Ladder or a Pole to be reared against a Wall, are also Leavers of the third Kind. But the Use of the Leavers of the third Kind is most beautifully shewn in the animal Body, where the All-wise Creator has supplied Animals with a Means to move the Limbs with great Velocity, by applying the Power of the Muscles very near the Center of Motion; but at the same time giving the Muscles such very great Force, as to perform their Office very suddenly, raising the Limbs even when great Weights hang at their Extremities; as for example, when we lift Weights with our Hands or Feet, or when we hold or break hard Bodies with our Teeth. There is scarce a Bone in the Animal Body but what is a Leaver of the third Kind. It is a delightful and curious Contemplation to consider what Proportion is observed in the Animal Body as an Engine from which Art only copies faintly: There we may see various Applications of Powers, and how they help each other in moving the Limbs, sometimes acting jointly at the same time, sometimes succeeding one another to change Directions, and sometimes acting against each other to stop and check Motion; at other times drawing over Pulleys to alter the Angle of Traction, as need requires. But this Subject is so copious, that we can only give one Instance or two here;

† Plate 15. Fig. 2.

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here; referring the Curious to consult *Alphonsus Borelli*, who has written a whole Book upon that Subject (*viz. de Motu Animalium*) and from whom we shall take only a few Examples. Here follows the 21st *Proposition* of his *first Book*.

"PROPOSITION XXI.

"*The absolute Power of every animal Muscle; must necessarily be greater* Plate 15. Fig. 3 and 4. "*than the Weight of the Limb suspended by it, but never less.* Plate 15. "Fig. 3 and 4.

"All-wise Nature has so contrived the Shape of Animals made up of "several Organs joined together, as to enable them to move from one "Place to another, and perform the several Operations required for the "Preservation of Life. But this could not be done by giving an Animal "an orbicular Shape like a Ball, but it was proper that he should be made "up of several Articulations, such as Hands and Feet, to walk about and "handle Objects. But these Limbs could not move about Joints, unless "they were drawn by muscular Cords, and those Cords were contracted "by a moving Force. We are going to shew, that that moving Force or "Power must not be less, but necessarily greater than the Weight and Re- "sistance of the Limbs suspended. Let us consider any Limb, for exam- "ple the whole Arm; it is plain that it was necessary for it to move round "every way about the Joint of the Shoulder, that it might be able to draw, "suspend, and impel the Resistances as well of the Weight of the Arm as "of external Bodies to be handled: Such Operations require a proper Fi- "gure, Forces and apt Instruments, and all fitted for that Purpose. The "Shape without doubt must be long like a Leaver moveable about a Cen- "ter, or the fixed Point or firm *Fulcrum* of the Shoulder. Then in the "Leaver must be considered the Positions in which the moving Power "and the Resistance are applied. The moving Power acts by contract- "ing the muscular Cords, which can only be fastened near the Center of "Motion of the Leaver, as has been said before *, whilst the Resistance "is

* Our Author in his 20th Proposition has ingeniously shewn, that the Tendons to move the Bones could not conveniently be fastened near the End of the Bone (at a great Distance from the Center of Motion, as we endeavour to do in mechanical Engines) but near the Articulation or Joint. These are his Words: 'Let there be two Bones * A B 'and G F joined and articulated at A F, namely in such Manner 'that A B may be drawn round C the Cen- 'ter of the Articulation, as the two Bones of 'the Arm; and let the Muscle D E be fastened 'to G the fixed End of the Shoulder at G, 'and let its End be joined to E, the End of 'the lower Bone of the Arm or Cubit A B; 'which End must be drawn round the Cen- 'ter of the Articulation C, describing the 'Arc B H: I say, that Nature neither could, 'nor ought to fasten the tendinous End E 'near the End B of the Bone A B. For if it 'could be, let us suppose the Connection E 'to be made near B the Wrist, and then the 'Tendon, and the Muscle D E is either loose 'and

* Plate 15. Fig. 3.

" is applied at its utmost Length or farther End; therefore *the Power* : " *will always be to the Resistance* : : *as the greater Distance of the Resistance* : " *to the lesser Distance of the Power from the same fixed Point.* Consequently " the moving Power is greater than the Resistance. Annotat. Lect. III.

" PROPOSITION XXII.

" *The first Enquiry of the absolute apparent Force, which can be exerted* " *by the two Muscles, the* Biceps *and* Brachiæus, *bending the Cubit (or lower* " *Bone of the Arm) when the whole Arm is in a supine and horizontal Situa-* " *tion; which is greater than twenty times the Weight that is sustained by* " *them, and exceeds the Force of 560 Pound Weight.* Plate 15. Fig. 4. Plate 15. Fig. 4.

" Let the *Humerus* E A, and the Cubit and Hand A B, be almost in " a right and horizontal Line, but supine (that is, with the Elbow down- " wards) and let the Cord G B fold about the Ends of the Fingers of the " expanded Hand G, to which Cord at G hangs the Weight R, which " must be increased by degrees, till the Excess of the moving Power of " the Muscles D C becomes wholly insensible, and they can sustain no " more Weight than R, but be just able to hold it with a Force brought " to be equal; then we may judge that the *Momenta* of the Powers of " the Muscle, and of the Weight, are wholly equal, neither of those " Forces overcoming the other. Now Experience shews us, that in a " robust

' and may be separated from the Limb and ' the Bones D A B; or it is bound down to ' it by some Ligament or *Fascia* as R. If ' the first, the Consequence will be as follows: ' Because the Bone A B cannot be turned up ' towards F G quite to the Situation A H, ' unless it be drawn by the Contraction of ' the muscular Cord D E, in which Case its ' Length D E, in order to be shorten'd to ' D M, must become less than the eighth Part ' of D E, which shortening in the Arm, will ' be of above a Foot and an half; this will ' not only be troublesome, but even impossi- ' ble. It would be troublesome, because the ' Breadth and Thickness of the Arm would ' be vastly increased to take in the Dimen- ' sions C M equal to C E; so that the Arm ' would upon this account only become as ' big as the Belly of the Animal, which mon- ' strous Thickness would hinder the rest of ' the Motions of the Arm and of the Animal. ' Then because the Structure of a Muscle is ' such, that it can be contracted but a little, ' seldom above two or three Fingers-Breadth; ' such a fastening of the Muscle, which re- ' quires so prodigious a Contraction (name- ' ly, of above a Foot and a half) would be ' altogether impossible. But the Absurdity ' of such a Position will most evidently ap- ' pear, if we suppose the Bone A B to be the ' *Humerus* (or upper Bone) of the Left Arm, ' which is to be moved every way round the ' Joint of the Shoulder C; that it may be ' brought to the Breast, it is plain that it ' must be drawn by the Muscle E D fixed to ' D, the right Side of the Breast. Another ' Muscle to raise it up must be fixed on the Top ' of the Head, and the Muscle to bring it ' down must have its Origin in the lower Part ' of the Belly; which Muscles, together with ' those of the right Arm, require a vast swelled ' Space like a great Tun; and the same would ' be required for the Muscles of the Feet, ' which would make a Man so far from being ' well jointed and clean limbed, that he would ' be a ridiculous unwieldy Mass, unfit for ' Motion and handling of Bodies; therefore ' such a Shape was entirely to be rejected.

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" robuſt young Man the Weight R does not exceed 26 Pounds, to which " muſt be added the whole Weight of the Cubit and the Head, which are " nearly equal to 4 Pounds, and this Weight acts, not in the End of the " Leaver as at B, but in the intermediate Place H, namely where its Cen- " ter of Gravity is; therefore if another Weight of 2 Pounds be ſuſpended " at B, which has the ſame Proportion to the Weight of the whole Cubit " as the Diſtance O H to O B, we ſhall have for our Leaver an indiviſible " Line and without Weight, at whoſe End B are ſuſpended two Weights, " namely R and the Weight of the Cubit, that is in all 28 Pounds; and " then becauſe the Direction C D of the Tendon of the Muſcle that draws " makes a very acute Angle with the Line C O, becauſe the Tendon of " the Muſcle exactly touches the Head of the Joint A, we muſt from the " fixed Point or *Fulcrum* O draw the right Line O I, perpendicular to C I " the Direction of the Tendon, and then from the Principles above demon- " ſtrated it will appear, that *the Power drawing the Muſcle* D C : *has the* " *ſame Proportion to the Reſiſtance of the Weight* R, *together with the addi-* " *tional Weight above-mentioned* : : *as the Diſtance* O B : *has to the Diſtance* " I O. But by a ſtrict Examination it appears that O B, the Length of " the Cubit and Hand, is more than twenty times greater than the Semi- " diameter of I O the Head of the Bone. Therefore the Strength and " Power drawing the Muſcle D C is above twenty times greater than the " Weight R and the additional Weight; and ſince the two Weights are equi- " valent to 28 Pounds, therefore the apparent Force with which the Muſcle " draws the Cubit and endeavours to bend the Elbow, is greater than the " Force of 560 Pounds.

" Proposition XXIII.

" *To find the Force which the ſaid Muſcles exert, when the* Humerus *or* " *upper Part of the Arm is perpendicular to the Horizon, and the* Cubit *is pa-* " *rallel to the Horizon.* Plate 15. Fig. 5.

* Plate 15. Fig. 5.

Secondly, " Let E A be the *Humerus*, and A B the *Cubit*, making " a right Angle with each other, the *Humerus* being perpendicular and " the *Cubit* ſtill horizontal: In that Poſition the Length of the Lea- " ver O B ſtill remains the ſame, and now at its End B is ſuſtained the " great Weight of 33 Pounds (as appears by making the Experiment) by " the ſame Muſcles D C; but becauſe the Angle I C O made by the Ten- " don with the Bone O C is leſs acute than in the foregoing horizontal " Situation of the ſame two Bones, becauſe when the *Humerus* E A is bent " towards the *Cubit* A B, the Tendon of the Muſcle D C adhering to " the *Humerus* is alſo bent; yet the Angle I C O does not become a right " one, becauſe the Tendon at I is firmly bound with membranous *Faſciæ* " and the outward Skin, which Ligaments ſerve as a Pulley to keep the

" Tendon

" Tendon towards A the Angle of the Joint; but yet the Tendon I C Annotat.
" is not so closely bound down at I, but that it rises a little, and there- Lect. III.
" fore the right Line O I perpendicular to the Direction of the Tendon
" C I becomes sensibly longer than in the foregoing Case, as we may find
" by feeling our Arm: and therefore the Distance O B will bear a less
" Proportion to I O, than it was found to do in the former Situation. But
" whatever Proportion the said Distances have, the same reciprocally will
" the Force contracting the Muscle D C, and drawing the Bone, have to
" the Resistance of the Weight R and the Weight of the *Cubit* together:
" Therefore that Force will bear a less Proportion to this Resistance, than
" 20 to 1; and if it appeared by the foregoing Enquiry that the greatest
" Force of the *Biceps* and *Brachiæus* Muscles was equal to the Force of
" 560 Pounds; it will appear from the present Enquiry, in which the great
" Weight R is of 33 Pounds, and, taking in the Weight of the *Cubit*,
" equal to 35 Pounds, that the Distance O I is only a sixteenth Part of
" the Distance O B, and not, as before, a twentieth Part of it; and there-
" fore that the Distance I O being sensibly increased, there must of conse-
" quence be raised a greater Weight now, namely 35 Pounds, by those
" Muscles.

" Here we must take notice, that though by reason of the bending of
" the Limb E A B the Muscles are not stretched as before, but must be
" in some measure relaxed; yet the moving Force of each Muscle has a
" less Power of contracting, because really the Muscles D C are not both
" fixed to the Top of the *Humerus*; for the *Biceps* is fastened to the *Scapula*
" or Shoulder-bone H L E at L, but the *Brachiæus* to the Middle of the
" *Humerus*. And because the *Scapula* H E L is always in the same and
" transverse Situation, the *Humerus* E A revolving about the Center E of
" its Articulation, must make the Angle L E O with the *Scapula* the less
" acute, the more the *Humerus* is bent downwards; and then the Origin
" of the *Biceps* Muscle D is more raised, and recedes farther from the Top
" of E the Head of the Bone, because the Length of the Line L D I sub-
" tending the Angle L E O is increased, and therefore the aforesaid Mus-
" cle is so much the more stretched as the *Humerus* is bent downwards.
" Therefore though by reason of the Angle E O B the *Brachiæus* Muscle
" be relaxed; yet the *Biceps* may be so much the more stretched by reason
" of the Elevation of the Point D above the Head of the *Humerus*.

" Proposition XXIV.

" *Hence probably may be singly found the apparent absolute Forces of the* Bi-
" ceps *Muscle, which is equivalent to* 300 *Pounds, and of the* Brachiæus,
" *which is equal to the Force of* 260 *Pounds.* Plate 15. Fig. 6. † Plate 15. Fig. 6.

" Let the *Humerus* O E be sent backward in order to make the Angle
" H E O as acute as may be, and likewise let the *Cubit* A B be inflected so

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" as to become parallel to the upper Line of the *Scapula* HL, and then " the alternate acute Angles HDI and CID will be equal to one another; " and then, as much as the *Biceps* Muscle DIC is relaxed on account of " the Acuteness of the concave Angle COE, just so much is it drawn and " stretched on account of the convex Angle HDO: therefore the natural " Tension of the *Biceps* Muscle is in no wise altered, and remains exactly " of the same Length as it had in the horizontal Situation of the whole " Arm; and as it suffers no Relaxation, it will have the same Force of " contracting itself as it exerted in the horizontal Position. But the *Bra-* " *chiæus* Muscle has not the same Advantage, its Origin being in the Mid- " dle of the *Humerus* at F, and its End or Insertion at I, near the Head of " the *Cubit*; and because the Angle EOC is acute, therefore the *Bra-* " *chiæus* Muscle must suffer the greatest Relaxation, and therefore exert " little or no motive Force. In this Case therefore one may find the mov- " ing Force of the *Biceps* alone; (that is, if the Distance OI from the " Tendon to the Center of the Head of the *Cubit* be not varied.) Let us " suppose the Weight R sustained in that Situation, together with the " Weight of the *Cubit*, to be equal to 25 Pounds; and because the Distance " IO is almost a twelfth Part of the *Radius* and the Hand BO, therefore " the absolute Force of the *Biceps* Muscle will be twelve times greater than " the appended Weight R together with the Weight of the *Cubit*; that is, " it will be equal to the Force of 300 Pounds, when the *Brachiæus* exerts " no Force by reason of its very great Relaxation. Then because the joint " Forces of the two Muscles, the *Biceps* and the *Brachiæus*, working toge- " ther in the first Experiment, were equal to the Force of 560 Pounds; if " from that Force we subtract the moving Force of the *Biceps* alone, just " found to be of 300 Pounds, the remaining Force of 260 Pounds will be " that which was exerted by the *Brachiæus* Muscle, and that was the Thing " to be enquired into.

" PROPOSITION XXV.

" *To find what Force the same Muscles exert when the* Cubit *hangs down-*
Plate 15. Fig. 7. " *wards, whilst the* Humerus *is kept perpendicular to the* Horizon. Plate 15. " Fig. 7.

" Now let the *Humerus* EA, and the *Cubit* AB be in one direct Line " and perpendicular to the Horizon; then the greatest Weight to be sus- " pended at B, might be almost immense, if the Strength and Tenacity of " the Ligaments could always resist and was wholly insuperable.

" If afterwards the *Cubit* be a little inflected, so as to make an obtuse " Angle EAB with the *Humerus* which is now perpendicular to the Hori- " zon, and an acute Angle BAK, with the horizontal Line OK, then " indeed may the great Weight R be very much increased, because if " from B the Line BK be drawn perpendicular to the horizontal Line " AK, then the Weight R drawing the Leaver AB obliquely, acts in

" the

" the same manner as if it had been suspended in the Point K of the Leaver Annotat.
" O K; and therefore we have now a new Leaver O K shorter than O B: Lect. III.
" but the Force of the Muscle raising the Leaver draws from the Point I,
" having I O for the Distance of its Line of Direction; and consequently
" the absolute Power that contracts the Muscle (which is always the same)
" has the same Proportion to the Resistance of the Weight R, as K O has
" to I O. Therefore if K O be only the double of O I, the Weight R,
" which is sustained in that Position, will be half of the whole absolute
" moving Force, and therefore equal to 280 Pounds; and if the Distance
" O K be less than O I, then also will the Weight R be greater than the
" moving Force of those Muscles.

" Hence may be gathered, that in the Flexion, or Elevation of the *Cu-*
" *bit*, the Effect of the same Force which draws the Muscle is continu-
" ally diminished; because the Length of the Leaver O K is successively
" increased, and therefore the Weight R must be decreased in the same
" manner.

" Proposition XXVI.

" *To find the Force of the same Muscles, when the Arm is placed in a supine*
" *horizontal Situation.* Plate 15. Fig. 8. Plate 15. Fig. 8.

" The Force of the Muscles bending the *Cubit* may be exerted in an-
" other manner, namely when the *Cubit* A B being in a supine horizontal
" Situation must be inflected downwards towards G by the Muscles D C,
" which are now below the *Cubit*; for the Cord B L G being thrown over
" the Pulley or Wheel M L, moveable about a fixed Axis M, it is evi-
" dent that whilst the Hand B descends, the Weight A is raised, A B being
" then the Leaver whose *Fulcrum* is O, and the Weight R draws the End
" B of the Leaver upwards towards L, and the Power of the Muscles D C
" draws down the Leaver A B from I towards D. And therefore those
" Things which have been said before, will also be verified here, only with
" this Difference, that in the former Case the End B was drawn down-
" wards, not only by the Resistance of the Weight R, but also the Weight
" of the whole *Cubit* and Hand; but on the contrary, here the Weight of
" the *Cubit* does not act counter to, but helps the Power of the Muscles
" to draw; because, that as in this Situation the Muscles draw down the
" *Cubit*, so the *Cubit* also acts downwards by its Gravity, and these two
" Powers taken together have a *Momentum* equal to that of the Weight R.
" And as in the first Case the Weight of the *Cubit* was added to the Resist-
" ance of the heavy Body R, now it is added to the Power of the Muscles;
" and because the greatest Power of the Muscles D C was shewn to be
" equal to the Power of 560 Pounds *, therefore if the Leaver A B was * By the 22d
" of no Weight, when the Distance O B is found to be twenty times the Prop. above
" Distance O I, the Weight R ought to be of 28 Pounds; but because the cited.
" two Pounds added to R are *in Æquilibrio* with the Weight of the *Cubit*

Annotat. Lect. III. " (that is, as they render it a Leaver without Weight;) therefore the " Weight with its Adjunct making up 30 Pounds, will be the greateſt " Weight that can be ſuſtained by the Force of the Muſcles in that Situa- " tion.

" This may be ſhewn another Way, becauſe the deſcending Weight of " the *Cubit* being as 2 Pounds hanging at B, is made to act equally with " a drawing Force of 40 Pounds applied at I, (by reaſon of the reciprocal " twenty-fold Proportion) and the proper Force D C I will be of 560 " Pounds; therefore this Force, together with the *Momentum* of the *Cubit* " acting with it, will produce an Effect equal to the whole Weight of 600 " Pounds."

But what appears moſt wonderful, is the Force of the Muſcles that move the lower Jaw, which the ſaid *Borelli* conſiders in the 87th and 88th Propoſition of the firſt Part of his Book abovementioned; where he ſhews, that thoſe ſmall Muſcles, all which taken together do not in a Man exceed the Weight of one Pound, do yet exert a Force equal to 534 Pounds; and in Maſtiff-Dogs, Wolves, Bears and Lions, have a Force vaſtly ſuperior, to enable them break large Bones, as they do daily in their Feeding.

7. [35. —— *And if the Arm* C W *be ſet ſtreight in a Line with* P C, &c. —— *the Inſtrument will plainly appear to be a Leaver of the firſt Kind.*] Though the bended Leaver is not an Inſtrument of common Uſe, except in the Hammer and Tools of that Kind, yet the Conſideration of it is very neceſſary in the Explanation of ſeveral Machines that virtually contain ſuch a Leaver; eſpecially in ſtatical Propoſitions, of which we will here give ſome Examples. The Caſes of the *Inclined Plane* and the Wedge * may be clearly ſolved this way. As for Example, when the Weight P † ſuſtains the Weight W upon the inclined Plane A B, drawing the Center of the Weight in the Line M W parallel to the Plane, one may conſider in the ſaid Weight the *bended Leaver* W T *n*, whoſe longeſt *Brachium* is W T, and the ſhorteſt T *n*. Now, as the Line of Direction of the Power is M W parallel to the Plane, T W acted upon at right Angles muſt be the Diſtance of the Power; and as W *n* is the Line of Direction of the Weight, *n* T at right Angles with it muſt be the Diſtance of the Weight; conſequently *as n* T *(the ſhort Arm of the bended Leaver) : is to* W T *(its long Arm) : : ſo is the Power* P *: to the Weight* W. Now, *as the Triangles* W T *n and* A B C *are ſimilar, the Power thus conſidered : is to the Weight : : as* B C *the Height of the Plane : to its Length* A B; which was proved in its Place *. But if the Power had drawn the Weight in a Line parallel to the Baſe of the Plane, which is the Caſe of the Wedge, its Effect might be thus explained: As the Power acts obliquely at the End of the long *Brachium* of the bended Leaver W T *n*, *its acting Diſtance* muſt be found by drawing from the Center of Motion T, T *o* perpendicular to the Line of Direction of the Power, which now muſt be conſidered as the long *Brachium*

* 48, 49, 50, 51, 52.
† Plate 10. Fig. 14.
49.

chium of the Leaver, whilſt *n* T ſtill is the ſhort Arm, and acted upon by the Weight at right Angles. Therefore the Quantity or Intenſity of the Power Π will now be found by this Analogy, *viz.* Annotat Lect. III.

As the long Arm of the Leaver, now T *o* :
Is to the ſhort Arm T *n* : :
So is W *the Weight* :
To Π *the Power*; *or the Baſe* A C : *to the Height* C B.

When I conſidered the *inclined Plane* as a mechanical Organ *, I only took notice of two Applications of the Power, the one with its Line of Direction parallel to the Plane, and the other (which reduced it to the Wedge) with its Line of Direction parallel to the Baſe of the Triangle, that is, inclined to the Plane as much as the Plane is inclined to the Horizon, the Angle W B A (called the *Angle of Traction* **) being equal to the Angle B A C, becauſe of the Parallels W B, A C; but as in compound Engines, and in the Uſe of Carriages, the Angle of Traction or Inclination of the Line of Direction of the Power to the Plane is very variable, we give this Way of conſidering a *bended Leaver* in the Body to be drawn up for the Solution of all Caſes, which will appear by the following Example. * 48, 49, 50, 51, 52.

Let D L B be the Angle of Traction, as when the Power Π draws over the Pulley ρ in the Line ρ L. W T *n* is a bended Leaver, whoſe Center of Motion is T, the Point where the Ball W touches the Plane; *n* T the ſhort Arm of the Leaver, on whoſe Extremity *n* may be conſidered the Weight, as ſupported and preſſing at right Angles, becauſe its Line of Direction goes through *n* *; and W T the long Arm of the Leaver to which the Power is applied obliquely. But as T *z* perpendicular to the Line of Direction of the Power is its *acting Diſtance*, we may conſider T *z* as the long Arm of the Leaver. Then the whole Caſe will be ſolved by this Analogy, *viz.* * L. 2. 47.

As the long Arm of the Leaver, now z T :
Is to the ſhort Arm T *n* : :
So is W, *the Weight* :
To Π *the Power.*

Since one may find this laſt bended Leaver, in any Direction of the Power, that is, in any Angle of Traction, this general Rule for all Directions of the Power may be deduced from it.

** The *Angle of Traction* is the Angle which the Line of Direction of the Power makes with the Plane; and conſequently when the Power draws in a Line of Direction parallel to the Plane, there can be no Angle of Traction.

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Lect. III.

As the Sine of the Angle of the Plane :
To the Sine Complement of the Angle of Traction : :
So is the Power :
To the Weight.

Plate 10. Fig. 14.

But to prove how the Rule is deduced from the bended Leaver, we must shew that *n* T, the short Arm of the said Leaver, is always to its long Arm *z* T, as the Sine of Inclination to the Sine Complement of the Angle of Traction; or that C B in the Triangle A B C : is to L *z* in the Triangle T L *z* : : as *n* T : is to T *z*; and then we shall draw some useful Corollaries from our general Rule.

DEMONSTRATION.

Since W T is perpendicular to A B, and the Angles W *q* T and A *q* E equal, (because they are vertical Angles) the Triangle A *q* E, (rectangular at E) having two Angles equal to two Angles in the Triangle W *q* T, the third Angle A must be equal to the third Angle *q* W T; consequently the Triangles will be equiangular, and therefore similar (by 4. 6. *Eucl.*) and as *q* E is the *Sine* of Inclination for the *Radius* A *q* (because in its similar B C is the *Sine* of Inclination for the *Radius* A B) so also will *q* T be the *Sine* of Inclination for the *Radius* W *q*. But as T *n* which is parallel to the Horizon falls upon W *q*, which being the Line of Direction of the Weight, is perpendicular to it, the Triangle W T *n* is similar to W *q* T (by 8. 6. *Eucl.*) and thus T *n* the short Arm of our *bended* Leaver becomes the *Sine* of Inclination for the *Radius* W T. *Which was one of the Things to be demonstrated.*

Further, as T *z* is by Construction perpendicular to L W, the Triangle W *z* T is similar to L W T (by 8. 6. *Eucl.*) therefore the Angle *z* T W is equal to W L T the Angle of Traction; so that as L T is the *Sine Complement* of the Angle of Traction for the *Radius* L W, T *z* (the long Arm of our *bended* Leaver) must be the *Sine Complement* of the Angle of Traction for the *Radius* T W, *Which was the other Thing to be demonstrated.*

COROLLARY I.

Hence follows, that when the Line of Direction is parallel to the Plane, the Power is the least that can be for that Inclination of the Plane, because then the *Sine Complement* of the Angle of Traction is changed into the *Radius* or *whole Sine*; that is, T *z* becomes T W, or the long Arm of the Leaver W T being acted upon at right Angles expresses the acting Distance of the Power; or, strictly speaking, the Angle of Traction vanishes. But if the Power should draw directly up in the Line W *e*, it must be equal to the

the Weight; becauſe then W *q* T being the Angle of Traction, its *Sine Complement* (*q* W being *Radius*) would be *n* T, which is equal to the *Sine* of Inclination here expreſſed by the ſaid Line. *Hence it is* plain, that if an Horſe draws a Burthen up Hill by means of a Cart or any rolling Machine, he will draw it with ſo much the more Eaſe, as the Line of Direction by which he draws the Load comes nearer to a Parallel to the Slope of the Mountain along which he draws. Annotat. Lect. III. Plate 10. Fig. 14.

COROLLARY II.

It follows alſo, that if the Line of Direction, as B W and *w* W parallel to A B, make the Angle *w* W B equal to the Angle *w* W D, the Power applied at B will be equal to the Power applied at D; becauſe in ſuch a Caſe the Angles of Traction W L B, W B L will be equal, ſince by 29. 1. *Eucl.* the Angle W L B is equal to its external oppoſite D W *w*, which is ſuppoſed equal to the Angle *w* W B, and conſequently to its alternate W B L. Whence it follows, that if the Line of Direction of the Power, as W *a*, and the Line W T perpendicular to A, make the Angle T W *a* equal to the Angle of Inclination A (or *q* W T) the Power applied at *a* will be equal to the Weight by *Cor.* 1. becauſe then the Angle of Traction W *a* L is equal to the Complement of the Angle of Inclination A; that is, W *a* L is equal to W *q a* = A B C.

COROLLARY III.

Laſtly, it follows alſo, that if the Line of Direction of the Power be W T at right Angles to the inclined Plane A B, which makes the Angle of Traction a right one; the Power applied at T, or in any Part of the Line *t* T, muſt be infinite. That is as much as to ſay, that a Power that ſhould draw the Weight W directly from the Plane, or directly againſt it, would not keep it there, let its Intenſity or Force be ever ſo great; becauſe in that Caſe the Sine of the Complement of the Angle of Traction is reduced to nothing, or being *infinitely ſmall*, the Power applied at T muſt be *infinitely great*; ſince, by what we have ſhewn, that Power : muſt be to the Weight : : as the *Sine* of the Angle of Inclination : is to the *Sine Complement* of the Angle of Traction.

Before I quit this Subject of Bodies ſupported or drawn on inclined Planes, I beg leave to apply what has been ſaid in the ſecond Lecture *, to ſhew generally why a Body will be ſuſtained on an inclined Plane by another Body of leſs Weight (if the former be drawn in a Line of Direction parallel to the Plane, and the latter hangs perpendicularly) when the Weight of the great Body : is to the Weight of the little one : : as the Length of the Plane : is to its Height. The whole is deduced from this Principle laid down and explained in the ſecond Lecture, *viz. That if the Center of Gravity of a Syſtem of Bodies does not deſcend, the Bodies cannot deſcend.*

* 28, 35, 47.

Now

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Now to apply this to our Purpose. A D B * is an inclined Plane, whose Height is D B. If by means of a Pulley P the Weight *w* hanging perpendicularly does by a String hold the Weight W on the inclined Plane, and those Weights are to one another, as the Length of the Plane to its Height (which here is as 2 to 1) they will remain at Rest (that is, keep each other *in Æquilibrio*) whatever Part of the Plane W is laid on. First, let the Situation of the Bodies be W and *w*; draw the Line *m n* which joins their Centers of Gravity, and having found their common Center of Gravity at C * (C *n* = 2 C *m*) draw H *h* an horizontal Line through that common Center; and then whatever Position the Bodies are in, or whatever Part of the Plane W be placed on, their common Center of Gravity will still be in the horizontal Line H *h*. If W be removed to V, *w* will fall to *u*, and the common Center of Gravity will be at *k*. If the Centers of the Bodies be at E and *e*, their common Center of Gravity will be at K, still in the same Line H *h*; which may be easily proved, because the Triangles *n h* C, *m* H C, *h q k*, *r o k*, *&c.* are all similar. *Since therefore there is no Position of* W *on the Plane, and of* w *in the Perpendicular* w q, *which can alter the Height of the common Center of Gravity of the Bodies, they must be* in Æquilibrio, *because they cannot fall unless their Center of Gravity descend.* Q. E. D.

* Plate 15. Fig. 9.

* By L. 2. N°. 39.

C O R O L L A R Y.

Hence also appears the Reason why two unequal Bodies will sustain each other upon unequal Planes of the same Height, whose Lengths are to one another reciprocally as the Bodies. For example, let the Weights F and G *, joined by the Cord F P G running over the Pulley P, be to one another in Weight as the Planes A B, B D (whose common Height is B E) on which they respectively rest. Find their common Center of Gravity C, and through C draw an horizontal Line; then you will find as before, that however you alter their Situations on their respective Planes, their common Center of Gravity will still be found in the same horizontal Line that goes through C, *&c.*

* Plate 15. Fig. 10.

8. [37.——*An upper or fixed Pulley adds no Force to the Power, but only prevents the Friction by making the Rope run easily; and so much the more, as the Sheever is bigger than the Center-Pin upon which it turns.*] How much the Friction of a Roller or upper Pulley is diminished in proportion as the Center-Pin, or the Gudgeons are less in Diameter than the Wheel or Roller, I shall exactly shew in the next Lecture. Now I shall take notice of the Diminution of Pressure upon the Center-Pin of a Pulley, which always happens in a certain Proportion of the Weights which hang on each side, when they are in motion, and that without regard to the Bigness of the Pin, which in this Case may be considered as a Line. The Thing in general is comprehended in the following

PROPO-

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PROPOSITION.

When a String or Rope runs over a single Pulley or Roller by the Descent of the preponderating Weight (the other Weight rising at the same time) the Pressure on the Axis of the Pulley is always equal to the Quadruple of the Product of the Weights multiplied into one another, and divided by the Sum of the same Weight.

PLATE XV. *Fig.* 11. Plate 15. Fig. 11.

The Pulley is D E, its Center C, the Weights p and q, and the running Rope p D r E q. I say, that the Pressure on the Axis or Center C is $= \frac{4pq}{p+q}$.

PLATE XV. *Fig.* 12. Plate 15. Fig. 12.

The Pulley being represented in this Figure by the same Letters as before, draw at pleasure the Lines D d and E e, in which the Weights rise or fall. Draw from the Center of the Pulley C k, parallel to the said Lines; then through the Point o taken any where in the Line C k, draw the horizontal Line p q, and taking the Distance p P equal to q Q, draw the oblique Line P o Q. From the Point c, the common Center of Gravity of the Weights, represented by the Letters p, q, and supposed to hang at those Points, let fall the Perpendicular c g, till it meets the Line P Q. If we consider p q as a Balance of unequal *Brachia*, whose Lengths are in the reciprocal *Ratio* of q to p, so that the Weights p and q shall upon it keep one another *in Æquilibrio*, we may call c q the Leaver of the greater Weight q, and p c the Leaver of the lesser Weight p; and if we suppose the Weight q to descend as far as Q, whilst the Weight p rises as far as P, (through an equal Space) because P g is to g Q as p c to c q, the Line c g will represent the Descent of the common Center of Gravity of the two Weights.

DEMONSTRATION.

The Sum of the Weights $p+q$: is to their Difference $q-p$: : as the Sum of their Leavers $pc+cq$: to their Difference $pc-cq$; and consequently : : as half the Sum of their Leavers oq : to half their Difference oc; and likewise (by 2 and 4. 6 *Eucl.*) : : as q Q the Velocity of the Weight descending : to c g the Velocity of the Center of Gravity descending. Now taking out of q a Weight equal to p, there remains only $q-p$

Annotat. Lect. III. $q-p$ to give Motion to the Weights by the natural Gravitation of it. But the *Momentum* of $q-p$ by the natural Gravitation is $qv-pv$ (v being taken for the natural Velocity of the Weight falling) therefore the Velocity expressed by q Q will be $\frac{qv-pv}{q+p}$. But before, we had this Analogy, $p+q : q-p :: q$ Q : cg; therefore we have $p+q : q-p :: \frac{qv-pv}{q+p} : cg$, the Velocity of the Center of Gravity, which will be $\frac{q^2-2pq+pp \times v}{p^2+2pq+q^2}$.

Therefore the *Momentum* of the two Bodies will be that Velocity multiplied by the two Bodies, and consequently the *Momentum* of the Fall of the two Bodies will be $\frac{q^2-2pq+p^2 \times v.}{q+p}$. Now if the two Bodies $q+p$ fell wholly, their *Momentum* would be $qv+pv$; but as we must subtract only that which falls off the Bodies, *viz.* $\frac{q^2-2pq+p^2 \times v,}{q+p}$ there is a Remainder of the Bodies which does not fall, and that Remainder of consequence must rest upon the Center and press the Axis. Now this Remainder, by making the Subtraction, appears to be $\frac{4pq}{p+q}$. *Q. E. D.*

SCHOLIUM.

Hence it follows, that if p be equal to q (in which Case there will be no Motion of the Weights) the Pressure will be $2p=p+q$; and if q be infinite, the Pressure is $4p$.

To try this practically, I made the following

Plate 15. Fig. 13.

EXPERIMENT. *Plate 15. Fig. 13.*

I screwed a very nice Pulley D to the Bottom of the Scale *b* of the Balance A B moveable round the Center C, and having balanced the Pulley by Weights in the Scale *a*, I fastened to a String the two Weights p and q, weighing 2 and 6 Ounces; then having put the String over the Pulley D, I placed 6 Ounces and one Penny Weight in the Scale *a*, kept from rising by a Thread, which joined in bottom to the fixed Hook H, whilst the whole Balance hung by another fixed Hook at M. Supporting the great Weight q by a Ruler held at *e* horizontally under it, I on the sudden removed *e* to give q leave to descend; which when it did, the Pressure on the Axis of the Pulley D was so much diminished (during the Rise of p

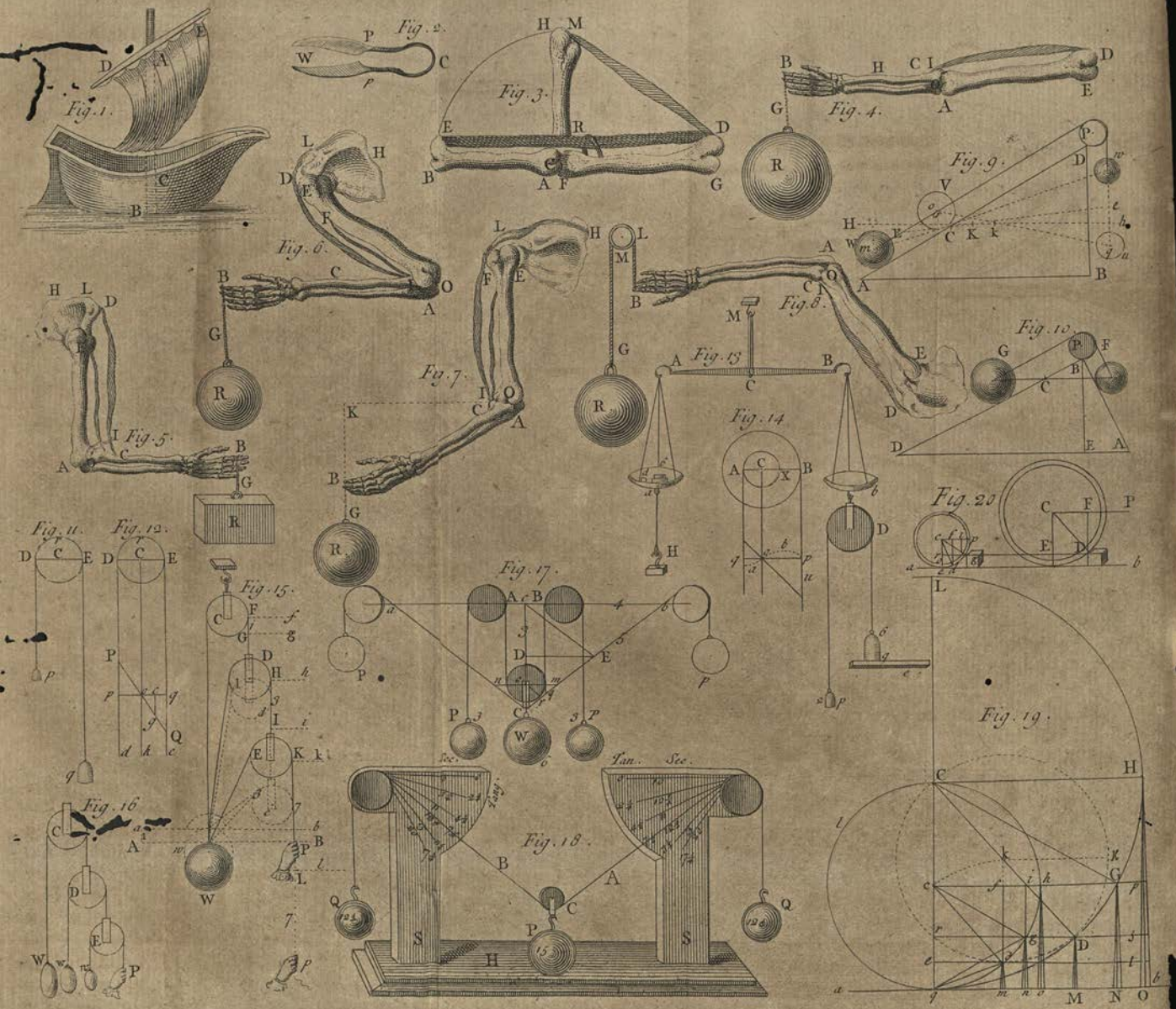
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.
Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17.
Fig. 18.
Fig. 19.
Fig. 20.
Sec.
Tan.
Sec.

and Descent of q) that the Scale a sunk by the Action of the little Weight d, even when it was a great deal less than a Penny-Weight, as plainly appeared by the Thread H a growing slack. Annotat. Lect. III.

When q was $= 12$ Ounces and $p = 3$, the Counterpoise in the opposite Scale was 9 Ounces and 12 Penny-Weight, according to the Theory, and the Experiment agreed with it; there being a visible Descent of the Scale a, by the Addition of the little Weight d, even when it was less than the 300th Part of q, *&c.* Take q and p in any other Proportion, and still the Experiments will agree with the Theory.

This Way of considering the Pressure upon the Axis, may also be applied to the Axle in the Wheel in the following manner:

Let A C B * be an *Axis in Peritrochio*, whose Wheel is A B, C the Center or Axis of Motion, and A X the Axle, q one Weight (commonly the greater Weight) and p the other; the Line $q\,p$, as a Leaver unequally divided at o, represents by its Part $q\,o$ the Arm or Semidiameter of the Axle A C, and by its Part $o\,p$ the Arm or Semidiameter of the Wheel C B. * Plate 15. Fig. 14.

The Bodies p, q, being in the Situation represented in the Figure, and $p\,b$ greater than $q\,a$, the Part of p which makes an *Æquilibrium* with q will be $\frac{q\,a}{b}$, and consequently there will be left of p only $p - \frac{q\,a}{b}$ or $\frac{p\,b - q\,a \times V}{b}$ (making V equal to the Velocity with which Bodies fall) to give Motion to the two Bodies; because the Remainder of the Weight of the two Bodies, $p\,V + q\,V$, namely $\frac{q\,b\,V + q\,a\,V}{b}$ of the two Bodies of *Æquilibrium* $q + \frac{q\,a}{b}$ will remain pressing on the Axis of the Machine. Let u be the Velocity of p, the Velocity of q will be $\frac{a\,u}{b}$, and its *Momentum* will be $\frac{q\,a\,u}{b}$; which added to $p\,u$ the *Momentum* of p, gives $\frac{p\,b\,v + q\,a\,u}{b}$ equal to the whole *Momentum* produced by the Force or *Momentum* $\frac{p\,b\,V - q\,a\,V}{b}$. Therefore since the *Momentum* produced is always equal to the *Momentum* which produces it, we have $\frac{p\,b\,u + q\,a\,V}{b} = \frac{p\,b\,V - q\,a\,V}{b}$ which gives $u = \frac{p\,b\,V - q\,a\,V}{p\,b + q\,a}$ and $\frac{a\,u}{b} = \frac{p\,b\,a\,V - q\,a\,a\,V}{p\,b\,b + q\,a\,b}$, which gives the *Momentum* of $q = \frac{q\,p\,b\,a - q\,q\,a\,a}{p\,b\,b + q\,a\,b} \times V$; which *Momentum* p will give it as it bears on the Axis, and consequently the Axis must suffer the Pressure of it. And as q, by its Reaction upon p, will make it lose

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as much (bearing also upon the Axis) or will as much retard its Descent, the Axis must of necessity by this equal Action and Reaction bear the double Pressure $\frac{2qpba - 2qqaa}{pbb + qab} \times V$; to which adding the Weight $\frac{qb+qa}{b} \times V$ of the two Bodies of *Æquilibrium*, the whole Pressure on the Axis will be equal to $\frac{qpbb + qqab + 3pbqa - qqaa \times V}{pbb + qab}$ or (dividing by the Denominator as far as we can) it will be equal to $q + \frac{3pbqa - qqaa \times V}{pbb + qab}$. Therefore the Axis bears as much Weight as if it sustained the Quantity of Matter $q + \frac{3pbqa - qqaa}{pbb + qab}$.

COROLLARY I.

If $pb = qa$, the Pressure is $q + p$, or $q + \frac{qa}{b}$ (according as pb is set down for qa, or the Reverse in the Form $\frac{qpbb + qqab + 3pbqa - qqaa}{ppb + qab}$) because in that Case $\frac{qa}{b} = p$.

COROLLARY II.

If p be infinite, the Pressure is $q + \frac{3qa}{b}$, or the Weight q together with three times the Weight that is able to keep it *in Æquilibrio* at the Distance of p, as in the Pulley above-mentioned, where the Pressure is $\frac{4qp}{p+q}$. If q be infinite, the Pressure then is $4p$; and if b be infinite, the Pressure is only equal to q.

N. B. *This may be given as a general Rule for the Pressure on the Axis, either of a Pulley, or of a Wheel and Axle, by the two Bodies acting against each other,* viz.

As the Momentum *of the two Bodies falling freely:*
To the Momentum *which is lost when they act on each other by means of the Machine::*
So is the whole Weight of the Bodies:
To the Weight pressing upon the Axis.

The

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The late Mr. *Peter Daudé*, from whom I had this Proposition concerning the Pressure upon the Center-Pin of an *Axis in Peritrochio*, not having at first considered, that the rising and falling Weight in the turning the *Axis in Peritrochio*, partake of the Nature of a Pendulum (because, as they have different Velocities, they mutually accelerate and retard one another) gave me afterwards the following Solution, wherein that Error is corrected. But, as it depends upon the Doctrine of Pendulums, which I have only considered towards the End of this first Volume, I thought proper not to give it the Reader till after I had treated of Pendulums. But, before I enter upon Mr. *Daudé*'s Solution, it is proper to premise something concerning the Center of Oscillation.

1st, The *Center of Gravity* of two Bodies joined together by a mathematical Line, is *that Point in the Line, where each of those two Bodies acts with its whole Weight in a State of Rest*; and this happens when their *Momenta*, which are in that Case the Product of their Quantities of Matter, by their Leavers, or Distances from that Center of Gravity, are equal. So that that Center feels the Weight of both Bodies.

2dly, The *Center of Oscillation* of two such Bodies, is *a Point in that Line, upon which the two Bodies in Motion act with all their Momenta*; and this happens, where the Products of their *Momenta*, by their Distance from that Center of Oscillation, are equal; in which Point their Forces being equal, and their Motions not being contrary to one another, they act, or fall, so that this Point moves with the natural Velocity of falling Bodies, and they strike an Obstacle with their whole Weight.

3dly, If we suppose these two Bodies revolving round the Center of Suspension, the Centers of Gravity and Oscillation will both be on that Side, where the greatest *Momentum* is.

4thly, If the Center of Suspension be between the two Bodies, the Center of Oscillation will be on the Outside of the two Bodies. But if the Center of Suspension is on the Outside of the two Bodies, the Center of Oscillation will then be between the two Bodies.

Now, the Proposition, according to Mr. *Daudé*'s last Solution, will be thus expressed.

PROPOSITION.

To find the Pressure, which two Bodies p *and* q *in their Motion, with contrary Directions, exert upon the Center-Pin of an* Axis in Peritrochio, *or* double Pulley *made of one Piece.* Plate 8. Fig. 15.

Let us suppose p greater than q, then the Center of Gravity of the two Bodies p, q, being beyond their Center of Motion c towards the Body a, the Body q will descend and cause the Body p to rise.

Annotat. Lect. III. Therefore to find their Center of Gravity s, we must make use of this Analogy;

$q : p :: b + a - z : z$, and $qz = pb + pa - pz$, which gives $z = \frac{pb+pa}{q-p}$; and taking that Value from a, we have $a, -\frac{pb-pa}{q+p} = \frac{qa+pb}{q+p} = cs$.

Now, considering the Motion of the two Bodies p, q, as that of a double Pendulum, whose Weights p, q, hang one below the Center of Suspension, and the other above it, we must find their Center of Oscillation o thus;

$qa : pb :: b + a + x : x$ (qo) and $qa - pb : pb :: b + a : x = qo = \frac{pb+pba}{qa-pb}$, and $qo + qc = co = a + \frac{pbb+pab}{qa-pb} = \frac{qaa+pbb}{qa-pb} = co$.

Therefore since the Center of Oscillation falls with the natural Velocity of heavy Bodies, which we will call $V = 1$, let us make this Analogy $co\left(\frac{qaa+pbb}{qa-pb}\right) : cs\left(\frac{qa-pb}{q+p}\right) :: V\,(1) : sg$, the Velocity of the Fall of the Center of Gravity s. Then reducing the two first Terms of this Analogy to the same Denominator, we shall have this, $co\,(qqaa + qpbb + pqaa + ppbb) : cs\,(qqaa - 2qba + ppbb) :: 1 : \frac{qqaa-2qpba+ppbb}{qqaa+qpbb+pqaa+ppbb} = sg$, the Velocity of the Fall of the Center of Gravity. Therefore multiplying that Velocity of their Center of Gravity by the Sum of the two Bodies $p + q$, we shall have the *Momentum* of their Fall (or so much of their whole Weight as actually falls) equal to $\frac{qqaa-2qpba+ppbb}{qaa+pbb}$, which being subtracted from their whole natural *Momentum*, namely from $qV + pV$, there remains $\frac{qpbb+2qpba+qpaa}{qaa+pbb} = \frac{qp \times b+a^2}{qaa+pbb}$; which is so much of their Weight as presses upon c their Center (or rather Axis) of Motion.

From the last Equation but one, may be drawn the following Corollaries:

COROLLARY I.

If $b = a$, the Pressure on c the Axis is $\frac{4pq}{p+q}$, as in the single Pulley.

CO-

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COROLLARY II.

If q be infinite, the Pressure upon the Axis will be $\frac{p \times \overline{a+b}^2}{a}$; whence if $a=b$, the Pressure will be $4p$, as in the single Pulley.

COROLLARY III.

If b be infinite, the Pressure upon the Axis will be q, because then the Leaver a is nothing in respect of the Infinite b.

COROLLARY IV.

If a be infinite, the Pressure upon the Axis will be p, because then the Leaver b is nothing in respect to the Infinite a.

SCHOLIUM.

It has been found above, that the Pressure upon the Center of the *Axis in Peritrochio*, is denoted universally by this Expression, $\frac{qpbb+2qba+qpaa}{qaa+pbb}$; but when $qa=pb$, there is an *Æquilibrium*, and then the Axis supports the two Weights $p+q$, and in that Case $\frac{qpbb+2qpba+qpaa}{qaa}$ must $=q+p$; which (supposing it so) gives $qpbb+2qpba+qpaa=qqaa+qpbb+pqaa+ppbb$; therefore by Reduction, $2qpba=qqaa+ppbb$, and $qqaa-2qba+ppbb=0$; and by extracting the Square-Root $qa-pb=0$, and consequently $qa=pb$. Therefore it is true, that when $qa=pb$, one may equally deduce from it the Truth of our general expression; namely, That the Pressure upon the Axis is $q+p$. *Q. E. D.*

The Weight, which presses upon c the Axis of the double Pulley cannot be increased by the Acceleration of the Weight q, because what makes the Weight of Bodies, which do not fall, by reason of an Obstacle hindering them, is only every single Impulsion of Gravitation, and not the Sum of many of those Impulsions; because each particular Impulsion is destroyed by the Obstacle as soon as it is given. But the Impulsions upon the descending Weight, do each of them give it a Velocity, which is not destroyed; therefore the given Velocities, being accumulated, cause an Acceleration of Descent in the Weight q, whilst the Weight, that presses on the Axis of the Machine or Pulley, is always equal and the same.

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9. [41. — *While* 2 *goes down to* a, 1 *goes up to* B *juſt twice as far*, &c.] There is another Caſe of raiſing a Weight by ſeparated Pulleys, which is omitted here. It is mentioned by Dr. *Pemberton* in his *View of Sir* ISAAC NEWTON's *Philoſophy*. I ſhall firſt give his Solution of it, and then ſhew how eaſily it may be reduced to our Rules, by proving, that however that Caſe is varied, there will be ſtill a reciprocal Proportion between the Power and the Weight, and their Velocities. The Weight W * is ſuſtained by the Power P, by means of the three Pulleys, C, D, E, of which C is fixed and the others moveable, and a Rope goes from the Weight to each Pulley as repreſented in the Figure. " To explain the Effect of Pulleys thus applied, " it will be proper to conſider different Weights hanging as in *Fig.* 16 *. " Here if the Power and Weights balance each other, the Power P is equal " to the Weight w; the Weight w is equal to twice the Power P, or twice " the Weight *w*; and for the ſame Reaſon the Weight W is equal to twice " the Weight w, or equal to four times the Power P. It is evident there- " fore, that all the three Weights *w*, w, W, together, are equal to ſeven " times the Power P. But if theſe three Weights are joined in one, they " would produce the Caſe of *Fig.* 15 †. So that in that Figure the Weight " W, where there are three Pulleys is ſeven times the Power P. If there " had been but two Pulleys, the Weight would have been three times the " Power; and if there had been four Pulleys, the Weight would have been " fifteen times the Power."

* Plate 15. Fig. 15.

* Plate 15. Fig. 16.

† Plate 15. Fig. 15.

To explain this our way, let us conſider the Weight W to be raiſed one Inch, as from the horizontal Line A B, to the horizontal Line *a b*, and from the Make of the Machine find what muſt be the Velocity of the Power. *Firſt* then, the Point F of the Rope going over the Pulley C, muſt deſcend one Inch, *viz.* from the Line F*f* to G*g* (becauſe W faſtened to the ſaid Rope, riſes one Inch by Suppoſition) and the Pulley D faſtened to the ſaid Rope D F, muſt alſo deſcend one Inch. From the Deſcent of the Pulley D one Inch, the Point H of its Rope muſt deſcend two Inches, as it is ſupplied from both Sides of the Pulley; but an Inch more of the ſaid Rope muſt be ſupplied by the Riſe of the Weight W; therefore the Point H will deſcend three Inches, or from H *h* to I *i*. *Laſtly*, As the Pulley E deſcends three Inches, becauſe it hangs by the Rope H I, the Point K of the Rope K P (being ſupplied from both Sides of the Pulley E) muſt deſcend ſix Inches on that account, and one Inch more on account of the Riſe of W: Therefore the Point K of the laſt Rope, by which the Power P pulls, will deſcend ſeven Inches, *viz.* from the Line K*k* to the Line L *l*, whereby the Power alſo will deſcend the ſame Diſtance, namely, from P to *p*. Conſequently one Pound at P inſtead of the Hand, will ſuſtain the Weight W ſeven times as big, 7 × 1 being equal to 1 × 7. *Therefore, in this Combination of Pulleys, as well as in all others, and indeed in all mechanical Engines (as we have often ſaid) where there is an* Æquilibrium, *there will be a reciprocal Proportion between the Intenſities of the Power and Weight and their Velocities.*

10. [41. —— *The Ropes* &c. (applied to Pulleys) *are always supposed parallel, except where it is otherwise expressed.*] Though in a Combination of Pulleys, where the last Pulley is a fixed one, as in the 4th, 5th, 6th, 7th, 8th and 9th Figures of Plate 10†; the Force exerted (supposing a Man or Men to draw) is the same in whatever Direction the Power draws the running Rope; yet if the Ropes that are applied to the Block or Blocks which come up with the Weight, are not parallel, Force will be lost in proportion to their Obliquity. Annotat. Lect. III. † Plate 10. Fig. 4, 5, 6, 7, 8, 9.

Suppose the Weight W*, together with the lower or moveable Pulley C, from whose Center it hangs, to weigh 6 Pounds; if it was suspended at *c*, it would require a Force equal to 6 Pounds to support it: and therefore if you suppose two upper, or fixed Pulleys, as A, B, to have over them a Rope, at each End of which hangs a 3 Pound Weight, whilst the Middle of the Rope comes under the Pulley C, it is evident, that the two Weights (or rather Powers P and *P*) being both together equal to the Weight, must also support and keep it *in Æquilibrio*. Now, since P and *P* balance one another; if P be taken away and the Rope made fast at *a*, *P* alone will support the Weight W, as we have said and explained already†. And this will appear more evident, if we reduce the Pulley C to a Leaver, after the manner shewn in the second Note of this Lecture*. In this Case *m n* is a Leaver of the second Kind, in which the Center of Motion or *Fulcrum* is at *n*, the Weight W draws at right Angles at *o* with the Distance *o n*, whilst the Power with the double Distance *m n* draws also at right Angles in the Direction *m* B. Now, if the Pulley B be removed to *b*, the Direction of the Power will be changed and become *b m*, consequently its Force will be diminished in proportion to the Obliquity of its Direction; that is, the Power able to sustain the Weight in the Direction *b m*: is to the Power which sustains it in the Direction *m* B : : as *b m*: is to B *m*†. * Plate 15. Fig. 17. † 57. * L. 3. Ann. 2. p. 136. † L. 3. Ann. 5. p. 146, 147.

From this Consideration may be deduced this general Rule for knowing the Intensity of the Power or Powers, which drawing obliquely over fixed Pulleys, cause a Weight hanging from the Center of a moveable Pulley to rise directly up.

As twice the Tangent of the Angle of Inclination (that is, the Angle made by the Line of Direction of the Power, which is the oblique Rope with the Horizon.:)
To the Secant of the said Angle : :
So is the Weight, when one End of the Rope is fixed:
To the Power drawing obliquely.

But if two Powers (one at each End of the Rope) be made use of, then the Analogy will stand thus:

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As twice the Tangent of the Angle of Inclination:
To twice the Secant of that Angle::
So is the Weight:
To the two Powers taken together.

PREPARATION.

* Plate 15. Fig. 17. Let the Pulleys A, B, * be removed to *a b*, and the Line *b m* and *a n* produced till they meet at C; join the Centers of the Pulleys *a*, *b*, by the horizontal Line *a b*; draw E D parallel to *a b*, from E so taken, that D *c* may be equal to D C; and making D *c* = D C, draw E *c*: draw *o r* perpendicular to C *b*.

DEMONSTRATION.

* L. 2. 22. Since *c* C is the Line of Direction of the Weight W, it must be perpendicular to *a b* * the horizontal Line, and consequently parallel to B *m*; therefore the Triangle *c* C *b* is similar to the Triangle B *q b* (by *x*. 6. *Eucl.*) and for the same reason D E C is similar to *c b* C, and *c* D E is also similar to them and equal to D C E, because of the right Angles at D, and the common Side D E and equal Sides D C = D *c*. Besides, the Triangle *a* C *c* is similar to the others above-mentioned, because as the Weight W (or its Center of Gravity) descends as low as it can, *a* C *b* must be an *Isosceles* Triangle bisected by the Line of Direction *c* C. Now if C *c* represents the Intensity of the Weight hanging on the Center of the lower Pulley C, its half D C will represent the Intensity of the Power drawing directly, or at right Angles to the Leaver *m n*, and C E its Intensity when drawing obliquely; and since the Angle D E C = *c b* C, E C the Secant of D E C will be the Secant of the Angle of Inclination; and D C the Tangent of D E C, will be the Tangent of the Angle of Inclination; and its Double is C *c*, representing the Intensity of the Weight. *Q. E. D.*

If the two Powers are used, P will draw with the same Obliquity as *P*, because the Angle *a* C *c* = *c* C *b*; therefore P + *P* : p + *p* :: D C + D *c* : C E + E *c*. *Q. E. D.*

This may be experimentally tried by taking the Weight W together with its Pulley equal to 6 Pounds, and the Weights P and *P* equal to 5 Pounds each; for then if the Pulleys A and B be fixed at the Distance of 8 Inches from each other at their Circumference, as at *a* and *b*, the three Weights will not rest till the Line C *c* be just 3 Inches long, in which Case the Triangles *a* C *c*, *c* C *b*, E D C, D E *c*, *m o* C, and *o* C *r* will have their three Sides in the Proportion of 4, 3, and 5. But the best Method of trying all Cases of this kind, is to make use of the Machine contrived by Dr. *s'Gravesande* for that purpose. (See his Introduction to Sir *Isaac Newton*'s Philosophy, *Part* I. N°. 205.) On the Plank or horizontal Board H † are fixed two Standards S, S, which have, each on its upper Part, a Sextant with several

Plate 15. Fig. 17.

† Plate 15. Fig. 18.

several Lines drawn from a Center taken on the upper Part of a Pulley, along which Lines the Threads going over the Pulleys may be stretched. In the Middle of the Lines are written the Numbers which express the Secants of the Angles which those Lines make with the Horizon, and at the Ends of the Lines are written the Numbers expressing the Tangents of those Angles. Now in making Experiments it will appear in every Case where there is an *Æquilibrium*, that the Weights Q and Q, are as the Numbers in the Middle of the Lines along which the Threads are stretched; and the Weight P as the Sum of the Numbers at the Ends of those two Lines. Annotat. Lect. III.

11. [48. ——— *A less Power will serve* &c. — *unless it pushes the Body against the Plane*, &c. *or draws it away from the Plane*, &c.] We have in the 7th Annotation considered all that relates to a Body moving on an inclined Plane, and therefore refer to that; but it will not be improper here to take notice of the Difference between high Wheels and low ones, as they roll over uneven Grounds or Rubs: Because though this Motion cannot be considered in every respect like the rolling of Bodies on inclined Planes; yet there are a great many Things alike in both Cases.

Let the Line *a b* * represent the horizontal Plane, or Way on which a Wheel, represented by the Circle *l C h g*, is to roll from *a* towards *b*. *m*, *n*, *o*, represent three immoveable Rubs, whose Eminences or Tops reach as high as the Points *d g h*; while the Power draws the Wheel in the Line of Direction *c G*. To know what the Intensity of the Power must be in proportion to the Intensity of the Weight (that is, in proportion to the Weight of the Wheel), we may suppose a bended Leaver in the said Wheel, and are to consider its Effect in the Operation, which being well examined, will give us this general Rule for all Cases of a Wheel going over a Rub on an horizontal Plane; the Line of Direction drawing along the Center of the Wheel being also supposed horizontal. * Plate 15. Fig. 19.

When the Circumference of a Wheel moving vertically on an horizontal Plane touches the Top of a Rub,

The Weight: (*taking in the Weight of the Wheel*)
Is to the Power that can draw the Wheel over the Rub : :
As the Sine of the Angle, which a Line drawn from the Center of the Wheel to the Top of the Rub, makes with the horizontal Line:
To its Cosine.

Preparation.

Through *d* the Top of the Rub *m d*, and through *g* the Top of the Rub *n g*, draw the horizontal Lines *e d t*, *r g s*; draw the *Radii c d*, *c g* and *c h*, the last of which is parallel to the Horizon, and must be produced as far as *p*; with the Radius *c d*, and Center *d* draw the Arc *c k*; then from the Points *d* and *g* draw *d f k* and *g i* perpendicular to *c h*, and round *k* for a Plate 15. Fig. 19.

Annotat. Lect. III. Center draw the Circle C x D *d* equal to C *l q*, which will represent the Wheel raised up upon the Top of the Obstacle or Rub *m d*.

DEMONSTRATION.

In respect to the Rub *d m*, *c d e* is a bended Leaver, whose *Fulcrum* is *d* and *Brachia c d*, *d e*; but as the Power draws obliquely to the *Brachium d c* in the Line *p c*, the *Brachium d c* must be reduced to *f d* perpendicular to the Line of Direction (being the acting Distance of the Power) but *d e* preserves its whole Length, because *c q* the Line of Direction of the Weight goes perpendicularly through its Extremity *e*. Therefore here the Weight and the Power will be to one another reciprocally as the *Brachia f d* and *d e*; but *f d* is the Sine of the Angle *f c d*, (= *c d e*) which the Line *c d* makes with the horizontal Line *d e*, or its Parallel *c f*, and *d e* its Cosine. *Q. E. D.*

SCHOLIUM.

If the Rub had been *n g*, twice as high, the Difficulty of drawing the Wheel over it would have been more than twice as great; because in considering the bended Leaver *c g r* acting in that Case, it must have been reduced to another bended Leaver *i g r*, in which the Power : is to the Weight : : as *r g* : to *g i*, where the Disproportion of the acting Distance is more than doubly increased to the Disadvantage of the Power.

COROLLARY I.

Hence follows, that the Difficulty of a Wheel to go over a Rub increaseth in a greater Proportion than the Height of the Rub; the Rubs of different Heights compared together being always as the versed Sines of the Complement of the Angle of Inclination, when the Power : is to the Weight of the Wheel and what it carries : : as the Sine Complement to the Sine of the Angle of Inclination, which last *Ratio* increases faster than the versed Sines.

COROLLARY II.

Hence follows also, that a Wheel cannot by any Power, how great soever, be drawn over a Rub whose Top is as high as the Axis (as for example the Rub *o b*) because in that Case the Sine Complement is become the whole Sine *c b*, and the right Sine is vanished; or, what is evident by the Scheme, the Power draws against the *Fulcrum* where it can have no Effect at all, let its Intensity be ever so great; unless the Direction of the Power

Power be made to alter, and it ſhould draw upwards: Therefore in Practice, eſpecially where Carriages are to go upon rough Ways, it is uſual to make the Horſes or Oxen draw a little upwards from the Center of the Fore-Wheels. Annotat. Lect. III.

COROLLARY III.

Hence likewiſe may be ſeen the Reaſon why high Wheels go over Rubs more advantageouſly than low ones, and that in proportion as they are higher: becauſe the Lengths of verſed Sines being (*cæteris paribus*) as the Diameters of the Circles to which they belong; that Rub, whoſe Height was the verſed Sine of an Arc of a certain Number of Degrees in a leſſer Circle, will be the Sine of an Arc of fewer Degrees in a greater, in proportion as it is greater: therefore the Sine Complement, or horizontal *Brachium* of the bended Leaver bearing the Weight, will be leſs, and the Sine of Inclination or perpendicular *Brachium* of the Leaver, to which the Power is applied, will be greater. Beſides, the high Wheel will not only go over Rubs impoſſible to the low Wheel, but ſeveral other Rubs ſtill higher, provided their Height be not equal to the Semidiameter of the great Wheel. As for example in *Fig.* 20 *, the Intenſity of the Power P drawing the great Wheel C D over the Rub D along the horizontal Line *a b*, is but half the Intenſity of the Power *p* drawing the little Wheel *c g* over the ſame Rub at *g*, ſuppoſing the Wheels of the ſame Weight; becauſe it is not the bended Leaver *i g r*, which is transferred from the little Wheel to the great one at F D E, but the Leaver *f d e*, whereby the little Wheel is drawn over a Rub only of half the Height. So in *Fig.* 19. †, the great Wheel, part of whoſe Circumference is repreſented by the Semicircle L H *q*, goes over the Rub M D with the ſame Eaſe, that the little one goes over *m d*; over the Rub N G (impoſſible to the little Wheel) with the ſame Eaſe that the little Wheel goes over *n g*, and has no Rub impoſſible to it (ſuppoſing the Intenſity of the Power ſufficiently great) till it becomes of the Height O H equal to its Semidiameter.

* Plate 15. Fig. 20.

† Plate 15. Fig. 19.

COROLLARY IV.

Laſtly, We may obſerve from what has been ſaid upon this Subject, that the greateſt Difficulty to bring a Wheel over a Rub is in the firſt Effort, and that the Action of the Power becomes continually eaſier as the Wheel riſes, whether the Wheel was at Reſt or in Motion juſt as it began to preſs upon the Rub; for the horizontal *Brachium* of the Leaver as *e d* (ſuppoſing *m d* the Rub) riſing continually round the *Fulcrum d*, when the Wheel is riſing from C *b g q l* to × D *d r*, diminiſhes its acting Diſtance as the Line of Direction *c q* of the Weight comes from *c q* to *k d*, whilſt the *Brachium d f*, by which the Power acts, continues the ſame, as the Line of Direction of the Power goes from the Situation *c p* into the Situation *k x*: for the Diſtance

Plate 15. Fig. 19.

of

Annotat. Lect. III. of the Horses or Oxen, *&c.* is so great in proportion to the Height of the Rub, that we need not look upon the Point x as raised at all above the Horizon; nay, if the Horses draw a little upwards, every Advance of the Line of Direction of the Power towards the Point x will be a more advantageous Situation of it.

From this last Consideration it appears, that when a Wheel, as C *l q*, goes over a Rub as *m d* or *n g*, we are not to consider it as if it rolled up an inclined Plane as *q d* or *q g*, where the Power drawing horizontally (as in the Case of the Wedge) acts uniformly, and that in the Ratio of the Base to the Height; the Power here being required to exert much more Force at first, and less afterwards.

12. [50. —— *How much the Power must be increased in proportion to the Angle which its Line of Direction makes with the Plane, will be shewn in the Notes.*] All this has been fully considered in the seventh Annotation to this Lecture.

15. [62. —— *This Sort of Thread is not used in Wood, but in Iron, and other Metals, it is of good service, being commonly more durable, and raising the Weight with more ease than the Sharp-Thread, as will be more fully shewn in the Notes.*] The Square-Thread is seldom or never made use of in Wood, because then the prominent Parts of the Screw, such as P, N, L, H, Q, O, M, K, I, (*Plate* 11. *Fig.* 11.) would have no more Strength than the lateral Cohesion of the Fibres of the Wood, for a Length very little greater than the Thickness of the Thread all the way; so that in very great Strains, the Arbor *a b c d* might be stript of its Thread all the way, and so let go what it was intended to hold. But to prevent this Inconveniency, the hollowed Part of the Screw is made sharp close to the imaginary Arbor, or included Cylinder, which thickens the Thread of the Screw next the Arbor; but then of consequence it must come to a sharp Edge on the out-side. This may be seen in the 14th *Fig.* where, instead of the hollow B A C D, you have *b a d.*

Plate 11. Fig. 11.

Plate 11. Fig. 14.

Though, by this means the sharp-threaded Screw is strongest in Wood, it is weakest in Metals, where the Thread is generally fine; for if the Male and Female Screws do not exactly fit, but have a little too much Play, the Sharp-Thread of the one will gull (that is, cut and wear away) the Sharp-Thread of the other. Whereas in a Square-Thread, though there should be some Play, there is no unusual wearing out, a Flat bearing upon a Flat. Besides, on the Sharp-Thread the Weight endeavours to descend (and therefore resists) with more Force than on the Square-Thread; because, besides the Endeavour of the Weight to slide back on that inclined Plane which makes the Ascent of the Screw (which has been already explained in describing the inclined Plane and Wedge) it has also an Endeavour to descend along another Plane (as *a d*, *Fig.* 14.*) which makes the Sharpness of the Thread, and consequently the Power must be increased on that Score.

* Plate 11. Fig. 14.

To

To estimate what is the Force which the Sharp-Thread has to bear the Weight on account of its Declivity, let us examine the Screw C D F, E M, * supposing a Female-Screw to be raised with all the Weight, which consequently must press upon the Thread of the Male-Screw here represented at C I K P G H, *&c.* all which we will reduce to the Pressure on the Point P, W A B P being supposed the Weight pressing, and W P its Line of Direction. Now, if we resolve the Force W P representing the Pressure of the Weight downwards into the two Forces W A and W B, of which the first represents the Force exerted against the Declivity A D, and the last that whereby the Weight endeavours to go along A D without pressing against it; it will appear (because Action and Reaction are equal) that the inclined Sharp-Thread only lifts the Weight with the Force represented by A W, whilst the Force left, *viz.* W B, carries down the Weight in the Direction A D, thereby drawing the opposite Side of the Thread of the Female-Screw to press the harder against the Thread of the Male-Screw at C, M, and E, *&c.* So that if the Weight does not happen to rise perpendicularly (which it does not in many Cases) there will be a great Increase of Friction on the upper Side of the Screw.

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* Plate 16. Fig. 1.

The Friction of the Screw being owing to many Causes, we shall consider it in another Place.

14. [70. ——— *Pretenders to* Perpetual Motions, *and those who promise greater Effects by Machinery than is conformable to the reciprocal Proportion between the Intensities of the Powers and Weights, and their Velocities.*] About the Year 1720, and 1721, the late *John Rowley*, Mathematical Instrument-maker, talked so much of the Wheel which he had seen at *Hesse-Cassel* (which he believed to be a perpetual Motion, as well as a great many Persons in that Country) that besides the common Herd of *Perpetual-Motion Men*, which every Age affords, some very ingenious Men made an Attempt that way, and were countenanced in it by some great Mathematicians, who, when the Scheme was laid before them, declared they knew no Reason why it should not do. But as I always declared against all Projects tending that way, I was desired at that time to publish my Reasons why the thing seemed impossible or impracticable; which I did in the *Philosophical Transactions* * * in such a manner, as might dissuade People at first from any such Attempts, in which so much Time and Money have been lost. I have here printed the whole Account again; but desire my Reader first to look over what I have said in these Notes, Pages 151 and 152.

* * N°. 369.

The Wheel at Hesse-Cassel *made by* Monsieur Orfireus, *and by him called* a Perpetual Motion, *has of late been so much talked of on account of its wonderful* Phænomena, *that a great many People have believed it to be actually a* self-moving Engine; *and accordingly have attempted to imitate it as such. Now, as a great deal of Time and Money is spent in those Endeavours, I was willing (for the sake of those that try Experiments with that View) to shew that the Principle which most of them go upon is false, and can by no means produce a perpetual Motion.*

They,

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They take it for granted, that if a Weight deſcending in a Wheel at a determinate Diſtance from the Center, does in its Aſcent approach nearer to it; ſuch a Weight in its Deſcent will always preponderate, and cauſe a Weight equal to it to riſe, provided that Weight comes nearer the Center in its Riſe; and accordingly, as itſelf riſes, will be over-balanced by another Weight equal to it: and therefore they endeavour by various Contrivances to produce that Effect, as if the Conſequence of it would be a perpetual Motion.

But I ſhall ſhew, that they miſtake one particular Caſe of a general Theorem, or rather a Corollary of it, for the Theorem itſelf. The Theorem is as follows:

If one Weight in its Deſcent does (by means of any Contrivance) cauſe another Weight to aſcend with a leſs Momentum *or Quantity of Motion than itſelf, it will preponderate and raiſe the other Weight.*

COROLLARY I.

Therefore if the Weights be equal, the deſcending Weight muſt have more Velocity than the aſcending Weight, becauſe the *Momentum* is made up of the Velocity multiplied into the Quantity of Matter.

COROLLARY II.

Therefore if a Leaver, or a Balance, has equal Weights faſtened, or hanging at its Ends, and the *Brachia* be ever ſo little unequal, that Weight will preponderate, which is fartheſt from the Center.

SCHOLIUM.

This ſecond Corollary cauſes the Miſtake; becauſe thoſe, who think the Velocity of the Weight is the Line it deſcribes, expect that Weight ſhall be overpoiſed which deſcribes the ſhorteſt Line, and therefore contrive Machines to cauſe the aſcending Weight to deſcribe a ſhorter Line than the deſcending Weight. As for example, in the Circle A D B α, (*Fig.* 2. *) the Weights A and B being ſuppoſed equal, they imagine, that if by any Contrivance whatever, whilſt the Weight A deſcribes the Arc A α, the Weight B is carried in any Arc, as B b, ſo as to come nearer the Center in its riſing than if it went up the Arc B D; the ſaid Weight ſhall be overpoiſed, and conſequently by a number of ſuch Weights, a perpetual Motion will be produced.

* Plate 16. Fig. 2.

This is attempted by ſeveral Contrivances, which all depend upon this falſe Principle; but I ſhall only mention one, which is repreſented by *Fig.* 4. where a Wheel having two parallel Circumferences, has a Space between them divided into Cells, which being curved, will (when the Wheel goes round) cauſe Weights placed looſe in the ſaid Cells, to deſcend on the Side A A A, at the outer Circumference of the Wheel; and on he

Fig. 1.

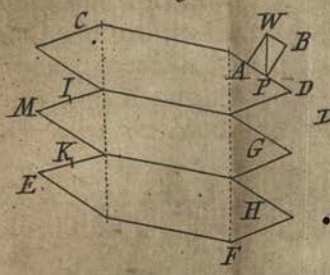


Fig. 2

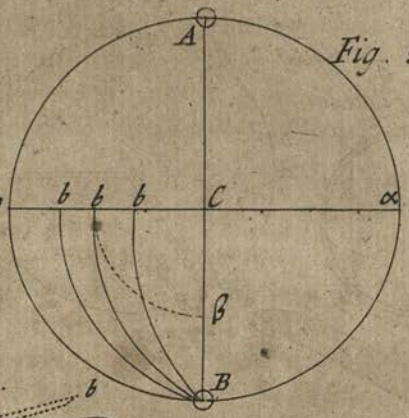


Fig. 9. Page 461.

Fig. 3.

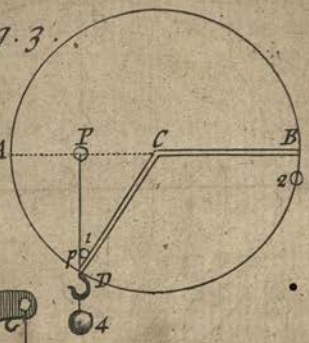


Fig. 8. Page 460.

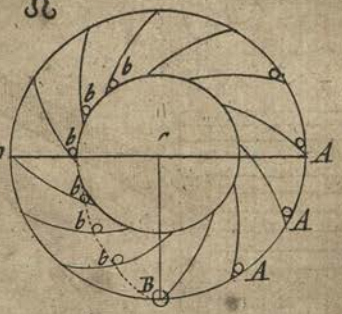


Fig. 4.

Fig. 5.

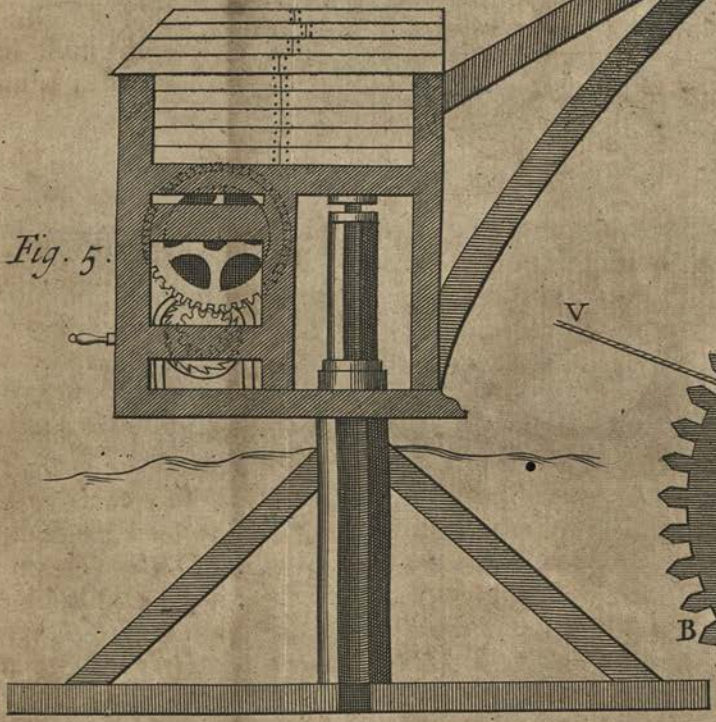


Fig. 6.

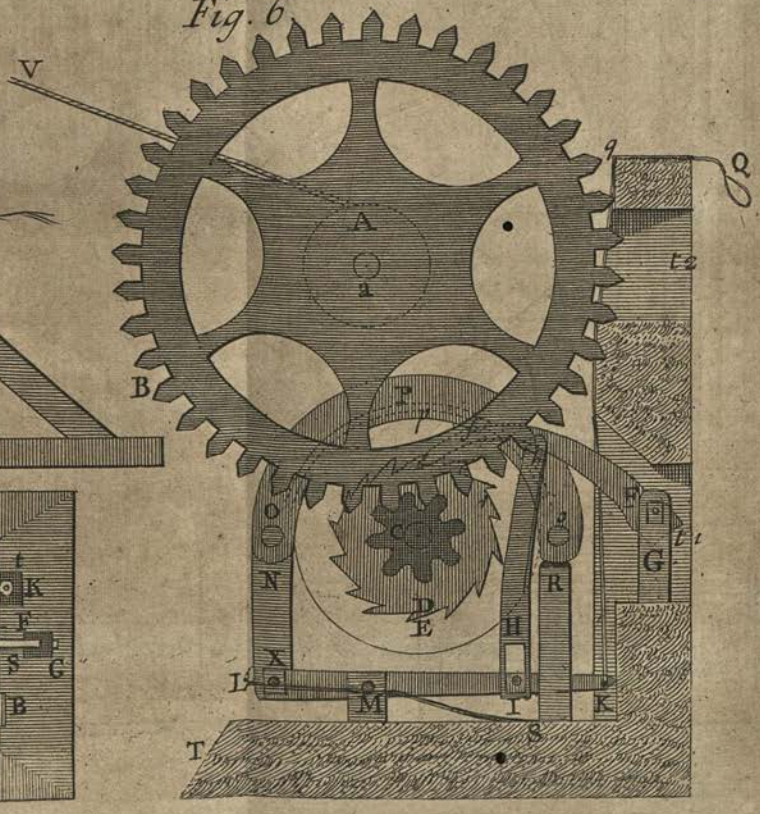
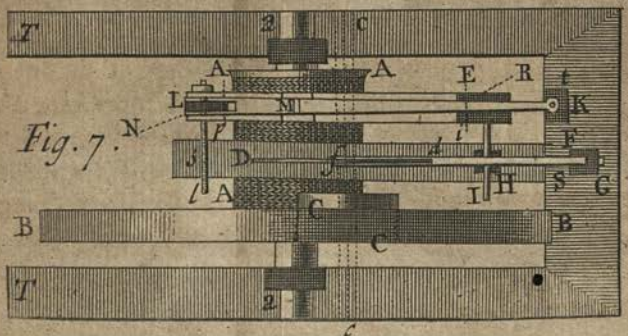


Fig. 7.



the Side D to ascend in the Line B *b b b*, which comes nearer the Center, and touches the inner Circumference of the Wheel. In a Machine of this kind, the Weights will indeed move in such a manner, if the Wheel be turned round, but will never be the Cause of the Wheel's going round. Such a Machine is mentioned by the Marquis of *Worcester*, in his *Century of Inventions*, in the following Words, N°. 56. Annotat. Lect. III.

" To provide and make that all the Weights of the descending Side " of a Wheel shall be perpetually farther from the Center than those of " the mounting Side, and yet equal in Number, and heft to the one Side " as the other. A most incredible Thing, if not seen; but try'd before " the late King (of blessed Memory) in the Tower by my Directions, two " Extraordinary Ambassadors accompanying his *Majesty*, and the Duke " of *Richmond*, and Duke *Hamilton*, with most of the Court attending him. " The Wheel was fourteen Foot over, and had forty Weights of fifty " Pounds a-piece. Sir *William Balfore*, then Lieutenant of the Tower, " can justify it, with several others. They all saw, that no sooner these " great Weights passed the Diameter of the Line of the lower Side, but " they hung a Foot nearer to the Center; nor no sooner pass'd the Dia- " meter Line of the upper Side, but they hung a Foot farther. Be pleas'd " to judge the Consequence."

Now the Consequence of this and such like Machines, is nothing like a perpetual Motion; and the Fallacy is this: The Velocity of any Weight is not the Line which it describes in general, but the Height it rises up to, or falls from, with respect to its Distance from the Center of the Earth. So that when the Weight (*Fig.* 2. *) describes the Arc A *a*, its Velocity is the Line A C, which shews the perpendicular Descent (or measures how much it is come nearer to the Center of the Earth) and likewise the Line B C denotes the Velocity of the Weight B, or the Height that it rises to, when it ascends in any of the Arcs B *b*, instead of the Arc B D: So that in this Case, whether the Weight B in its Ascent be brought nearer the Center or not, it loses no Velocity, which it ought to do, in order to be raised up by the Weight A. Nay, the Weight in rising nearer the Center of a Wheel, may not only not lose of its Velocity, but be made to gain Velocity in proportion to the Velocity of its counterpoising Weights, that descend in the Circumference of the opposite Side of the Wheel: for if we consider two *Radii* of the Wheel, one of which is horizontal, and the other (fasten'd to and moving with it) inclin'd under the Horizon in an Angle of 60 Degrees (*Fig.* 3. †) and by the Descent of the End B of the Radius B C, the Radius C D by its Motion causes the Weight at D to rise up the Line *p* P, which is in a Plane that stops the said Weight from rising in the Curve D A, that Weight will gain Velocity, and in the beginning of its Rise, it will have twice the Velocity of the Weight at B; and consequently, instead of being rais'd, will overpoise, if it be equal to the last mention'd Weight. And this Velocity will be so much the greater, in proportion as the Angle A C D is greater, or as

* Pl. 16. F. 2.

† Pl. 16. F. 3.

Annotat. Lect. III. Pl. 16. F. 2. the Plane P *p* (along which the Weight D must rise) is nearer to the Center. Indeed if the Weight at B (*Fig.* 2.) could by any means be lifted up to β, and move in the Arc β *b*, the End would be answer'd; because then the Velocity would be diminish'd, and become β C.

EXPERIMENT. *Fig.* 3.

Pl. 16. F. 3. Take the Leaver B C D, whose *Brachia* are equal in Length, bent in an Angle of 120° at C, and moveable about that Point as its Center: In this Case, a Weight of two Pounds hanging at the End B of the horizontal Part of the Leaver, will keep *in Æquilibro* a Weight of four Pounds hanging at the End D. But if a Weight of one Pound be laid upon the End D of the Leaver, so that in the Motion of D along the Arc *p* A, this Weight is made to rise up against the Plane P *p*, (which divides in half the Line A C equal to C B) the said Weight will keep *in Æquilibrio* two Pounds at B, as having twice the Velocity of it, when the Leaver begins to move. This will be evident if you let the Weight 4 hang at D, whilst the Weight 1 lies above it: For if then you move the Leaver, the Weight 1 will rise four times as fast as the Weight 4.

Pl. 12. F. 3. 15. [80.———*The Weight and Gibbet would run back and rest over* W, &c.] In the third Figure of Plate 12, C G *r g s* represents the Top of the Gibbet with its Pulley at *r* its Extremity, and its Center (or the Top of its Axis) at C. It is to be observ'd here, that the horizontal Part of the Rope C *r* is in the same Plane with the middle Line of the Gibbet, or that the Rope C *r* is parallel to the Line C *r* (under it) in the Gibbet. Now if it was possible for the Rope to continue parallel to the said Line C *r* in the middle of the Top of the Gibbet, whilst it is mov'd to the left successively in the Situations C 6, C 7, C 8, or to the right in the Situations C 1, C 2, C 3, C 4, C 5, the Guider, or Person who pulls and directs the Guide-Rope fasten'd at the End near *g*, would bring the Weight to any Place on the Wharf, on either side the Crane, without any more Labour than what overcomes the Friction of the Axis of the Gibbet. But when the Pulleys P and Q are placed at *p* and *q*, the middle Line of the Gibbet advances towards P and Q faster than the Rope does, which being obliged to fold about the Pulleys, makes an Angle with the Line above-mention'd; as for Example, the Angle *p* 8 *n* on the left Side and *q* 4 *t* on the right, the Angle being so much greater as the Gibbet is more drawn back on the Wharf towards the Crane. The Consequence of this is, that the Weight (in this Motion of the Gibbet) must be rais'd in proportion as the Rope is lengthen'd by folding about the Pulley, or in the Proportion of C *p* 8 to C 8 on the left, and C *q* 4 to C 4 on the right. Now if the Excess of the lengthen'd Rope above the Length of the middle Line of the Top of the Gibbet be (for Example) one tenth of the whole, then a Person holding the Gibbet by the Guide-Rope, in a Position which makes that Difference, must sustain a tenth Part of the whole Weight, *viz.*

viz. 224 lb. in a Ton, too great a Weight for a Man thus employed to move with the Gibbet. Now as this Force of the Weight gradually increases by drawing round the Gibbet, it deceives the Man who pulls the Guide-Rope; and as by a sudden Jerk he has brought on the Weight, he is often forced to let it go when it comes beyond his Strength; which sometimes proves of dangerous Consequence, as well to the Persons loading or unloading, as to the Goods craned up. Annotat. Lect. III.

To prevent these Inconveniencies, Workmen have plac'd the Pulleys in a different Situation, *viz.* in the Situation P, Q; so that in turning the Gibbet to the right, no Part of the Weight will be felt by the Guider till the Gibbet comes beyond C 4; and in moving to the left, till it comes beyond C *c*. But then another Inconveniency arises from this Construction, which is the Reverse of the other Thing consider'd; namely, that the Weight descends, and consequently brings on the Gibbet with a Swing, which (if unexpected) may likewise do Mischief. As in large Weights these Inconveniencies are most sensible, Mr. *Padmore* of *Bristol*, making a Crane for Mr. *Allen* (Postmaster of *Bath*) to raise Stone out of a Quarry, contrived an Application of the *Axis in Peritrochio*, which takes off this Danger and Inconveniency. In the first Figure *x* in a Wheel with Arms, Pl. 12. F. 1. whose Axis *x u* has on it a Pinion at *u*, which takes the Teeth of an horizontal contrate Wheel fasten'd to the Axis of the Gibbet; whereby one Man standing out of Harm's way close to the Wheel, has such Advantage of Power by means of the long Arms of the Wheel, as to move the Axis of the Gibbet with great Ease, notwithstanding the Inequalities above-mention'd, and also to hold the Gibbet in any Position without Difficulty.

The same ingenious Workman has made another considerable Improvement in another Crane fix'd by the River-side, whereby Mr. *Allen* lets down his Stone into such Vessels as come to fetch it away.

The Crane itself is not of an uncommon sort, but a Rat-tail'd Crane with a double Axis *in Peritrochio* and two Handles, whereby four Men may raise very great Weights; and then turning the whole Crane about upon its upright Shaft, can fix it in any Position, and let down the Weights speedily into the Boats or Barges which come near the Wharf to receive them. See the 5th Figure of Plate *16*, where you may observe it to dif- Pl. 16. F. 5. fer from that represented in Plate 12. Fig. 4. because the long Neck of Pl. 12. F. 4. the Crane is here of one Piece, and the Power differently applied. But this Construction is not new. Neither is it a new Invention to let down Goods after they have been raised by a Crane, by pressing an Arch of a Circle strongly upon a Wheel fix'd to the principal Axis, in order to retard and regulate the Descent by a Friction increas'd or diminish'd at pleasure, as is done in stopping Wind-Mills. The Catch also, that hinders a Crane or Capstane from going back, is of common Use; but I don't know that any one has applied them both together in the same Crane, so as to depend upon one another, before Mr. *Padmore* did it, though many

Annotat. Lect. III. 11. 16. F. 6.

have done it since. Therefore I shall give a particular Description of this Contrivance, whose chief Intent is to prevent the great Mischiefs which often happen by the Carelessness of the Men employ'd to raise and let down heavy Burthens by the Use of the Crane. The sixth Figure represents an upright Section of so much of the Crane as the Contrivance above-mention'd is apply'd to.

A B is the great Wheel, whose large Axis A moving on two Iron Centre-Pins such as *a*, receives the Rope, or lets it run down according as it is turn'd, by means of the Handles fasten'd at C to the lesser Wheel or Pinion C, or as it is suffered to turn the other way by the Gravity of the descending Weight when all Obstacles are removed. Upon the Axis of the Pinion is the Ratchet-Wheel D *d*, whose Teeth successively receive the Iron Catch *f* F (moveable on a Pin at F on the Iron-Standard G, and to be rais'd up occasionally by the upright Iron H *h*) to hinder the Weight from going back when the Handles are loosed. Upon the same Axis behind the Wheel D *d* is a wooden Wheel E *p*, over which hangs the Half-Ring of Iron O P *o*, with a Groove or Hollow made in it to fit the Circumference of the said Wheel, so as to retard, or stop, or any way regulate the Motion of the Wheel, (and consequently of the Axis and Pinion C, and the great Wheel and Axis A B, which has the Rope V A) according as it is more or less strongly press'd down to make a Friction on the Wood, as it moves after the Catch is rais'd out of the Teeth of the Ratchet. The horizontal Leaver K L governs all these Motions in the following manner, *viz.* When the String Q *q* K, fasten'd to the said Leaver at K, is pull'd, the Leaver moving on its Center M, does, by an horizontal Pin fixed at right Angles to its Side at I, raise the Piece H *h*, and consequently release the Ratchet by raising the Catch at *f* out of the Teeth: Then the Weight descends swiftly, moving the Wheel and Pinion round by its Force; but to prevent the too swift Descent, the Leaver is pull'd up a little more strongly by the Guider who holds the String Q *q* K, which brings down the contrary End of the Leaver L, and consequently the Iron N, so low as to make the semicircular Ring O P *o* press hard upon the Wheel E *p*, which it did not do when the Catch was rais'd but just out of the Ratchet. N. B. *A strong Pull stops the whole Motion, and a more gentle one regulates the Descent.* And if the Guider should be careless and let go the String; then immediately the Spring S *s*, whose End *s* had been depress'd by the End L of the Leaver, will raise it up again (by its lateral Pin X) and restoring the whole Leaver to its first horizontal Position, the other lateral Pin I in the long Arm M K of the Leaver, will through the Notch H press upon I, the lower End of the upright Piece H *h*, and so bring down the Catch F *f* into the Ratchet Wheel at *f*; the curved Piece O P *p* at the same time flying up, and no longer pressing the wooden Wheel E *p*.

Thus will Mischief never be the Consequence of Carelessness, because of the Catch; nor will the Weight go down by Jerks, which would have

been the Consequence of the Catch * used without the Half-Ring, because the Catch is lifted quite out of the way when the Half-Ring is brought down and applied by pulling the String at Q. *N. B.* T, *t* 1, *t* 2, *is part of the upright Section of the Timber of the Frame.* Annotat. Lect. III. * Pl. 16. F. 6.

To make this the plainer, let us examine the 7th *Figure* of *Plate* 16; Pl. 16. F. 7. where we have an horizontal Section of the Parts above-mention'd. T *t* T is Part of the Timber of the Frame. B B is the great Wheel, whose Axle that holds the Rope is mark'd A A A, and its Iron Axis goes thro' Bell-metal Boxes at *a a*. C C is the small Wheel or Pinion, whose Axis is *c c*. D *d* is the Section of the Ratchet Wheel made of Iron. Between the prick'd Lines A *p* and E *i* is supposed the wooden Wheel upon the Axis of the Pinion, (not drawn here to avoid Confusion, no more than the semicircular pressing Piece mark'd O P *o* in the last Figure) the Basis of whose upright fix'd Supporter is represented by R, and the End of the Piece which brings it down upon Occasion is shewn at N. K L is the horizontal Leaver, whose Center is at M, moving vertically by a Pull of the String fasten'd to it at K. I *i* is the first lateral Pin of the Leaver, which at I goes through the Bottom of the Piece H, Raiser of the Catch F *f*, already describ'd with its Supporter G, on whose Top the Catch moves by a Center Pin. A *p* A is the second lateral Pin of the Leaver, whose Office is to press upon the End *s* of the crooked Spring S *s*, fastened to the Bottom of the Frame at the farther End S. So that when the End K of the Leaver is pull'd up, the End L, which is depress'd, must be lifted up again into its Place by the Force of the Spring restoring itself.

N. B. *The crooked Figure of the Spring, and the Manner of its lifting the Pin, is best shewn in the last Figure.* Pl. 16. F. 6.

There is a fine Contrivance of an inclin'd Waggon-way made of Timber to bring down the Stone from Mr. *Allen*'s Quarry to the River's-side, the Distance of near a Mile, in Waggons which come down the Declivity on the artificial Way by their own Gravity, as the Coal-Carriages do near *Newcastle*. But this Waggon-way differs quite from those in the North, every Part being very much alter'd for the better, and the Carriages themselves contrived to carry much more Weight without Danger. *The Description of this Contrivance, wherein Mr.* Allen *and his Workmen have shewn great Skill and Ingenuity, I must now omit, and give it in another Part of this Book.*

LECTURE IV.

Concerning the Friction in Mechanical Engines.

WHAT I have said hitherto in the Three first Lectures, and their Notes, is sufficient for explaining the Principles of Mechanicks (strictly so call'd) enough to set People to work, who have a Genius for Practical Arts; but as they may be guilty of considerable Errors without making proper Allowances for Friction, and knowing how to find out (nearly) what it is in Engines already made, and such as they intend to make; I thought proper in this Place to give a Lecture on what I have been able to discover of the Nature of Friction, by reading all the Accounts I could meet with on that Subject, by repeating several Experiments already made, the better to consider their Circumstances, and whether the Accounts of them were true; and likewise by making a great many new Experiments myself.

Tho' there are so many Circumstances in the Friction of Bodies, that the same Experiment does not always succeed with the same Bodies, so that a Mathematical Theory cannot be easily settled; yet we may deduce a Theory sufficient to direct us in our Practice from a great Number of Experiments, always taking a Medium between Extremes.

FIRST then, it is observ'd that Wood, Iron, Brass, Copper and Lead, when greased or oiled (as is done in Engines) have nearly the same Friction, and therefore the same Rule will serve for all those Substances. For tho' one may at first imagine that Metals must needs slip over one another more easily, because they may be made smoother and will take a better Polish; yet it is found by Experience, that the flat Surfaces of Metals or other Bodies may

may be ſo far poliſh'd as to increaſe Friction; and this is a me- Lect. IV.
chanical Paradox; but the Reaſon will appear when we conſider
that the Attraction of Coheſion becomes ſenſible as we bring the
Surfaces of Bodies nearer and nearer to Contact. This is very
evident in drawing Glaſs Plates over one another, which requires
more Force than if they were of Wood; and the ſame is true in
Metals, for tho' the Preſſure of the Air is ſometimes a little con-
cerned in this Phænomenon, yet the Attraction of Coheſion does
ſo far exceed it, that we need not take notice of the Effect of that
Preſſure in the Action of the Parts of an Engine on one another.
It is true indeed, that a couple of Marble Plates ſtick to each other
by the Preſſure of the external Air, when being firſt oil'd, to
exclude the Air from between them, they have been ſlipp'd upon
one another; and that when ſuſpended in the Receiver of an Air-
Pump, they fall aſunder as ſoon as the Air is drawn out of the Re-
ceiver. But if they be made very ſmooth, they will cohere ſtrongly
even after the Preſſure of the Air is wholly taken off by the Air-
Pump. I have applied together the flat Surfaces of two ſmall
Cryſtal Buttons, without wetting or oiling, which have coher'd ſo
ſtrongly as to hold 19 Ounces *Troy* before they were ſeparated,
when their Contact was but a Circle of one 12th of an Inch in
Diameter; in which Caſe the Preſſure of Air could not be greater
than the Weight of an Ounce; becauſe a Column of Air, whoſe
circular Baſe is one 12th of an Inch in Diameter, weighs no more.
In Metals the ſame is evident, but more eſpecially in Lead; for
two Balls of Lead of about one or two Pound Weight each, if
pared clean with a Knife and applied cloſe together, ſo as to touch
in a Surface of about one 10th of an Inch in Diameter, will ſtick
together, ſo as not to be ſeparated by a leſs Weight than of 40 or
50 Pounds, tho' the Preſſure of the Air in that Caſe could not
amount to one 4th of a Pound. *See Phil. Tranſact. N.* 389. But
to return to our Experiments of Friction.

EXPERIMENT I. *Plate* 13. *Fig.* 18. Pl. 13. F. 18.

C *b* is a ſmooth Piece of Wood one Inch thick, four Inches
wide and ſix long, with a little Hook in its Fore-part *b*, and weigh-
ing ſix Ounces. If it be drawn along an horizontal Plane, repre-
ſented by the Line A B, over the Pulley P, by means of the String
H W, the Weight W equal to two Ounces and as much more as
will overcome the Friction of the Pulley, will draw it along whe-
ther

Lect. IV. ther it slides on its greatest Surface or flat as at *c*, or on its Edge as at K, tho' in the last Case there is only a fourth part of touching Surface. From hence it appears, that *the Friction is equal to about one 3d of the Weight* *, *and arises from the Weight that presses the Parts together, and not from the Number of Parts that touch:* for whether the Parts of the Surface be considered as Springs to be bent, or as small Eminences over which the Body drawn must be lifted, it is plain that the Difficulty will be the same, whether the Surface be greater or less, provided it be pres'd in a reciprocal Proportion of its Bigness: for though the Base of *c* be four times broader than the Base of K, every Inch of it in Breadth is pres'd but by one Inch of Wood in Thickness; whereas the Surface on which it rests in the Position K, which is four times narrower, is pres'd by a Thickness of Wood of four Inches. The Friction therefore, or Impediment to the drawing the Body in the Line A B, arises not from the Number of Parts which touch, but from the Pressure on the touching Surface, as appears yet more plainly from the following

* Ann. 1.

Pl. 13. F. 18. EXPERIMENT II. *Plate* 13. *Fig.* 18.

LET the Pulley π (over which the Weight w (= W) drew along K, when it was raised up to the Level of *m* the Hook of K above A B) be brought down to the same Level as P; then W=w will not draw K along without the Help of the little additional Weight *q*; because the Traction made in the oblique Direction *m p* makes K press more against the horizontal Plane than when the Draught is in the Direction *m* π: but if the Hook be fix'd at *n*, so as to make the Line of Direction parallel to the Plane A B, then will the Weight W, without the Help of *q*, draw K as W did *c*.

Pl. 13. F. 19. EXPERIMENT III. *Pl.* 13. *Fig.* 19.

UPON the Table or horizontal Plane T M N O, let a circular Plane or Plate of about two Foot Diameter move, bearing on a Pin round the Center C so as not to touch the first Plane; then let a Brass or Leaden Plate, such as A, B or D, be so laid betwixt the circular Plane and the Table, as to press upon the Table with all

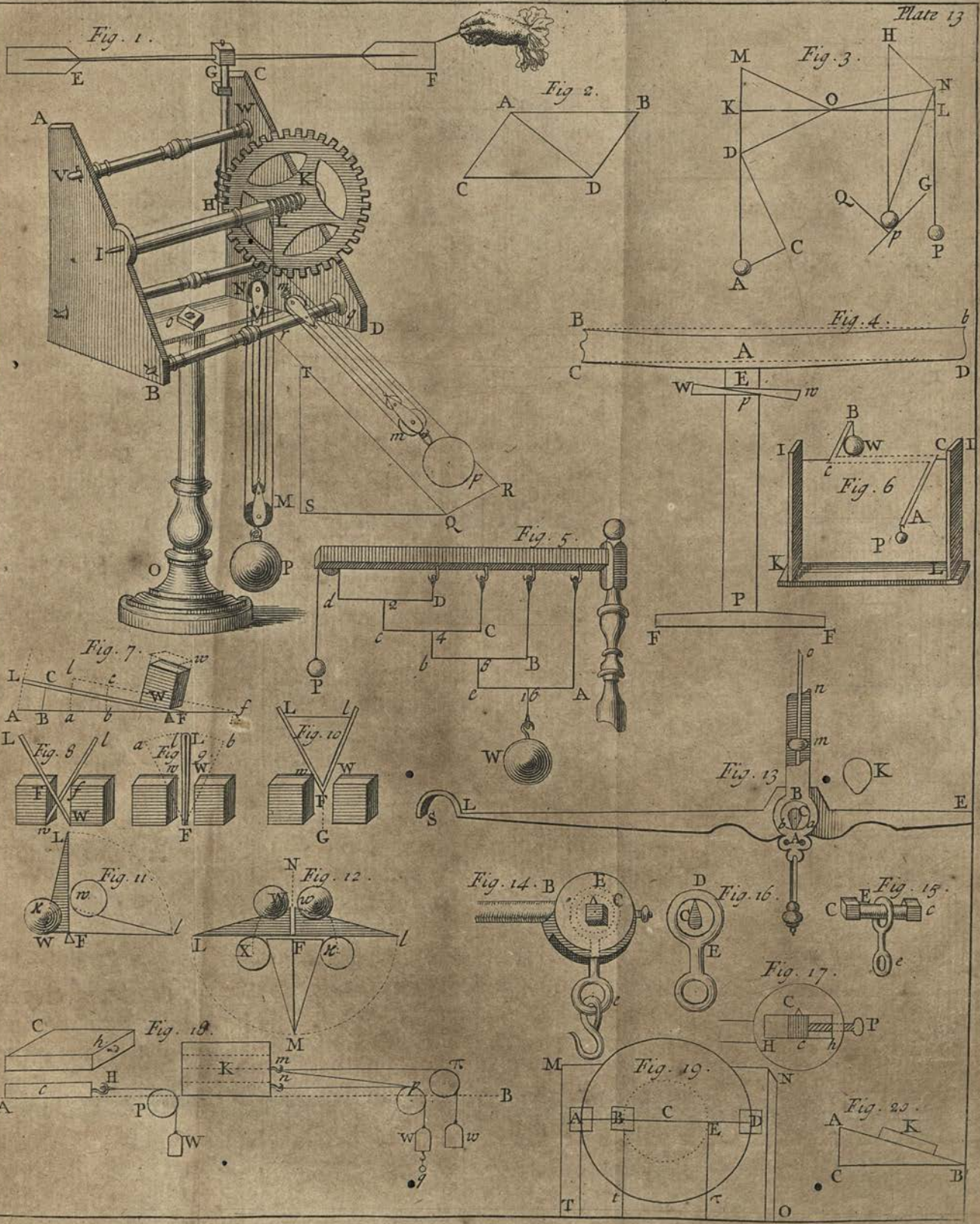


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6
Fig. 7.
Fig. 8
Fig. 10
Fig. 11
Fig. 12.
Fig. 13
Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17.
Fig. 18.
Fig. 19.
Fig. 20.

all its Weight, and at the ſame time (by means of a Pin riſing from its Middle) be carried round along with the circular Plate under the Point A, B or D. Then if the Weight of the Plate A be equal to 30 lb, a Force equal to 10 lb will draw the Plate round, pulling in the Direction A T, when the ſaid Plate is plac'd at A or D; but if the Power draws in the Line B *t* or E *τ* Tangent to the little Circle B E, it will require 20 lb to draw A round: Laſtly, if the Power draws from A towards T in the Tangent of the great Circle A *t τ* D, whilſt the Plate is fix'd at B, then a Force equal only to 5 lb will move round the Plate. Lect. IV.

THIS ſhews that the Friction is increaſed when the rubbing Parts move faſter, and decreaſed when they move ſlower than the Power. Therefore we may ſay, that *the Reſiſtances from Friction are to one another in a Ratio compounded of the Preſſures of the rubbing Parts, and the Times or Velocities of their Motions.*

WHEN a ſmooth Body is laid on an inclined Plane, according to the Laws of Mechanicks, it ought not to reſt upon it, but muſt ſlide down. But the Friction being equal to one 3d of the Weight, as we have already ſhewn, the Body K * laid on the inclined Plane A B will not ſlide down 'till the Perpendicular A C be the third part of the Length of the Plane A B, becauſe then the Body's Tendency downwards in a Line parallel to A B is equal to one 3d of the Weight of the Body. *Pl. 13. F. 20.

HENCE it appears how neceſſary it is to conſider the Friction in mechanical Performances; as for Example, if a Beam A B † lying on the riſing Ground D *c* is to be drawn by a Power at P in the Direction A P parallel to D *c*; when A *a* the Riſe of the Plane above the horizontal Line H *h* is one 30th of A *c*, a Power equal to a little more than one 30th of the Weight would be able to draw it along, according to the Laws of Mechanicks excluſive of Friction; and therefore a little more than 100 lb would draw along a Beam 3000 lb along the Plane A *c*: But as the Friction is equal to one 3d of the Weight, we muſt have a Power equal to 100 lb + 1000 lb, before we can draw the Beam along, if the Plane was as ſmooth as a marble Floor. But along the Ground, where even the ſmootheſt Earth is rough, a much greater Force will be required. Monſ. *Amontons* of the Royal Academy at *Paris* found that a Force equal to 2250 lb was but ſufficient to draw ſuch a Beam up a Hill of the Aſcent above mention'd. † P. 17. F. 1.

Lect. IV.

EXPERIMENT IV. *Pl.* 17. *Fig.* 2, 3, *and* 4.

*Pl. 17. F. 2.

LET A B, a wooden Cylinder of an Inch Diameter, smooth and well polish'd, turn freely on two Supporters S, *s*, likewise smooth, well polished and greased. When at each End of a silken Thread or String thrown over the said Cylinder, or wound about it two or three times (for that makes no Difference if the Parts of the String do not touch one another) suspend a Weight of 9 lb, which Weights will keep each other *in Æquilibrio*. Then adding several Weights to the 9 lb Weight P, here made use of as a Power, to find exactly what Addition it must have to raise the Weight W, which is also only a 9 lb Weight, that Weight will not be raised up by the turning of the Cylinder till P and all the Weights added to it make all together 18 lb. This is agreeable to what was shewn before by the first Experiment of this Lecture. For as the whole Weight that presses the Cylinder on its Supporters is W + P or 18 lb, 6 lb must be added to P in order to overcome that Friction; but that additional Weight adds also a Friction of 2 lb the third part of its Pressure, for the overcoming of which Friction 2 lb must be added, whose Friction requires also two 3ds of a Pound, and so on, in a Series of Fractions as $\frac{2}{9} + \frac{2}{27} + \frac{2}{81}$, *&c.* Now as the Weight of the Cylinder it self was not considered, the proper way to allow for its Friction, if it be not very heavy, is to double the Power at once *, as appears in *Fig.* 4. where A represents the Section of the Cylinder above mentioned.

*Pl. 17. F. 4.

IF the Cylinder has Gudgeons or a small Axis, such as that represented in *Fig.* 3. † it will move as much more easily on its Supporters as the Diameter of the Axis at G and *g* is less than the Diameter of the Cylinder *a b*; as here the Weight added to *p* need be but $1\frac{1}{2}$ lb, when the Diameter of the Axis is but $\frac{1}{6}$ of an Inch; because then the rubbing Parts move slower than the Power, in that Proportion, agreeable to what was shewn by the third Experiment of this Lecture. The fourth *Figure* ‖ represents the Section of the Cylinder at *a*, and of its Axis at *g*.

† Pl. 17. F. 3.

‖ Pl. 17. F. 4.

HENCE we may know what Difference there is between the Use of Sledges and Carts for drawing along heavy Goods. If the Ground was perfectly even like a Marble Floor, it would require $\frac{1}{3}$ of the Weight of the Sledge, and all that is laid upon it, to draw it along; * but as such a Plane is not used in Practice (and if it could be had,

Ann. 2.

Horses

Plate 17.

Fig. 1.

Fig. 2.

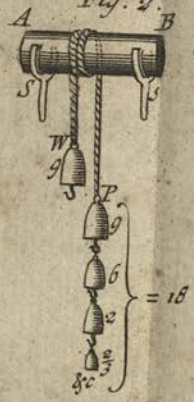


Fig. 3.

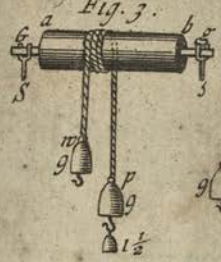


Fig. 4.

Fig. 5.

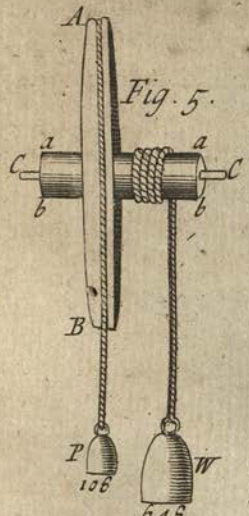


Fig. 6.

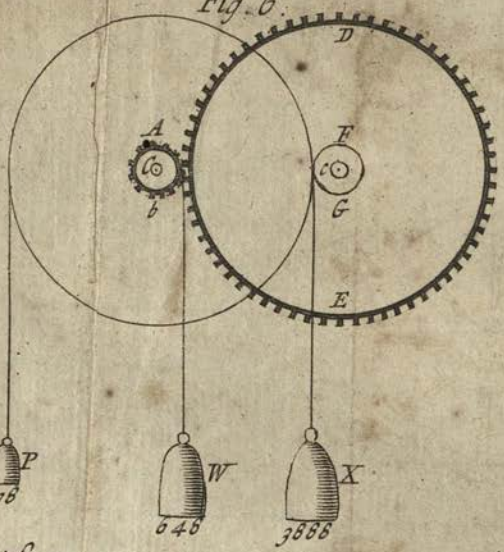


Fig. 7.

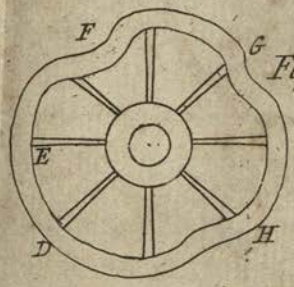


Fig. 8.

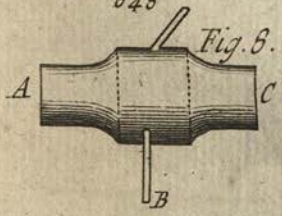


Fig. 11.

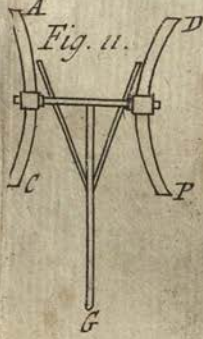


Fig. 12.

Fig. 9.

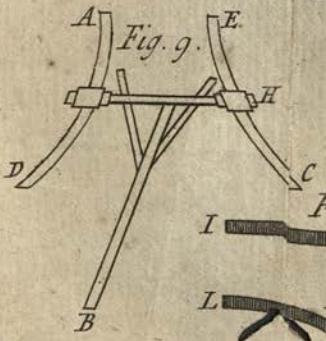


Fig. 14.

Fig. 15.

Fig. 13.

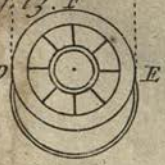


Fig. 16.

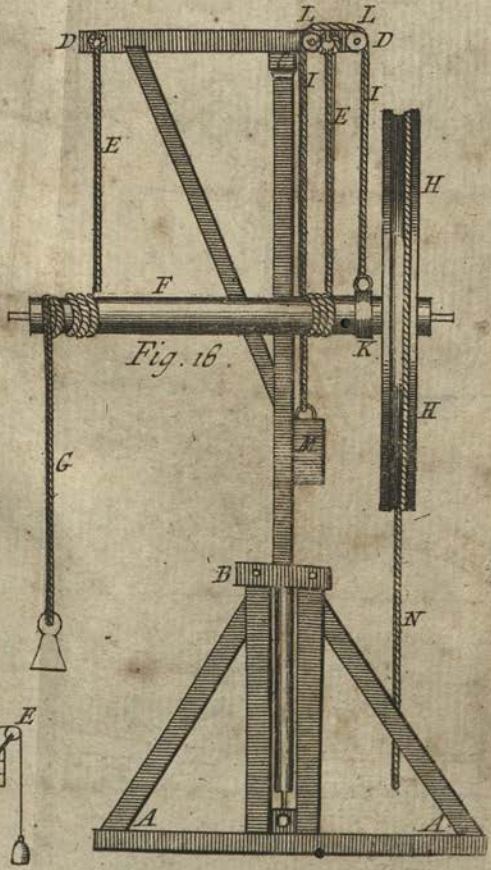


Fig. 10.

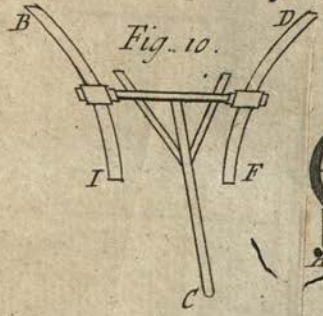


Fig. 16.

Fig. 17.

Horses could not pull well on a slippery Pavement) we must al- Lect. IV.
ways make use of a Power greater than one 3d of the Weight. But in a Cart, if the Wheel be six Foot high, and the Hole in the Nave for the End of the Axle four Inches, the Friction will be diminish'd 18 times; therefore a Power equal to $\frac{1}{54}$ Part of the Weight will be able to draw it along upon an horizontal Plane; because the 18th Part of $\frac{1}{3}$ is $= \frac{1}{54}$. Sledges therefore should only be used where Streets are too narrow for Carts, and where Goods in small Parcels are to be laid on and taken off often, for the Conveniency of the Men; but the Horses must work the harder. This also shews that high Wheels in Carriages are preferable to low ones on account of their having less Friction, besides the mechanical Advantage which we have considered in another Place; tho' it is not always expedient to make use of them, as for Example, in Beer-Carts high Wheels would be inconvenient, because of the Necessity of often loading or unloading.

To reduce all this to Practice, take the following Rules for calculating the whole Friction of Engines.

RULE I.

In examining all the Frictions, begin with that which is nearest to the moving Power.

RULE II.

To find the first Friction, consider the Spaces gone through in the same time by the Power and by the rubbing Part, and accordingly take a proportional Part of $\frac{2}{3}$ of the Power. As for Example, in equal Velocities of the Power and rubbing Part, the Friction is $\frac{1}{3}$ of the Pressure which is made not only by the Power, but by the Resistance equal to it; and therefore the third Part of the whole Pressure must be equal to two 3ds of the Power.

RULE III.

The Power being unknown, must first be found by knowing the Velocity of the Weight, and thence deducing that of the Power by mechanical Principles: Then, by what has been shewn here, finding the Friction, there must be so much added to the Power to make the Engine work; always observing that what has been

Lect. IV. added for the Friction first found, does still further increase the Friction, and therefore there must still be allowed something for the Increase of Friction caused by every new Addition to the Power.

N. B. We must remember, as has been said before, that if the Power be an heavy Body, its Velocity is its perpendicular Ascent or Descent; but if of another kind, the Velocity is its Space gone through.

RULE IV.

The Friction of the several Parts of an Engine, however remov'd from that Part to which the Power is applied, may be found by comparing their Velocities with that of the Power, and thence deducing the Friction, as has been shewn; and when all the particular Frictions have been found and added together, their Value added to the moving Force will not enable it to overcome the Resistance in a compound Engine any more than a simple one; for that Addition superadds a Friction to every rubbing Part, therefore we must still increase the Power, as shewn in the last Rule. An Example will illustrate both Cases.

* Pl. 17. F. 5. The Diameter A B of an *Axis in Peritrochio* * is equal to 3 Foot, and that of its Axle *a b* only to six Inches; therefore if the Power P be = 108 lb, the Weight or Resistance W will be = 648 lb. Here the Friction will be 12 lb or $\frac{1}{6}$ of 72 lb, if the † Pl. 17. F. 2. Axle be supported on its Surface, as in *Fig.* 2. †. Now as 12 is $\frac{1}{9}$ of the Power 108, there must be added $\frac{1}{9}$ of 12 to overcome the Increase of the Friction caused by the 12 lb, *viz.* 1 $\frac{1}{3}$ lb; and then $\frac{1}{9}$ of this last and so on; so that to overcome the Resistance = 648 lb, the Power must be = 108 + 12 + 1 $\frac{1}{3}$ + $\frac{4}{27}$, *&c.* lb, or 121 $\frac{1}{2}$ lb. But if the Axle had Gudgeons of Iron C C of only one Inch Diameter, the Friction on them would be six times less, the Velocity of their Surface being diminish'd in that Proportion.

‖ Pl. 17. F. 2. Now if there be join'd to the Axle *a b* another Wheel D E ‖ likewise of three Foot Diameter, with an Axle of six Inches F G, about which the Rope or Chain that supports the Weight winds instead of winding about *a b*, then by mechanical Rules the Weight X = 3888 lb will be sustained by the Power P of 108 lb.

N. B. *We here suppose the last Axle* F G *to have also Iron Gudgeons as* c, *of an Inch Diameter, and that (by means of a Strap of Leather, or a Rope or Chain going round the Axle* A b *and*

and the Wheel D E) *the second Axis* in Peritrochio D F c G E *is carried round in the Direction* A D, *so as to draw up the Weight* X, *six times more slowly than it would rise if at* W. *But the best way is to make use of a Pinion with Leaves (that is, a small Wheel with Teeth) to carry round the great Wheel by its Teeth, as here the Pinion of* 12 *Leaves on the Axis* A b *will carry round the Wheel* D E *which has* 72 *Teeth (its Diameter being six times greater than that of* a b:) *Because a Rope, Strap, or Chain cannot perform its Office without being made so tight as not to slip, in which Case the Pression caused by the Elasticity would occasion an additional Friction, besides the additional Resistance made by the Difficulty there is to bend the Rope,* &c. Lect. IV.

To find the Friction of this compound Engine, we must consider all the Parts that rub, *viz.* first C the Gudgeon with $\frac{1}{36}$ of the Velocity of the Power; then the Teeth or Strap at A B, with $\frac{1}{6}$ of the Velocity of the Power; and lastly, the Gudgeon *c* of the Wheel D E, with $\frac{1}{216}$ of the Velocity of the Power; which gives for the several Frictions $\frac{72}{36} = 2$ lb, $\frac{72}{6} = 12$ lb, and $\frac{72}{216} = 0{,}333$ &c. lb. The Sum of all the Frictions therefore is 14,333 &c. lb; by which Number if you divide the Intensity of the Power or 108, you will have for your Quotient 7, 5 &c. which will give you a Divisor for every Weight to be added to the Power on account of the Friction. When therefore you have added 14,333 &c. for the Sum of the Frictions, you must superadd $\frac{14{,}333}{7{,}5} = 1{,}91$ for the Friction of the first additional Weight, and $\frac{1{,}91}{7{,}5} = 0{,}25$ for the Friction of the second additional Weight, &c. Therefore in order to raise X or 3888 lb by means of the compound Engine here mentioned, the Power must not be 108 lb +, but 124,61 lb +.

N. B. *All other Cases may be deduced from this.*

Tho' this may serve for finding the Sum of the Frictions to direct our Practice; yet as I have in the Calculation omitted several Decimals, whereby the Solutions might have come nearer to the Truth; for the sake of the Curious, who may expect a Solution entirely exact, I give here a Method to find it.

Since the Weights, that are to be added to the Power in order to overcome Friction, continually decrease, and always in the same Ratio, they may be consider'd as the Terms of a Geometrical Progression, whose last Term is 0.

In order to find the Sum of the Terms of such a Progression, the two following lemmatory Propositions must be premis'd.

I.

Lect. IV.

I.

In a geometrical Proportion or Progression, the Sum of the Antecedents : is to the Sum of the Consequents : : as any one of the Antecedents : is to its Consequent. (See *Euclid.*)

II.

In a geometrical Progression, the second Term Minus *the First : is to the First : : as the Last* Minus *the First : is to the Sum of all the Terms preceding the last.*

LET the Progression be $\div\!\div\, a \,.\, b \,.\, c \,.\, d \,.\, f \,.\, g$. we must prove, that $b - a : a :: g - a : a + b + c + d + f$.

THE Progression may be thus always express'd, $a : b :: b : c :: c : d :: d : f :: f : g$; but we have by the last $a : b :: a + b + c + d + f : b + c + d + f + g$.

And by Inversion,

$$b : a :: b + c + d + f + g : a + b + c + d + f;$$

By Division,

$$c - a : a :: b + c + d + f + g - a - b - c - d - f : a + b + c + d + f.$$

And taking away the Expressions with contrary Signs, it will be,

$$b - a : a :: g - a : a + b + c + d + f. \qquad Q.\ E.\ D.$$

COROLLARY 1.

So that calling the Exponent of the Progression universally n, the first Term a, the last Term z, and S the Sum of all the Terms, excepting z, we have by the second *Lemma* $na - a : a :: z - a : S$, which gives $\frac{za - aa}{na - a} = S$, which being reduced, gives $\frac{z - a}{n - 1} = S$. That is, in Words at length;

The last Term Minus *the first, divided by the Exponent of the Progression, lessened by the Number* 1, *is equal to the Sum of all the Terms but the last.*

COROLLARY 2.

AND in Progressions, whose Terms continually decrease (as those of the Weights to overcome Frictions do) the last Term being equal

to

to *o*, instead of $\frac{z-a}{n-1} = S$, we shall have supposing *z* equal to the first Term, $\frac{z-o}{n-1} = S = \frac{z}{n-1} = S$. So that dividing the first Term by the Quantity equal to the Friction, the Exponent of the Progression will be had; and dividing the first Term by that exponent *minus* 1, the Quotient will give the total Sum of all the Additions to be made on account of Frictions.

To give Examples of it.

In the 4th Experiment of *Lecture* 4. *Page* 194, the first Term of the Progression is 18, the second 6, whence the Exponent is 3, which Exponent being lessened by 1 is 2; by which Number, dividing the first Term 18, the Quotient 9 is the Sum of all the Terms but the first, which is the Power.

So in the Example of *Rule* 4. *Lect.* 4. *Page* 196, the first Term is 108, the second 12, whence the Exponent is 9, and $\frac{108}{9-1} = \frac{108}{8}$ = 13,5, which is equal to all the Additions.

So in the other Example, *Page* 197, the first Term is 108, the second 14,333 *&c.* when the Exponent is 7,535 *&c.* and $\frac{108}{6,535} =$ 15,526, *&c.* which is the exact Sum of all the Super-additions.

In the Text I have only taken 7,5 for the Exponent, having neglected the two last Decimals, which gives $\frac{108}{6,5} =$ 16,61+ as in the Body of the Lecture, *&c.*

Tho' I have, at a Mean, taken the Friction to be equal to a third Part of the Weight; yet by making many Experiments since, I found it much to vary, being sometimes greater and sometimes less; so that it is scarce possible to come at an exact Theory: Yet for Practice, as I have already said, it is useful to know what Experiments have been made with any particular View to direct us in the like Cases, by taking the Mean of the different Effects we find. As Carriages are Machines of the greatest Consequence for the Uses of Life, I thought it would not be unwelcome here, to enlarge upon the Friction of such Instrument

by

Lect. IV. *by giving an Account of several Experiments made upon Sledges and Wheel-Carriages by Mr.* Camus, *a Gentleman of* Lorrain, *which I have since tried, and found to succeed in the same manner in almost every Trial: The little Difference I observ'd being unavoidable in Things of this Nature, even when made by the same Hand. Therefore I chuse to give the Account of them in the Author's own Words, translated from the* French, *in which Language he published his Book entitled,* Traité des Forces mouvantes, pour la Pratique des Arts & Métiers, *&c.* Par M. de *Camus.*

As there will be a Difference in the Friction of the same Body drawn over different Bodies, it is necessary as much as possible to know what that Difference is, in order to know how to manage or apply Forces. For this Reason the following Experiments have been made, in order to find out nearly what Forces or Powers are required to draw Weights, or what Force is lost by Friction, and what Metal is most proper to be used, or what Materials; that we may lose the least we can of the Force of the Power; and what Effect Water, Grease or Oil have upon different Substances.

For this purpose, we must take three Sledges, each of them an Inch and an half wide and three Inches long, each Side or bearing Part being two Lines wide. Different Weights must be laid upon each Sledge, and they must be drawn over flat Bodies of different Substances or Metals; and the following Effects will be produced.

Take three Plates 2 Inches wide and 4 or 5 Inches long, one of Iron, one of Brass, and one of Copper; let them be draw-fil'd, that is, fil'd longwise, without being polish'd, and rubb'd upon such a Stone as the Streets are pav'd with, that they may be like the Sledges drawn along the Streets, and also their Grain may lie the same way as it does in the Holes for Pivots and Gudgeons in Machines, in regard to the Direction of the Motion. Let these Plates be fastened upon a little Plank of Oak with a Nail whose Head is off, that they may be put on and taken off easily one after another by means of an Hole in the End of the Plate. At the End of the Board fix a little Pulley with very fine Pivots, and let the Copper and Brass Plate be rubb'd smooth on one Side with Pumice-Stone.

Then

THEN take a light silken Purse with a strong Silk to run over the Pulley, and draw an Ounce Weight, (including the Sledge;) then prepare twenty leaden Balls weighing all together an Ounce, that you may put into the Purse such a Number of them, that the Sledge and its Load may be drawn upon the different Bodies and Metals. Let there be also prepared 20 Balls, weighing all together one Pound, to draw such a Sledge as shall, together with its Load, weigh one Pound; as also a third, weighing three Pounds, with 20 Balls equal in Weight to three Pounds. Let the Sledges be one of Iron, or arm'd with Iron, as most of the Sledges are, which are drawn along a Pavement of Pebbles; one of Wood unarm'd; the third of Lead, or arm'd with Lead; and a fourth, if you will, of Brass or Copper. Lect. IV.

IF you put the Balls gently into the silken Purse (or the great ones in a Bag of Linnen) stopping it so that it may not move, and then the Board be lifted up at the Pulley End, so as to rise an Inch in the Length of two Foot, that the Sledge being once in motion may not run swiftly, but on the contrary may continue at rest when it has been stopp'd in the running softly at first; but it must not be kept long in one Place, because there it would stick or sink in, and more Force would be required to set it going at one time than at another. If the Board was set horizontal, or inclining towards the Pulley, the Experiments would not succeed; because the Sledge being once set a-going, would run all at once: And therefore we found the best Situation of the Board was to have it rise a little towards the Pulley, and to set the Sledge a-going. Things being thus prepar'd, the Effect of the Experiments was as is express'd in the following Tables.

THE first Column shews the Number of Balls requir'd to draw the Sledge dry upon different Substances or Metals, as set down. Thus *Iron upon Wood*, signifies the Sledge arm'd with Iron, sliding upon the wooden Board: *Iron upon Iron*, the same Sledge sliding upon the Iron Plate; and so of the rest: *Upon polish'd Brass*, signifies the same Sledge sliding upon the Brass rubb'd with the Pumice-stone: And *upon polish'd Copper*, the Copper Plate rubb'd with the Pumice-stone.

THE Columns over which are the Words—wet—greas'd—oil'd—signify that the Plates and Sledges being *wet*, *greas'd*, or *oil'd*, the Sledges were drawn along by the Number of Balls specified according to the Columns. Thus the first Column, where

Lect. IV. you find theſe Words *Iron upon Wood* 5, above which Number is written *Balls*, ſignifies that the Sledge which weighs three Pounds, ſlides upon a Board of Oak with 5 of thoſe Balls, 20 of which weigh 3 Pounds; ſo that there is only required one Quarter of its Weight to make it ſlide along a plain'd oaken Board: But if that Board be wet, 8 Balls will be required; which ſhews the Friction to be increaſed the Value of 3 Balls. If the Wood be greaſed, 4 Balls and a half will do, which is but little more than half of what is required when the Wood is wet: If the Wood be oiled, there muſt be 5 Balls; and ſo on for all the Columns upon different Metals.

We have not ſet down the Number of Balls required for drawing the Weight of one Pound, becauſe the Proportion was found the ſame as in the 3 Pound Sledges; at leaſt the Difference was ſcarce perceivable. But for the Sledges weighing one Ounce, we give an Account of their Effects, becauſe as they differ from thoſe of 3 Pounds, we may the better from their Compariſon deduce the Cauſes of Friction and Reſiſtance.

A Table

A TABLE of FRICTIONS. Lect. IV.

*A Load of three Pounds is drawn upon a Sledge one Line * and an half wide in the Under-Plates, and three Inches long, with*

	Balls		Balls		Balls		Balls
Iron upon Wood	5	wet—	8	oil'd—	5	greas'd—	4½
Iron upon Iron	3	wet—	3	oil'd—	3	greas'd—	3½
Iron upon Brass	3½	wet—	4	oil'd—	4	greas'd—	4½
upon polish'd Brass	3	wet—	4	oil'd—	3½	greas'd—	4
upon polish'd Copper	3½	wet—	4½	oil'd—	4½	greas'd—	4½
Wood upon Wood	7	wet—	14	oil'd—	3	greas'd—	3½
Wood upon Iron	5	wet—	11	oil'd—	4	greas'd—	4
upon Brass	4	wet—	6	oil'd—	3	greas'd—	4½
upon polish'd Brass	4½	wet—	7	oil'd—	3½	greas'd—	4
upon polish'd Copper	5	wet—	8	oil'd—	4	greas'd—	4½
Lead upon Wood	5	wet—	11	oil'd—	5	greas'd—	4
Lead upon Iron	7	wet—	6	oil'd—	6	greas'd—	5
Lead upon Brass	5	wet—	7	oil'd—	5	greas'd—	5
upon polish'd Brass	6	wet—	8	oil'd—	6	greas'd—	4½
upon polish'd Copper	6½	wet—	9	oil'd—	7	greas'd—	5

A Sledge weighing an Ounce with its Load is drawn along, with

	Balls		Balls		Balls		Balls
Iron upon Wood	6	wet—	5	oil'd—	8	greas'd—	10
Iron upon Iron	4	wet—	9	oil'd—	7	greas'd—	13
upon Brass	5	wet—	6	oil'd—	7	greas'd—	13
upon polish'd Brass	6½	wet—	7	oil'd—	8	greas'd—	13
upon polish'd Copper	6	wet—	7½	oil'd—	9	greas'd—	13

* *N. B.* A Line is one 12th Part of an Inch.

Lect. IV.

	Balls	Balls	Balls	Balls
Wood upon Wood	7	wet— 16	oil'd— 6	greas'd— 12
Wood upon Iron	6	wet— 15	oil'd— 8	greas'd— 11
upon Brass	5½	wet— 11	oil'd— 8	greas'd— 12
upon polish'd Brass	6	wet— 11	oil'd— 8½	greas'd— 11
upon polish'd Copper	7	wet— 13	oil'd— 9	greas'd— 12
Lead upon Wood	7	wet— 10	oil'd— 9	greas'd— 11
Lead upon Iron	7	wet— 8	oil'd— 9	greas'd— 11
upon Brass	6	wet— 6	oil'd— 8	greas'd— 10
upon polish'd Brass.	7	wet— 8	oil'd— 9	greas'd— 9
upon polish'd Copper	8	wet— 8	oil'd— 10	greas'd— 11

The same bigger Sledges with the Weight of Three Pounds were drawn upon a Pebble, such as the Streets are pav'd with, by

	Balls	Balls
Wood upon Pebble, or Paving-stone	8	wet— 13
Iron upon Pebble — —	10	wet— 9
Lead upon Pebbles — —	16	wet— 15

The little Sledges of an Ounce.

	Balls	Balls
Wood upon Pebble	13	wet— 18
Iron upon Pebble	9	wet— 12
Lead upon Pebble	15	wet— 15

A Load of Three Pounds upon a Sledge, whose Sides are brought to a cutting Edge where they bear, instead of the common Way, was drawn along with

	Balls	Balls	Balls	Balls
Iron upon Wood	4	wet— 7	oil'd— 4	greas'd— 3
upon Iron	3½	wet— 3½	oil'd— 3	greas'd— 3
upon Brass	3	wet— 3½	oil'd— 3½	greas'd— 3½
upon polish'd Brass	3	wet— 3½	oil'd— 3½	greas'd— 3½
upon polish'd Copper	3½	wet— 3½	oil'd— 3½	greas'd— 3½
Wood upon Wood	10	wet— 16	oil'd— 5	greas'd— 3
upon Iron	3	wet— 7	oil'd— 3	greas'd— 3
upon Brass	3	wet— 6	oil'd— 3	greas'd— 3
upon polish'd Brass	4	wet— 5	oil'd— 3½	greas'd— 3
upon polish'd Copper	5	wet— 5	oil'd— 4¼	greas'd— 3½

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An Ounce Load upon a Sledge brought also to an Edge, Lect. IV.
drawn by

	Balls		*Balls*		*Balls*		*Balls*
Iron upon Wood	5	wet—	8	oil'd—	6	greas'd—	7
upon Iron	4	wet—	4½	oil'd—	5	greas'd—	7
upon Brass	4	wet—	7	oil'd—	6	greas'd—	8
upon polish'd Brass	5	wet—	7	oil'd—	6	greas'd—	8
upon polish'd Copper	6	wet—	8	oil'd—	6	greas'd—	8
Wood upon Wood	9	wet—	16	oil'd—	5	greas'd—	9
upon Iron	4	wet—	10	oil'd—	5	greas'd—	9
upon Brass	5	wet—	8	oil'd—	6	greas'd—	9
upon polish'd Brass	6	wet—	8	oil'd—	7	greas'd—	9
upon polish'd Copper	7	wet—	9	oil'd—	8	greas'd—	9

With the same cutting Sledges, loaded with three Pounds.

Wood upon Pebble	11	wet—	16
Iron upon Pebble	14	wet—	12
Lead upon Pebble	15	wet—	18

The little Weight.

Wood upon Pebble	9	wet—	8
Iron upon Pebble	14	wet—	13
Lead upon Pebble	16	wet—	8

Upon Clay.

For one Pound Weight with the cutting Sledge, 8 Balls.
For the broad wooden Sledge, only 6 Balls.
For the broad Iron Sledge, 4 Balls.
For the cutting Iron Sledge, 5½ Balls.
For an Ounce Weight with the broad Sledge, 13 Balls.
For the same Weight with the cutting Sledge, 18 Balls.

COROL-

Lect. IV.

COROLLARY I.

THE different Effects of theſe Experiments will alſo admit of a different Way of explaining them. As for Example, there is proportionably more Force required to draw along the little Sledge than the great one; and that probably becauſe it glues itſelf or ſticks to the Parts to be overcome, and alſo the Oil or Greaſe ſticking to it become an Obſtacle, inſtead of helping it as they do the great Weight, by filling up the Holes in the Wood or Metals, their Parts perhaps as they are ſpherical may help as ſo many Rollers: As Oil is not ſo hard as Greaſe, it does not ſo much hinder the little Weight, which requires leſs Force to overcome its Reſiſtance than that of the Greaſe.

COROLLARY II.

FOR this Reaſon the great Weight runs more eaſily over the greas'd than the oil'd Wood; becauſe the Greaſe being firmer fills the Pores of the Wood better, and keeps together the little Parts that riſe when it is not greas'd, and which muſt be broken or preſs'd down when there is neither Greaſe nor Oil. This obliges us to employ more Force, and much more when it is wet; becauſe the Water penetrating into the Wood, cauſes the rough Parts to riſe, which are like ſmall Muſhrooms or Grains of Corn, that one muſt either run over or preſs down flat.

COROLLARY III.

AND therefore when Wood is wet, as in the wooden Sledge, there muſt be as much again Force applied as when the Wood is dry; and a Force more than double would be requir'd if the Wood had been long imbib'd in Water, or was of a nature to ſuck in a great deal of Water and to ſwell upon that account. And though Water does not ſeem to be imbib'd by Metals, yet it is very reaſonable to think from theſe Experiments, that it does in ſome meaſure join and glue them together; which perhaps would not happen when heavy Loads are drawn by Horſes, where this

this Adhesion would hinder but little in proportion to the great Weight * Lect. IV.

COROLLARY IV.

As Iron sometimes appears to run more easily over Iron than Brass, and over Brass than Copper, the Reason seems to be, that Iron being harder than Brass it is less liable to yield and sink in; and as Copper is softer than Brass, it sinks more than Brass, and consequently has more Resistance and Friction than Brass; as this appears in several Cases and several Sledges; and may perhaps do the same in the large ones, of which ours are but as Models.

COROLLARY V.

We must not conclude from thence, that Grease or Oil should not be used in Engines, tho' here they have seem'd of little Use, and sometimes have been an Hindrance, as here in the little Sledge; because we know them to be of use in great Machines, for two Reasons; because they fill up the Holes as we see, or else rolling under the Weight facilitate its Motion, and they hinder the Parts from wearing or flying off; and besides, they hinder such Parts as are broken off, from sticking to the Work: For if large Machines, as large Presses and Screws for coining Money, were used without Grease, they would move much harder on account of the small Parts that would wear off and stick on in other Places, and by that means break off more, which would not only make the Engine harder to move, but wear it out much sooner.

COROLLARY VI.

But tho' Grease or Oil seems prejudicial to small Machines, their Motion is thereby render'd more equal; and tho' one may perceive the Motion to be more difficult, as in Pocket-Watches when they are oil'd, because then they generally go slower, yet then they go more equally: And it is always an Advantage to have them clean'd

* This is a plain Consequence of the Attraction of Cohesion; for since the Attraction of Cohesion is proportionable to the Surface or the Number of the touching Parts, and the Friction proportionable to the Weight, the Hindrance or Loss of Force on account of the said Attraction will always be less in proportion to the whole Friction, as the Weight increases.

Lect. IV. clean'd and oil'd; for then they go better, and the Holes do not so soon wear wider; the Balance plays better, and is not liable to vary. One might indeed use no Oil in some small Machines which go easily, and are but seldom to be put into Motion.

COROLLARY VII.

THE Use of Grease is visible, especially when Wood acts against Wood; for then Grease makes the Motion as easy again; nay two 3ds easier if the Parts be sharp and cutting, as appear'd from the Experiments made with the flat and the sharp Sledge of Wood drawn along on Wood not greas'd.

COROLLARY VIII.

FROM hence also it appears how useful Grease is for Coach and Waggon Wheels, especially in rainy Weather; for if the Naves were wet, or if there was no Grease to hinder the Water from penetrating into the Wood, four times the Force would be required to draw a Burthen, than when it is drawn with Grease in dry Weather, as may be seen by the Sledge in respect only of the Friction made upon the Axle-tree, which is but little when compar'd with the Resistance made upon the Ground or the Pavement: But besides the Obstacle, the Hole of the Nave would grow considerably bigger; for by its swelling the Axle-tree would no longer be free, and must break off the Parts in its Way: Then as the Nave grows dry, the Axle will no longer fill the Hole; the Wheel will vary and be more liable to break, in those Jolts that happen as it goes over a sudden Rise, or falls into a Hole. For these Reasons therefore it is always well to grease Machines.

COROLLARY IX.

THO' these Experiments do not wholly determine every thing that relates to very great Weights to be drawn upon Pebbles, and the Motions of great Machines; yet they shew us which are the easiest Metals for moving on one another: As also that there are Parts to be overcome or broken off in Frictions: That it is merely the Weight and the Motion which cause the Resistance and Friction, and that the Quantity of Surface does not increase it when the Parts that rub and bear do not move swifter; for tho' the sharp Sledge

Sledge goes with less Difficulty in several Cases, it is not to be considered as if that Way of making it would serve to avoid or ease the Friction, for it works as an Edge or a Saw, which penetrates into the Body of Metals, or Bodies to be cut; and if it appears to slide more easily upon Wood, it is on account of the Grain of the Wood, and the Way being once marked it follows the Grain as in a Path, and then it has fewer Paths to press down or overcome; but on Clay, which has no Grain, or upon Pebbles, it goes much harder, because the Obstacles to be removed are greater, as it sinks more when sharp than when flat. Lect. IV.

COROLLARY X.

By this one may see that it is possible to increase Friction, by making the moveable Parts so small that they will penetrate and rub off Parts of what they bear upon, as it may happen to the Pivots in Watches, if they should be too small; for then they would soon make the Holes too big; as likewise in Sledges, if too narrow a Bar of Iron be put on, thinking thereby to lessen the Friction on the Pavement, and so in other Cases.

COROLLARY XI.

Hence may be easily conceived that the Bottom of Sledges ought to be broad, and it is better to shoe them with two broad Plates of Iron under each Piece, than only one narrow one; and when the Streets are dry, it would be more advantageous to have no Iron at all (the Experiment shewing that Wood will slide better) which still appears more probable, because the little Sledge of one Ounce (with its Load) slides more easily upon the paving Stones than the great one; whereas it goes with more Difficulty on other Occasions, by reason of its being in some measure glued by its close Cohesion.

COROLLARY XII.

Besides this Experiment, one may also infer that the Draught will be the easier in dry Weather with the Wooden Sledge; because when Stone-dust is strewed before Sledges, the Draught is easier by one 4th Part, whether it be upon Wood, upon Iron, or upon Brass: And as there are always small Parts rubbed off from

Lect. IV. the Stones of the Pavement by the Nails of the Wheels, and the Horſes Shoes, beſides the Duſt at that Time; all which Things will facilitate Motion, as may be known by making a Compariſon with the Experiment in which neither Stone-duſt, nor other Duſt was uſed.

COROLLARY XIII.

As for the Iron Sledge, it is evident that it would go moſt eaſily in rainy Weather; but if it was dry Weather, and the Pavement not wet, it would be worth while to try with the Sledge itſelf, (not in Model) whether then it might not be as well not to wet the Pavement, as is commonly done by a little Barrel full of Water laid upon the Sledge with two little Holes for the Water to run out and wet the Pavement and the Sledge: For though it is certain, that if the Sledge and Pavement were ſufficiently wet, all would go eaſier; it is very probable that it is better not to wet them at all, than to wet them ſo imperfectly, as only to make a kind of Dirt and Lumps that may prove an Hindrance. There would not be ſo many Parts for the Iron of the Sledge to remove in running its Length, as was obſerved by the Experiment on the ſmall one, which running twice over the ſame Track, goes much more eaſily the ſecond time than the firſt. This may alſo be obſerved in whetting a Knife, which ſlides eaſily over the Whetſtone after having rubb'd it over once or twice when the Stone is dry, and ſo wears but a little; whereas it rubs hard and wears much when the Stone is wet.

COROLLARY XIV.

We do not ſpeak of the Leaden Sledge, which we only uſed for Curioſity, and to ſhew that Parts are broken off by Friction, and that Lead cauſes moſt Reſiſtance in many Caſes, where Pivots or Arbors are made to turn upon Lead or Pewter.

All the foregoing Experiments are of Uſe to direct us how to chooſe Matters or Subſtances to make uſe of, and how to calculate the Force according to the Weight and the Motion which the rubbing Parts will have. Thoſe that would try theſe Experiments over again out of Curioſity, muſt take care to make them in the ſame Order: Firſt, all thoſe that are dry, then thoſe that are

are wet, next those that are oiled, and lastly those that are greased; because if the Machines were greased before they were oiled, the Grease filling up the Pores, the Effects of the Oil would only be like those of the Grease, especially when we make use of Wood, which must, after having been wet, be suffered to be quite dry, and new planed again before it be anointed with Oil or Grease. Lect. IV.

THO' I have in the Notes of my last Lecture * mathematically considered the Effects of high and low Wheels, and compared them together; yet as Coaches, Waggons and Carts, and other Wheel-Carriages are so necessary for the Uses of Life, that only the Disuse of them for one Month would be enough to put a whole Nation in Confusion, I shall here again give an Account of some more of M. *de Camus*'s Experiments and Reasonings about them, being well satisfied of the Truth he asserts from my own Experiments; and having found his Experiments to answer as near as can be expected in Machines that have so much Friction as Models of Carts and Waggons. I have indeed a Machine with Brass Wheels, whose Steel Axes have very small Pivots so nicely made, that any of the Wheels once set in Motion will turn upon their Axes for the Space of more than half an Hour, making several hundred Revolutions before they stop; but the Use of my Machine being chiefly to shew how near those kinds of Experiments may be brought to agree with a mathematical Theory; we cannot expect that any Carriage to bear Weight can have so little Friction †: Therefore I choose to relate M. *de Camus*'s Experiments made on Models of Carriages of an Inch to a Foot, every way representing Carts and Waggons, and liable to as much Friction in proportion to this Bigness; because it shews us directly what is the real Friction in the Carriages at present in use. In the manner of remedying Friction, I shall add some of my own Considerations and Observations to his.

* Lect. 3. Ann. 11.

† Ann. 3.

I BEGIN here with his 24th Proposition, Sect. 5. of his Book above-mentioned.

PROPOSITION XXIV.

The Wheels of Carriages must be exactly round, and the Fellies at right Angles to the Naves, according to the Inclination of the Spokes.

IT is a general Rule in all Cases, that the Wheels be exactly round; for if they were not so, but like E F G H ‡, and the Nave

‡ Pl. 17. F. 7

Lect. IV. out of the Center; if is certain, that such a Wheel, in turning, would be affected in the same Manner upon plane Ground, as other Wheels are when they rise and fall, and would not be *in Equilibrio*; the Wheel turning towards H, would move with as much Difficulty, as if there was a Rise to ascend, and that Height being passed, it would fall on the sudden as if a square Stone was rolled along, and the Jolts of the Wheels would precipitate and push the Horses at one Time, and immediately increase their Difficulty of drawing the next Moment; and that in Proportion to the Wheels being out of round: Suppose the Wheels to be without Angles, nay truly round; yet if the Nave should not be in the Middle, the shortest Part, as * F, being upon the Ground, when such a Wheel begins to turn, the Weight must be raised in the same Manner, as when another Carriage is going up an Hill, and from F to D, or quite to G, the Wheel would act like a Wedge, and at D or G it would fall and drive on the Horses as in a steep Descent; so that Horses or Oxen would be as much tired with such Wheels upon even Ground, as in straining to climb up Hill, or to bear the Shock of a steep Descent; and this would mostly affect the Tiller or Horse next to the Wheels: Therefore Wheels ought to be exactly round, and the Naves and Holes of the Naves exactly in the Center of the Wheels.

* Pl. 17. F. 7.

Secondly, The Fellies must not cross-wind, but be at right Angles with the Naves according to the Inclination of the Spokes, that is, the Plane of the Circle of the Wheel, which passes through the Fellies, must cut the Nave at right Angles, though it need not pass through the Place where the Spokes are inserted into the Nave; for otherwise the Wheel, in turning, would find Inequalities, as it happens when the Hole of the Nave is too big, and the Wheel moves from Side to Side, which comes to the same as if the Wheel was out of round; and then the Inequality of the Spokes which would be too leaning, or too strait upon the Nave descending into an Hole, or rising upon an Eminence opposite to their Inclination, would cause them or the Fellies to break. Therefore the Wheels of Carriages must be exactly round, and the Curves of the Wheels, or Fellies, at right Angles to the Naves.

COROLLARY I.

Hence follows, that where Wheels are not shod with Iron, great Care must be taken to fix Wood to the Fellies, in order to keep

keep them round; and that nothing is ſaved by uſing no Iron; for if in ſuch Places Wheels wear but little, becauſe they only go upon Earth, it would only be making the Iron Plates thin, which would then coſt but little; and that Expence once over, the Wheels would be preſerved longer and more Work would be done, ſo as to make the Gain far exceed the Expence. Whereas in Countries where they do not ſhoe Wheels with Iron, or ſecure them with Wood, the Wheels wear ſo as at laſt to be rather ſquare than round, and thereby the Horſes or Oxen being very much fatigued, hardly do half the Work that they would do if the Wheels were kept to their Roundneſs. Lect. IV.

PROPOSITION XXV.

The Spokes muſt be inclined to the Naves, that the Wheels may be diſhing, or concave.

IF Wheels always turned upon ſmooth and even Ground, it is certain that the Spokes ought to be ſtrait upon the Naves, that is, at right Angles to their Axes; becauſe then they would bear perpendicularly, like the Spokes B of the Nave A C *, which is the ſtrongeſt Way for Wood: But becauſe the Ground is unequal, and when the Wheels fall into the Ruts, that Wheel which is in the Rut bears a greater Part of the Weight than the other, becauſe it is lower (as we have demonſtrated it) in ſuch a Caſe, the Spokes of a diſhing Wheel become perpendicular in the Rut, and therefore have the greateſt Strength; whilſt the oppoſite Wheel being upon higher Ground, bears a leſs Part of the Weight, and conſequently the Spokes need not be at their full Strength, and ſo will have a ſufficient Force, tho' that Force be leſs than what they have upon even Ground. The Spokes, therefore, ought to be inclined to make the Wheel diſhing, as is the uſual Practice. * Pl. 17. F. 8.

PROPOSITION XXVI.

The Axle-trees muſt be ſtreight in all reſpects, and at right Angles to the Shafts, or to the Pole.

IN the Motion of all Bodies, there is one Way of moving, which is the eaſieſt of all the reſt; and that happens here when the Axle-tree is every way ſtreight: For if its Ends ſhould bend backward,

Lect. IV. ward, so as to bring the Wheels nearer together behind as A E *, and spread them much before as D C, it is certain that they could not go into the Ruts nor turn in going forward, or at least with great Difficulty, dragging instead of rolling: There would be the same Inconveniencies in bending the Axle-tree forward, so as to bring them nearer the Pole as I F †, and make them spread behind as at B D. The less the Axle is bent, the less the Inconveniency; but there will always be some, when the Wheels are not parallel, therefore the Axle ought not to be bent at all.

* Pl. 17. F. 9.

† Pl. 17. F. 10.

‖ Fig. 9. * Fig 10. † Fig. 9. * Fig. 10. † Fig. 11. ‖ Fig. 9.

IF the Wheels spread outwards as D C ‖, or inwards as I F *, there will still be three other Inconveniencies. If the Axle bends outwards so that D C † bears upon the Ground, the Way will be too wide; it will be hard to turn; and the Weight being drawn forward will crush the Wheel, the Length of the Spoke C H in that Case acting as the Arm of a Leaver to break the Axle or the Spokes, C being the long Arm, the Center of Motion at one End of the Nave, and the short Arm at the other. If the Axle be so bent as to bring the Wheels inwards as at I and F *, the same three Inconveniencies would happen; the Way would be too narrow, and the Weight would tend to crush the Wheels, and there would be a Difficulty in turning: Besides, they would bear but on the Edge of the Iron, and become cutting by their small Bearing. Therefore as such Inconveniencies will happen more or less according to the bending of the Axle, it should not be bent at all. But there will be no Inconveniency when the Axle is streight and the Wheels are in the Situation C P, A D †. By this means the Wheels will have Liberty as they go along; but otherwise, tho' a Wheel when off of the Ground might turn upon the Axle, yet when on the Ground and drawn at H ‖, it would only drag.

THE Axle must also be at right Angles to the Pole or Shaft; for if the Pole or Shafts were on one Side as at B, the Coach or Carriage would be drawn on one Side, and almost all the Weight would bear upon one Horse; but it must be at right Angles like the Pole G *, as has been said before. Therefore the Axle-trees of all Carriages must be streight, and at right Angles to the Pole or Shafts.

* Fig. 11.

COROLLARY I.

THIS shews the Inconveniency of those Coaches, whose Axles are bent so as to make the Wheels spread upwards that they may not touch

touch the Braces; for this brings on all the Inconveniences above- Lect. IV.
mentioned, and the Coaches are more liable to turn over, becauſe the Way is narrowed, and they go into the Coach-houſes with more Difficulty ſpreading at top, the Tops of the Wheels being more aſunder than if the Axle was ſtreight, and when they ſtrike againſt any thing at top, they are more liable to break; and then likewiſe the rolling of the Wheels is hindered; therefore it would be better to bring in the Braces nearer together, or ſet the upright Wheels farther aſunder.

COROLLARY III.

THIS Way of bending the Axle does alſo render uſeleſs the Advantage gained by diſhing the Wheels, as has been explained: For in this Caſe the Spokes of both Wheels bear perpendicularly at the ſame time as if they had not been inclined to the Nave; and when a Wheel comes into an Hole or deep Rut, the Spokes being no longer perpendicular, it is more liable to be cruſhed by the Weight. This makes the Wheels and Axle-trees more ſubject to break, and the Spokes to be looſe or break: Nay, in even Ground they cut more, and bear on the Edge of the Iron Plate, as one may obſerve in ſuch Wheels, that the ſhoeing is more worn outwardly than inwardly; and this occaſions their ſliding the more upon paved Streets.

PROPOSITION XXVII.

The Hind-wheels do not drive or impel the Fore-wheels, let the Hind-wheels be ever ſo high, and the Fore-wheels ever ſo low.

IN ſome of the Antients Pictures one may ſee Chariots repreſented with four Wheels all high and equal. In ſome Countries, where Faſhions ſeldom change, they ſtill retain thoſe Sorts of Wheels. People, in all Probability, lowered the Fore-wheels, in order to turn the more eaſily becauſe of the Shafts being in the Way. The Fore-wheels have alſo been lowered ſtill more in Coaches, by reaſon of the Braces, which, in ſome meaſure, hinder the turning ſhort, and are liable to be cut by the Wheels: Afterwards Crane-necks have been invented for turning yet ſhorter, and by degrees the Wheels made low enough to go quite under the Bend

of

Lect. IV. of the Crane-neck. Then several Coachmen pretend that their Horses tire because the Fore-wheels are too high, so as not to be driven on enough by the Hind-wheels: And this false Principle has been followed even to Children's Carts and Toys, which are made with very low Wheels before. It is very likely, that they would be made still lower, if it was not evident that by making them a little lower they would not go at all over some Rubs, nor get out of deep Ruts. Coachmen generally encourage lowering the Wheels, not only because of the easy Turning, and their Imagination that the Hind-wheels drive on the Fore-wheels if they be much smaller; but chiefly because of their getting up easily into the Box. Now if the four Wheels are upon smooth and even Ground, we shall find that the Fore-wheels will be at Rest, though ever so low, the Center of Gravity of each Pair of Wheels being so placed that the Line of Direction falls between the Wheels, both in the Fore and Hind-wheels. Therefore the Carriage cannot move of itself in that Situation, only the Fore-wheels are more loaded because they are lower than the Hind-wheels; but it does not follow from that, that they roll more easily; for if it was so, Carriages would be more apt to roll when most loaded, which is contrary to Experience. A Consequence also of this Principle of *Wheels driving*, would make a Carriage with very high Hind-wheels, and low Fore-wheels go of itself upon even Ground, which never was; therefore the Principle is false.

SCHOLIUM.

If the Hind-wheels could, by their greater Height, drive the Fore-ones, it would follow that a Coach or Carriage would go with most Difficulty when the High-wheels go foremost, by making the Horses draw behind. Now let a Carriage be made in Model with Hind-wheels of five Inches, and Fore-wheels of two Inches and three Lines, which is the common Size, if we take an Inch for a Foot (though some Hind as well as Fore-wheels are less in Proportion) and let this Carriage or Waggon be set upon a smooth Board, and loaded in the Middle with five Pound Weight of Lead; let a small Pulley be fixed at the End of the Board, over which runs a silken Thread fastened at one End to the Waggon, and at the other to a Scale, or a little Linen Bag to receive Leaden Balls to draw the Loaded Waggon by their Weight. The same Weight that draws the Waggon with the small Wheels foremost, will also draw

draw it with the great Wheels foremost, provided the Line of Direction of the Draught be in the same Situation in both Cases: This shews that there is no driving, whatever Difference of Height there be in the Wheels, even upon plane, horizontal and smooth Ground. Lect. IV.

COROLLARY I.

IT follows, that this Notion of *driving* must have been from some Workmen who thought the Case to be parallel between a Carriage upon an inclined Plane, and one with high Hind-wheels, and low Fore-wheels, though on an horizontal Plane: But the Case is very different; for on an inclined Plane, the Line of Direction falls out of the Base, and the *Æquilibrium* is lost; therefore the Carriage will roll till it finds it, or meets with some Obstacle that reduces it to an *Æquilibrium*. This would do the same in one or two Wheels as well as in four, if one Wheel was broad enough to support itself. If this be called *driving*, a Cart, though it has but two Wheels, will have it as well as a Coach or Waggon.

COROLLARY II.

IT is not a good Objection to say, that the great Wheel being in motion, continues to move longer than the small Wheel, and so drives it; for, upon the Ground, the Resistance is much greater than such an Impulsion. For if we give by Force some Degree of Velocity to a Waggon on plane Ground, but such as may let it sink in ever so little, as in wet Weather, or upon soft Ground, not so hard as Pavement, as soon as the Force ceases to act, the Waggon will stand quite still, which shews the Hind-wheels do not drive even in that Case.

COROLLARY III.

IF we consider the Thing upon a Pavement, be it even, or uneven, or upon rough Ground with Rises and Falls; let the Waggon be so placed as to have the high Wheels upon the higher Ground, then indeed the Waggon will run down, and they will seem to drive the low Wheels; but then in rising again, the Fore-wheels will drive the Hind-ones, and the Waggon will run backwards:

Lect. IV. wards: To say that the great Wheels turning more easily will drive the better, only shews that the Waggon will go better if all the Wheels are large, and that they will altogether roll better than if there were two little Wheels. So that little Wheels before do not (whatever way you consider them) facilitate the Motion of a Carriage.

PROPOSITION XXVIII.

Great Wheels are always more advantageous for rolling than little ones, in any Case, or upon any Ground whatever.

THE Wheels of Carriages are considered according to the Velocity and Friction they have upon the Axle-tree, and likewise according to their Resistance or Sinking in upon the Ground. If we consider them according to the Friction, it is certain that a Wheel, whose Diameter is double that of another, will make but one Turn, whilst the little one makes two, for the same Length of Way, the Circumference (which is in Proportion to the Diameter) being double: Therefore in respect to Friction, a Wheel of double the Diameter will have a double Advantage, there being but one Turn instead of two, which doubles the Friction in the small Wheel.

* Pl. 17. F. 12, and 13. The Wheel A B C *, being twice as big as the Wheel D E F, will have twice the Advantage in respect of the Friction, the Holes of the Naves and the Axles being equal.

IF we consider the Wheels according as they sink into the Earth or fall into Holes, there will be the same Advantage for the one and Inconveniency for the other: If we consider the Bearing, it is double in the great Wheel, therefore it will sink but half the Way; and if we consider Hollows, it will give the same Advantage in some Cases; but then in others (as for Example, where the Holes are deep) the little Wheel will have much more Disadvantage †; for if it should fall into a great Hole as D E ‖, of a Diameter equal to that of the Wheel, it would wholly sink in, whilst the great Wheel would only fall in the Depth of its Segment A B *, which would not be half the Wheel, as is easy to be understood by the two parallel Lines A D and B E †: We may suppose the same to happen in marshy Grounds where the little Wheel would sink wholly in the same Hole that the great one would sink but in part.

† L. 3. Ann. 11. p. 181.

‖ Fig. 13.

* Fig. 12.

†Fig. 12, & 13.

3

IF

IF we consider an Eminence to go over upon even Ground, as a Pavement, and that it is the same at B as it is at E *, the Segment, or the Chord of the little Wheel will be one Third nearer the Top than the Segment, or the Chord of the great one, and there must be a Third more Force to overcome the Rub. If the Rub be something which must be broken or crushed, wholly or in part, there will be the same Proportion, and the Circumference of the Wheel making a sort of Wedge, or inclined Plane, it will be shorter or less acute in the small than in the great Wheel; so that the Effort must be greater to overcome it all at once: And if the Rubs are only Rises and Falls of Ground, there will, for the same Reason, be more Difficulty for the *little* than the *great* Wheel. Consequently great Wheels are better for rolling than little ones on any Occasion, or upon any Ground whatsoever. Lect. IV. *F. 12, & 13.

COROLLARY I.

HENCE it follows, that if a Wheel be only one Inch in Diameter, or in Height, more than another, it will have more Advantage; and that the higher the Wheels are, the more advantageous they are in Proportion, provided they are not too high, that is, not above five or six Feet high: For if they should exceed that Proportion, they would themselves become a great Weight; or, if made light, then they would be too weak and subject to break on account of the great Length of their Spokes; besides, with such Wheels Horses would be hindered from exerting their utmost Strength, by having the Axle-tree higher than their Breast; so that they would draw downwards, especially if the Horses are not very tall; as in little Wheels the Draught is made more difficult by the Horses drawing upwards: For to deviate from an horizontal Line of Direction, by drawing either upwards or downwards, is inconvenient for the Horses, as will appear more plainly by the following Experiments.

PROPOSITION XXIX.

CARRIAGES with four Wheels, as Waggons or Coaches, are much more advantageous than Carriages with two Wheels, as Carts and Chaises.

Lect. IV. WHAT we are to conſider in Carriages is the Advantage which they have one more than another in rolling, and the Manner of applying Horſes or Oxen in ſuch a Way as they may be the leaſt ſubject to tire, and that they may draw with the greateſt Advantage. Now in applying Horſes to a Cart with two Wheels, it is plain that the Tiller carries Part of the Weight, in what manner ſoever the Weight is *in Æquilibrio* upon the Axle; for in going down an Hill the Weight bears upon the Horſe, and in going up the Hill the Weight falls the other way, and lifts the Horſe (that is, pulls him upwards) which takes away great Part of his Force: and if to avoid this laſt Inconveniency, the worſt of the two, the Weight be put forward, the Horſe will the ſooner tire for carrying as well as drawing. Beſides, as in the Holes in the Road, ſometimes one Wheel ſinks in, and ſometimes another, the Shafts ſtrike againſt the Tiller's Flanks, which is the Deſtruction of many Horſes.

MOREOVER, as in a Cart the whole Weight bears intirely upon two Wheels, when one of them ſinks into an Hole or Rut, half the Weight falls that way; and to draw the Wheel out of the Hole, half the Weight muſt be drawn out: If it be upon ſoft Ground, where both the Wheels ſink in, and muſt be drawn out, the Labour is greater than when the four Wheels of a Waggon ſink upon the ſame Ground, becauſe the Weight being diſtributed upon four Wheels, muſt make them ſink leſs than if it was only ſupported by two; and in Holes, where one Wheel only ſinks in at a time, there is only a Quarter of the Weight to be drawn out, but half when we uſe the Cart; in which therefore, a double Force muſt be uſed to draw out half the Weight. If two Wheels of a Waggon fall at once into an Hole, then only half the Weight is to be drawn out, but the whole Weight when we uſe the Cart: And in the Riſes and Falls upon Pavement, as in croſſing a Kenuel in a Waggon, an *Equilibrium* is often made between the Hind and Fore-wheels, thoſe in coming down helping theſe to riſe as they are juſt got over the Kennel; and if this happens only on one Side, there will be the ſame Help: But in a Cart it would happen otherwiſe; for one of the Shafts would ſtrike the Tiller in the Flank. As for the Objection, that there is leſs Friction upon two Wheels than four (which very likely has been the Reaſon for preferring Carts to Waggons) it is wholly falſe; for we have ſhewn that there is as much Friction upon two Wheels as upon four, if there be the ſame ſized Hole in the Nave, and the Weight be the ſame.

ſame. On the contrary, there will be rather more in the Uſe of the Cart, becauſe as all the Weight bears upon two Points, the ſmall Parts will be more liable to be torn off, the wearing being double; and if the Load on the Waggon be not greater than on the Cart, by making the Axles and Holes of the Naves leſs, it will have ſtill leſs Friction; but the Friction (or at leaſt that Difference of it) being but little when the Wheels are well greaſed, it is not worth Notice. Beſides the Advantage ſhewn in the Uſe of four Wheels, we muſt have Regard to the Till-Horſes which carry as well as draw in the Cart, but in the Waggon exert more Strength to draw, and yet laſt longer, becauſe they are not banged on the Sides. Therefore Four-wheeled Carriages, as Coaches and Waggons, are more advantageous than Carts and Chaiſes. Lect. IV.

PROPOSITION XXX.

IT would be much more advantageous to make the four Wheels of a Coach or Waggon large and nearly of a Height, than to make the Fore-wheels of only half the Diameter of the Hind-wheels, as is uſual in many Places.

IF there be ſome Conveniency for turning in making the Fore-wheels of Coaches or Waggons as little again as the Hind-wheels, there is a very great Diſadvantage, becauſe half the Force is loſt that would be effectual if they were large, according to the 26th Propoſition. The Jolts alſo are greater when we uſe little Wheels, becauſe they ſink as low again in the Holes and Hollows of the Pavement, and therefore muſt jump as high again; and this, no doubt, has brought People to contrive Springs to avoid the Jolting; but at the ſame time it has made Coaches more apt to overthrow, by raiſing the Body of the Coach the higher, to place the Springs under it.

BESIDES theſe Diſadvantages, Horſes that draw upwards tire ſooner, and grow more ſtiff in the Hams; and this is the Reaſon that Horſes that have been uſed to a Coach are no longer fit to be ridden, tiring their Riders very much; which would not happen if the Fore-wheels were high, and the Points where the Traces are fixed were as high as their Breaſt, ſo as to draw in Lines parallel to the Ground, as Cart-horſes commonly do, and thereby gain ſo much as to overcome the Diſadvantages that Carts otherwiſe

Lect. IV. This Advantage of Carts would cease if the four Wheels of a Waggon were equal, and then one would have the above-mentioned Advantages also of four Wheels over two. Some object, that the Horses drawing upwards, lift the Coaches out of the Dirt, and ease the Weight; but if they do, then they carry a Part of the Weight; and as Horses, one with another, are able to carry but 200 lb, but can draw near 1000 lb on a Waggon, this Way must tire them more than fair Drawing when the Traces are parallel to the Ground; therefore it is best to have all the Wheels of a Coach high and equal, *&c.*

SCHOLIUM.

The following Experiments will confirm what I have been explaining. Let us make use of a little Waggon or Model of an Inch to a Foot, represented in the seventeenth Figure of the seventeenth Plate, with the four Wheels of five Inches and nine Lines, and so contrived, that one may put on Wheels of different Diameters: As for Example, Four of 5 Inches, Two of 2 Inches 3 Lines, Two others of 3 Inches; and let them have Naves, Spokes and Fellies in proportion, to represent the Wheels of a Coach or *Pl. 17. F. 17. Waggon. Let them be changed one after another, the Waggon* D B being always loaded with the same Weight A of 5 lb, and drawn by means of a silken Thread running over a Pulley, with a little Bag, or Scale of a Balance to put in Balls for the different Wheels, according as they are to run on even Ground upon Earth, Sand, or Pavement. The Board A F must be of Oak, three Feet long, plained on one Side, and carved on the other to imitate the Pavements and the Channels of Streets: The paving Stones must be of 7 or 8 Lines instead of 7 or 8 Inches, reducing them from Inches to Lines, as the Wheels are reduced from Feet to Inches. It must be so contrived, that the Pulley may be turned to either Side of the Board. The Whole being so disposed, the Experiments will answer to the following Table.

To represent a Cart we hang *in Equilibrio* under an Axle-tree, the same Weight A of 5 lb, and a Pole only is made fast to the Axle-tree to tie the String to it, which makes the Cart three times lighter than the Waggon in making the Experiments; for the Waggon has an Axle-tree, and two Wheels and Shafts more than the Cart; and the full Wheels of 5 Inches and 9 Lines weigh twice as much as the Five-inch Wheels with Spokes.

To

To draw the Load of 5 lb upon the ſmooth Side of the Board laid level with the four great Wheels, each of 5 Inches, 9 Lines in Diameter, there is occaſion but for three Quarters of a Ball. Lect. IV.

For the Weight of Five Pounds upon the Waggon.

	Balls.
With the Four Wheels of Five Inches	1
With the Two little Wheels before	2
With the Wheels of Three Inches before	$1\frac{1}{2}$

For the Cart and the ſame Weight.

With the Wheels of Five Inches	2
With the Two little Wheels	$3\frac{1}{2}$
With the Two Wheels of Three Inches	3

With the Waggon upon very moiſt Earth.

With the Four biggeſt Wheels, 2 Lines wide or thick	12
With the ſame narrower and almoſt cutting	16
With the Four of Five Inches, Three Lines wide	6
With the Two leaſt before	12
With the Two of Three Inches before	8

For the Cart upon the ſame Earth.

With the Two great Wheels	$9\frac{1}{2}$
With the Two leaſt	18
With the Two of Three Inches	13

For the Waggon upon dry Sand.

With the Four Wheels of Five Inches	28
With the leaſt Wheels before	46
With the Wheels of Three Inches before	35

For

For the Cart upon dry Sand.

	Balls.
With the Two Wheels of Five Inches — —	40

When it stopped with 39 Balls, I was obliged to add 10 more Balls to make it move from that Stop.

For the Waggon upon wet Sand.

With the Four Wheels of Five Inches — —	14
With the Two least Wheels before — —	28
With the Wheels of Three Inches before — —	17

For the Cart upon wet Sand.

With the Two Wheels of Five Inches — —	17
With the Two Wheels of Three Inches — —	24

To overcome an Eminence, or Rub of Two Lines, for the Waggon.

With the Four Wheels of Five Inches — —	20
With the Two least Wheels before — —	30
With the Wheels of Three Inches before — —	25

Half the Number of Balls will do when only one Wheel goes over the Rub.

For the Cart to go over the same Rub.

With the Two Wheels of Five Inches — —	35
With the Two least Wheels — — —	60
With the Two Wheels of Three Inches — —	48

For the Waggon to overcome an Height of one Line.

With the Four Wheels of Five Inches — —	15
With the Two least Wheels before — —	21
With the Two Wheels of Three Inches before —	17

For

For the Cart to go over the same Rub.

	Balls.
With the Two Wheels of Five Inches	27
With the Two least Wheels	38
With the Two Wheels of Three Inches	31

For the Waggon to come out of a Hole, as if a paving Stone was wanting under each Wheel.

With the Four Wheels of Five Inches	9
With the Two least Wheels before	18
With the Two Wheels of Three Inches	14
If the Weight be laid upon the little Wheels before, to come out of the same Hole, we must put in	34
But if it be laid upon the Hind-wheels, and the Ground even, only	3

For the Cart to be drawn out of the same Hole.

With the Two Wheels of Five Inches	18
With the Two least Wheels	34
With the Two Wheels of Three Inches	25

For the Waggon to be drawn from an Hole, as out of the Channel or Kennel of the Pavement.

With the Four Wheels of Five Inches	$4\frac{1}{2}$
With the Two least Wheels before	8
With those of Three Inches	$6\frac{1}{2}$

For drawing the Cart out of the same Hole.

With the Two Wheels of Five Inches	10
With the Two least Wheels	15
With the Two of Three Inches	11

For the Waggon upon the Pavement.

	Balls.
With the Four Wheels of Five Inches — —	$2\frac{1}{2}$
With the Two least Wheels before — —	$4\frac{1}{2}$
With the Wheels of Three Inches — —	$3\frac{1}{2}$
When the Wheels of Three Inches are behind, and those of Two before — — —	$4\frac{1}{2}$

For the Cart upon the Pavement.

With the Wheels of Five Inches — —	$4\frac{1}{2}$
With the Two least Wheels — —	8
With the Two Wheels of Three Inches — —	$6\frac{1}{2}$

If the Board be rais'd an Inch at the End where the Pulley is.

For the Waggon.

With the Four Wheels of Five Inches — —	$4\frac{1}{2}$
With the Two least Wheels before — —	$6\frac{1}{2}$
With the Two Wheels of Three Inches before —	$5\frac{1}{2}$

For the Cart in the same manner.

With the Two Wheels of Five Inches — —	6
With the Two least Wheels — — —	11
With the Two of Three Inches — —	8

If we make use of the Four great Wheels of Five Inches and nine Lines, about one Quarter more of Force is required than for the Wheels of Five Inches, which are three Lines wide, and that as well for the Waggon as the Cart; because as those largest are very narrow, and almost cutting, they run into the Separations of the Pavement, and after sliding down between two Pavements, to rise up again, there is more Force required, and they go less swift than the broad Wheels, even when that Quarter of Force is superadded, and though they are so much bigger; but upon plane and smooth Ground, where they don't sink in, they go much easier, and have more Advantage than the others.

COROLLARY I.

WHENCE it is easy to judge how much those Carters are deceiv'd who would have the Irons or Shoings of the Wheels to be made very narrow, that they may the better come out of the Ruts, and cut the Ground the more easily: For if the Wheels have no Friction on the Sides of the Fellies, being narrow they sink deeper and spoil the Ruts the more; and if they go where there are no Ruts, as on Earth, they will tire the Horses much more, one fourth Part more Force being requir'd. Such kind of Wheels are very disadvantageous to every body, because they cut the Ruts the deeper.

COROLLARY II.

THE same Inconvenience happens upon Pavement, and the Irons of the Wheels being narrow they wear out the faster, bearing in some measure but upon one Point; and as the Iron Plates wear, they grow round, and slide more strongly between the Pavement, which also breaks them easier than those that are wider.

COROLLARY III.

BY the Experiments upon the smooth Board, it appears that the Friction upon the Axle-trees is inconsiderable; for with the Waggon that had the four Wheels of five Inches and nine Lines, one Ball, twenty of which weigh a Pound, draws a Weight of six Pounds, or a Weight of five Pounds, with the Wheels of five Inches: The Waggon together with the Wheels weighing besides about one Pound and a Quarter, which makes the whole Weight taken together equal to 130 Balls. So that in this Case, one Pound would draw 130 Pounds; or what amounts to the same thing, the Friction on the Axle would be only equal to the 130th Part of the Force with Wheels in this Proportion *. For when the Ground is perfectly even, all the Resistance arises from the Friction, which is but small in comparison with the sinking in of the Wheels in Earth and Holes, from whence they must be raised.

* Ann. 4.

COROLLARY IV.

By the Experiment of the Cart, whoſe Weight only bears upon two Wheels, it appears that the Friction is double when compared with that of the Waggon with four Wheels: For there is required double the Weight for the Cart; and even then it does not go till it be put into Motion; whereas the Waggon goes of itſelf, and double the Weight, or thereabouts, is required for Wheels of half the Bigneſs. This, perhaps, is not altogether owing to the Friction; for it ought to be double in the little Wheels, compared with the great ones, ſince they go twice round while the great ones go but once round; and though the little Wheels in the Cart muſt be ſet a-going as well as the great ones, yet they go a little faſter than the two great ones, and the two great ones in the Cart faſter than in the Waggon, though they move in zigzag: This may alſo partly be owing to their not being perfectly round, nor *in Æquilibrio* upon the Axle, which is not ſenſible in the Waggon: And this ſhews that it is a great Diſadvantage to make uſe of Carts, even in reſpect of Friction.

COROLLARY V.

It is eaſy to conceive, from the Experiments made upon hard Clay, and upon Sand, that half the Force is generally loſt in a Waggon, when little Wheels are before inſtead of great ones: For though it does not appear that half is loſt upon the Ground when the Earth is firm, there would be much more than half loſt, if the Earth was ſoft, as we ſee more is loſt on dry Sand. Beſides, one would often be mired with little Wheels in thoſe Places where great ones would go through.

COROLLARY VI.

And though half the Force be not loſt upon the Pavement, eſpecially when a Waggon or Coach is drawn by the Horſes in a Trot, becauſe as the Wheel goes down the Declivity of one Pavement, it acquires a Force to riſe up the next; but yet if we obſerve the Horſes drawing, we ſhall ſee that they grow heavy or ſtiff in the Hams, drawing upwards; and on that account, in theſe Circumſtances, we may alſo reckon half the Force, even upon a Pavement; but there will be more loſt on ſtiff Clay and Sand.

CO-

COROLLARY VII.

IF we add to this the Conſideration of the hinder Axle being bent in a Coach, which makes the Wheels be leſs free, we need not wonder that vigorous Coach-Horſes that are well looked after, will be fatigued when they have drawn two or three Hours in the Streets, and have gone four or five Miles upon the Pavement: And for the Country, we muſt uſe four or ſix Horſes, and we find that if they be backed a few Steps they'll be out of Breath, by reaſon that the End of the Pole is low, and has a Tendency to break in ſuch a low Direction; whereas the Force and Direction would be wholly employed in going forward or backward, if the Pole was as high as the Breaſt of the Horſes, by having high Wheels before.

COROLLARY VIII.

HENCE we may find, that Horſes pay very dear for the Conveniency of a ſhort Turn, and that it would be better to go to the End of a Street for the Conveniency of turning: For ſince half the Streets are inconvenient, even for a ſhort Turn, what would it ſignify if the Wheels were high, and the Pole had not the Crane-neck, or arched Piece behind, to go a little farther, or croſs a few more Streets? It would be better for the Horſes, and even for the Coachmen, who would be leſs fatigued with the Jolts; and that would be ſufficient Amends for a little more Trouble to get up into their Boxes, which happens but once; whereas they are ſhaked with a thouſand Jolts. Their Maſters' would be leſs interrupted in their Buſineſs, by ſuch Jolts as happen in the ſhort Turns, and the Coach itſelf would be leſs liable to overthrow.

COROLLARY IX.

THEREFORE the Contrivance of a circular Piece of Crane-neck for turning, which obliges one to have little Wheels, has much more Inconveniency than Uſefulneſs, both for the Horſes and Coachmen, beſides the greater Expence of the Crane-necks, and the Springs, which are often the Occaſion of overturning, by raiſing up the Coach too high, as has been ſaid.

COROLLARY X.

CONSIDERING theſe Experiments made with great and little Wheels, it is not hard to conceive that the Berlins are harder for the Horſes than Coaches; beſides the Lowneſs of the Fore-Wheels, the Shafts do not yield, and the Pole that bends ſends back the Wheel a little when there is a Rub to be overcome, and then draws it on more ſwiftly; this makes the Wheel act backwards like a Wedge. If Berlins are leſs ſubject to overſet, when they do, the Fall is the greater: If they coſt leſs, the Wheels muſt be oftner repair'd, becauſe in the Berlins the Wheels cannot have long Naves; and then the Shafts break oftner than Poles.

COROLLARY XI.

THO' it may ſeem that there is not ſo much Advantage in going up and down for great Wheels as for little ones, becauſe as they roll eaſily they are troubleſome to the Horſes when they are going down, and that the little ones not rolling ſo eaſily are not ſo difficult to ſtop; and beſides, that in going up Hill, the Fore-Wheels are not ſo much loaded, and therefore leſs Force is required in proportion to the great ones than upon plain Ground: Yet it appears that in going up there is always more Force required for little Wheels than for great ones, and that it will always be ſo proportionably; for as the Direction is lower, the Horſes are thereby more tired, and tho' the great Wheels are more rolling, the Horſes too have the full Advantage of their Strength to ſtop them, the Pole being then Breaſt high, whereas it is very low when we uſe ſmall Wheels, and it tends to break, as often happens in going down Hill: So that all being well weigh'd, there is at leaſt as much Advantage in Proportion for the great Wheels as the little ones in going up and down Hill: Beſides, in travelling we go an hundred Steps upon plain Ground for one up or down Hill.

COROLLARY XII.

THERE is another Diſadvantage for little Wheels, which is, that they break the Pavements and ſpoil the Ways more than great ones: Beſides, they bear more Weight, and having leſs Bearing they ſink deeper and jump up higher; which hurts the Houſes which they ſhake

ſhake as they go by; they make more Noiſe, and alſo ſplaſh more. Lect. IV.

COROLLARY XIII.

ONE may ſee the Difference of ſtiff Clay, Sand and Pavement, that it is always moſt advantageous to draw on the Pavement, and that Horſes muſt needs tire very much on Sand; but that in rainy Weather it is often better to go on the Sands than the common Earth when it is a ſtiff Clay: But in dry Weather Earth is better than Sand, Sand being more eaſy to draw upon when Earth is the moſt difficult, *&c.*

COROLLARY XIV.

ACCORDING to the Experiment of bringing forward the Weight upon the little Wheels, where twenty-four Balls were required to draw the Load out of a Hole, inſtead of three when the Weight was behind, and not in a Hole, it appears that there would be required much the ſame proportional Force, on Pavement or Earth. This ſhews, that going into the Country, it is better to put Boxes, Portmanteaus, and Footmen behind than before; which is the Reverſe of what moſt Coachmen do, imagining, according to their Notion of *great Wheels driving*, that the Coach will roll the better, the more it is loaded before. Whereas the Pages who are before fatigue the Horſes twice as much as the Footmen that are behind.

COROLLARY XV.

BY obſerving the Waggon or Cart ſtopped on the Sand, for which a Quarter of Force muſt be added to each of them when they had time to ſink, one may conclude, that if a Carriage be mired, and the Horſes are baulked in their drawing; we muſt not ſtand long, but put on the Horſes behind to draw it out, and then go through another Place if poſſible; if not, the Horſes muſt from ſome little Diſtance be driven briſkly, that the Wheels may not have time to ſink, and the Horſes may have acquired ſome Swiftneſs, as when we go to jump over a Ditch. In making the Experiments, we muſt not give the Wheels time to ſink on the Sand or Earth, but lift them up every time when we put Balls into the Bag if there are not

Lect. IV. not enow. One may obſerve that even upon Pavement we muſt uſe more Weight when the Load has ſtood ſome time, the Adheſion of Parts (or rather *taking* like the Teeth of Wheels) becoming greater both on the Axle-tree and on the Pavement, as has been obſerved in the Conſideration of Friction.

COROLLARY XVI.

It is alſo well when we travel upon Sand, either in dry or in wet Weather, to go in the Ruts, whereby we may avoid the Friction on the Sides of the Fellies or Curves of the Wheels, and have no Earth to turn up, and alſo the Ground is firmer there. It appeared by the Experiment, that when the Waggon or Cart had gone twice thro' the ſame Ruts, if it was then drawn in the ſame Rut the third time, it required ſcarce half the Force this laſt time, whether upon Sand or upon Clay, becauſe the Wheels then did not ſink above half a Line: Therefore in making Experiments to be compared together, we always filled up the Ruts of the Sand and of the Clay, that the Difficulty might be the ſame in the Caſes compared: For without that, when the Cart went after the Waggon, it always had the Advantage in the Experiments on Sand; and the Waggon would ſtill have loſt more if it had gone before the Cart upon the Clay.

COROLLARY XVII.

Several other Things might alſo be obſerved concerning little Wheels, as to Holes, Heights to be overcome, and other Caſes, wherein there will always be found a great Diſadvantage: If it be not ſo great in the Heights to be overcome, becauſe there appears to be only the Loſs of a third Part, as the little Wheels then do not ſink deeper than the great ones; yet in other Caſes more than half the Force is loſt: So that which way ſoever we conſider that Matter, there will always be more Diſadvantage than Conveniency.

COROLLARY XVIII.

As to the Carts with two Wheels, we ſee ſufficiently what Diſadvantage they have when compared with Waggons of four equal Wheels; and if they have any apparent Conveniencies, as that of loading

loading and unloading more easily, much more is lost than gained by saving that Labour, which ought not to be considered, any more than the greater Ease of going into a Chaize with low Wheels; for we suffer for it in another Case by the rough Jolts, and the greater Labour of the Horse that draws. Lect. IV.

COROLLARY XIX.

This Reason of loading more easily might at most obtain in Cities, where we use small Carriages and often load and unload; but for Carriages that hold their Load a Week or two without unloading, the greater Ease of loading should not be considered. At that rate we might alledge that we should only use Sledges, rather than load upon a Waggon with four high Wheels: But what would be the Difference of the Profit?

COROLLARY XX.

Do not those Carters who use Carts to carry Wines with very low Wheels, that they may load with the more Ease, and then also ride upon the Horse, deserve to be made to draw the Cart themselves instead of killing Horses by needless Labour that might do great Service if worked moderately? That Laziness is so much the more to blame, because a Turn or two more of the Handle of the Jack, or of the Windlass, would raise up a Vessel of Wine into a high Carriage, with spending very little more time, and scarce taking any more Pains.

COROLLARY XXI.

If the Cart has some Advantages on account of the Direction of the Fore-Horses, and the Height of the Wheels, it has a great Disadvantage for the Tiller, especially upon Pavement; one Wheel coming down from a high Pavement drives one Shaft against the Horse's Flanks, then the other Wheel falling drives the other Shaft against the Horse's other Side, so that the poor Horse being banged about, especially in great Jolts, is soon worn out, or killed; so that there must be a great Force to draw the Cart upon the Pavement on this account; and even when it is drawn by a String over a Pulley it goes in zigzag.

COROLLARY XXII.

BESIDES this Disadvantage, the Tiller carries part of the Weight; as he goes up, the Weight falls back and pulls him, and in going down Hill the Weight comes upon his Back: Besides he is forced to stop alone an immense Weight that is laid upon the Cart; so that it is a Wonder no more Tiller Horses are kill'd, tho' People generally make Tillers of the strongest Horses they can get. A Horse thus harrass'd every way cannot employ so much Force to draw as if he was before, or between the Shafts of a Waggon, where he is not banged upon the Sides, and has nothing to carry.

COROLLARY XXIII.

WHEN we consider how much of the Tiller's Force is lost, how much deeper the Wheels sink in than in the Waggon, how much more Force is required upon Sand, Clay and Pavements; we may judge, that there is a double Advantage, or nearly, to make use of a Waggon with four equal Wheels and as high as those of Carts; for the Direction for the Horses would then be the same; and the Experiment shews us how much more Force is requir'd for a Cart upon Sand and Clay, where it goes streight like a Waggon.

COROLLARY XXIV.

SOME Carters, when they have new Wheels in their Carts falsly attribute to the Friction of the Naves the tiring of their Horses, which are twice as much fatigued without appearing to do any more Work; for when the Wheels have had a few Turns they go free upon the Axle-trees; and when they are well greased they go as easy the second Day as any time afterwards; besides we have shewn that the Friction on the Axle-tree is very little: But the Cause of this Labour to the Horses is the Quantity of Nails in the Iron Plates round the Wheels, and the great Height of their Heads, which is about an Inch. This Difficulty of rolling along when there is a Space between the Nails, is in the Experiment represented by an Height of one Line to overcome, reducing the Feet of the Wheels to Inches, *&c.* And since it appears that there must be twenty times more Force, or thereabouts, to overcome this Height of one Inch; though the Nails should be but half an Inch high, one may guess

gueſs how much Labour muſt be employed to raiſe the Weight ſo at every Nail when the Shooing of the Wheels bears upon the Pavement, and then the Wheel is raiſed up upon the Head of a Nail all the while the Wheel goes round, and eſpecially where there are narrow Gutters. This is the true Cauſe of the extraordinary Labour of the Horſes. Lect. IV.

COROLLARY XXV.

THIS ſhews that it is neceſſary the Wheels ſhould be round, and the Carters who think they ſave Money in uſing many great Nails (becauſe the Iron Plates or Shooing does not wear ſo faſt when there are ſuch Nails) are in an Error, and loſe double by doing leſs Work, and fatiguing their Horſes.

COROLLARY XXVI.

THESE Nails alſo have the ſame Effect as narrow Plates; they ſlide between the Pavement, and from thence they have a greater Height to riſe in lifting the Weight up, and ſometimes ſliding down again, and that wears the Plates round, which makes them ſlide more between the Pavements and into the Ruts, which the Kennels make in the Middle of the Streets; and it is alſo upon this account that narrow Plates ſoon grow round; and, being round, tire the Horſes almoſt as much as if there were Nails.

COROLLARY XXVII.

THIS ſhews plainly enough, that it is advantageous to have the Plates upon the Fellies of the Wheels wide, as well on the Pavement as on the Ground, as it appeared by the Experiment of the great narrow Wheel upon the Pavement; and alſo conſidering the Strength that the Horſes muſt exert to draw out the Wheels from the Kennels, the Damage that is done to the Pavement, and the quick wearing of narrow Plates, which alſo ſoon become round and inconvenient. It would be better to have the Kennels like Troughs, wide and ſhallow *, as I E, *Fig.* 14. and not like L M, *Fig.* 15. which is the common Make of the Chanels in the Streets.

* Plate 17. Fig. 14 & 15.

COROLLARY XXVIII.

If Kennels were made in this Shape, ſeveral Advantages would be gained: The Plates or Shoeings of Wheels would not ſo ſoon grow round, nor break the Paving ſo eaſily: The Pavement of the Kennel itſelf would laſt longer, becauſe it would not receive ſuch great Shocks, every Part ſupporting more equally; whereas now the loweſt Place bears much a greater Part of the Weight, whereby it ſinks moſt and ſooner breaks. This ſinking in makes thoſe Places lower when they are mended, and the Rain-water ſtays in them, ſo that the Wheels or the Horſes Feet ſtriking into them, ſplaſh the People that walk along; whereas the Water would always run down into the Middle of the Kennel, were it not for theſe Inconveniencies: Neither would the Stream of the Kennel-water grow ſo wide as it does in great Showers, becauſe in the Form we propoſe, the Kennel is wide enough at Bottom to carry off the Water; and it would be eaſier in walking to ſtep over them.

COROLLARY XXIX.

Thus alſo might the little Channels, that bring the Water from the Houſes to the Kennels in the Middle of the Street, be made leſs: Nay it would be better to be without them, only making a gentle Declivity from the Houſes to the Middle of the Street. Theſe little croſs Kennels are very inconvenient for thoſe that walk the Streets; nay for the very Horſes and thoſe that go in Coaches: People don't conſider that one muſt ſtop a little at every one of theſe Channels and go irregularly, taking firſt a little Step, and then a great one to ſtride over; and a little Stop of the Center of Gravity, or of a Body in Motion, tires one very much: This is the Reaſon why People are more tired to walk an Hour in *Paris* than two or three in the Country, even upon Pavement, becauſe there the Body always continues its Motion without Interruption. The ſame might happen in *Paris* if there was only one Kennel pretty wide and ſhallow in the Middle of the Streets; then alſo Perſons that go in Coaches would be leſs jolted.

COROL-

COROLLARY XXX.

THE Uſe of Waggons with regard to paved Streets and to publick Roads would alſo be of publick Advantage, inſtead of Carts, which are very weighty in proportion to their Bigneſs, and which are generally loaded ſo heavy as to endanger the breaking of all the Geer. The vaſt Weight which they carry being ſupported on two Points, as they have but two Wheels, ſinks deep and makes great Ruts in the Ground; and upon Pavement when the Wheel is born up on a Nail Head and comes to fall down again, it preſſes down or breaks that Pavement worſe than if ten or twenty Waggons with as heavy a Load had paſſed over that Place. For a Waggon Wheel, that falls upon the ſame Place, and from the ſame Height, comes down but with half the Weight that the Cart Wheel does, and therefore ſtrikes it but with half the Force. Now the Pavement being ſomething of the Nature of Glaſs, will not break unleſs it receives a ſufficient Blow; and an hundred Blows, if each of them be leſs than that ſufficient Blow, will not break it. The Blow from the Cart in thoſe Places is often ſufficient to break the Pavement where an hundred Blows from a Waggon Wheel, as they ſtrike but with half the Force, will not be able to hurt it; ſo likewiſe will a Waggon paſs over as often as you will where a Cart would ſink in. Carts therefore, little Wheels, narrow Plates for ſhooing the Fellies, and great Nails, are to be avoided as much as can be.

COROLLARY XXXI.

BY the Experiment made upon Clay and upon Sand, we may ſee how uſeful it is to have the Roads firm and ſolid, ſince there is ſo much Difference between the one and the other. And certainly the Carriage of Goods, by Land or Water, contributes much to the artificial Riches of a Kingdom or State: And all things well conſidered, we ſhall find that the Time and Expence employed in mending Roads, is the moſt for the publick Advantage.

COROLLARY XXXII.

In this, and the other three following Corollaries, Mr. CAMUS *finds fault that the Laws and Orders concerning mending Roads and Matters relating to Carriages are no better obſerved, and takes*

Lect. IV. *takes Notice that a Farmer that uses many Horses to draw a large Cart very heavy loaded, may spoil the Roads for many Miles, worse than an hundred others with the common Carriages, and therefore proposes in*

COROLLARY XXXVI.

THAT Orders should be made for the publick Good, That only two Horses should draw in a Cart, and four in a Waggon, which four would do more Work than five in a Cart.

COROLLARY XXXVII.

THAT Wheels of Carriages never be made less than four Feet and a half, or five Feet in Diameter; that the Plates of Iron never be less than three Inches wide; the Fellies three Inches and a quarter deep, at least for Coaches, and four Inches for Waggons or Carts; which, by that means, would be stronger as they are broader.

COROLLARY XXXVIII.

THAT the Nails for the Plates be made without Heads, and that a great many be made long enough to go quite through to be rivetted, which will keep the Irons from rising up: For two Nails screwed behind, or with a Counter-Rivet will hold the Irons on faster than six with Heads. One may as well make the Plates three Inches wide as two; and as they wear out more at the Edge, they may be made thinner in the Middle next the Fellies, in the Shape
* Pl. 17. F. 16. shewn in the Section, *Fig.* 16. at B C *, which may be easily done in the forging by help of a Swage of that Shape. These Plates being put on hot will preserve the Fellies more from cracking, last as long as others, and neither be heavier or cost more. By this means the Roads would not be so soon spoiled, and all that use Carriages would find their Advantage.

COROLLARY XXXIX.

THE Naves of Coach-Wheels should be made a little thicker in the Middle, and not be bored quite thro' where the Spokes are let in; for then their Ends would not receive the Grease, and thereby be so apt to get loose: The Naves might also be made fifteen or sixteen Inches long,

long, with the great End a little leſs, ſince the greater Length of the Nave does not encreaſe the Friction, and it holds the Wheel ſtreighter. The Spokes ſhould alſo be made an oblong Square at Bottom, to have a Shoulder to keep them firmer than when they are round, as they are uſually made. The Spokes thus fixed would not be ſo apt to fly out, nor make that Rattling which they do, eſpecially in Summer-time. The Workmen may work out theſe Shoulders with a Bevil at the ſame Angle that the Spokes are let in, to mal- the Wheels diſhing out, after the manner the Joiners make their Skew Tenons, and the ſame Inſtrument would direct them in cutting the Mortaiſe, *&c.* Lect. IV.

Mr. CAMUS *ends his Conſiderations upon this Subject with obſerving, that a Poſt Chaize with two Wheels has all the Inconveniencies obſerved in Carts; and beſides has the Inconveniency of the ſecond Horſe which draws on one Side; therefore, for the Eaſe of the Horſes, as well as the Travellers, he would have all thoſe Chaizes to have four Wheels, and the Driver not to ride, but ſit on a Box like a Coachman; and the Fore-wheels as high as the hind ones.*

BESIDES the Friction already explained, there is another Impediment to Motion in ſeveral Engines, and that is the Difficulty with which Ropes are folded, which encreaſes according to the Bigneſs and Stiffneſs of the Rope, the Weight which they bear, and the Smallneſs of the Diameters of the Bodies about which they wind. This Impediment we ſhall alſo call Friction, becauſe we muſt make Allowance for it as well as for the rubbing of the Parts of the Machine, otherwiſe we ſhall always find the Effect of Powers by means of Engines to be leſs than we expect.

MONSIEUR *Perault*, in his Comment upon *Vitruvius*, deſcribes an Engine of his contriving, whereby he thinks to avoid all Friction. This Machine is an *Axis in peritrochio* applied a new Way: But as he is not aware of the Friction or Hindrance that ariſes from the Difficulty of bending the Ropes, the Effect will by no means anſwer as he propoſes: For, upon Tryal, his Machine appears to have more than double (ſometimes than triple, or quadruple) the Friction of the ſame Engine uſed in the common Way, when the Pivot or Iron Axis is in Diameter the 12th Part of the Roller or wooden Axle. He does, indeed, ſay, that the Machine was tried and ſuc-

Lect. IV. ſucceeded; but having carefully examined it, I found it impoſſible to anſwer according to the Deſcription; and leſt at any time any body ſhould be at the fruitleſs Expence to make the Machine in large, I demonſtrated the Diſadvantage of it before the *Royal Society*, and ſhewed that beſides the Friction (overlooked by its Author) there was a great Inconvenience in the Application: And this I confirmed by Experiments made on a Model of an Inch to a Foot, the large Pulley or Wheel being ſuppoſed of five Feet.

MONSIEUR *Perault*'s Account of his Engine is as follows: "In Imitation of the (modern) Crane, I have invented two Engines "for raiſing Weight. The firſt is made of that Organ which is "the moſt advantageous of any in Mechanicks for facilitating Mo- "tion; becauſe it is free from that Inconveniency which we meet "with in all others; namely, the Friction of the Parts of the Ma- "chine, which renders their Motion more difficult. This Organ is "the Roller, which *Ariſtotle* prefers to all other Organs, becauſe "all the others, as Wheels, Capſtanes, and Pulleys, muſt neceſſa- "rily rub in ſome of their Parts. But the Difficulty was to apply "the Roller to an Engine that raiſes Weights, its Uſe having only "been hitherto to cauſe them to roll on an horizontal Plane. The *P. 1. F. 18. "Engine which I propoſe has a Baſe A A B*, (*Plate* 17. *Fig.* 18.) "ſomething like the Crane: This Baſe has, in its upper Part, the "horizontal Piece B, which claſps an upright Shaft C O, ſupported "under its Pivot O, on which the whole Engine moves in the ſame "manner as the Crane, when the Weight is to be lowered. This "Shaft ſupports on its Top a croſs Piece D D, to which are faſten'd "the Ropes E E, which wrap round the Barrel, Axle, or Roller F, "which has another Rope G, that alſo wraps or winds round one "of its Ends. This laſt Rope is that which raiſes the Weight. At "the other End of the Axle there is a great wooden Wheel like a "Pulley H H, about which is wound a long Rope N. To work "this Engine, one muſt pull the long Rope N, which cauſing the "great Wheel to turn, does alſo carry round the Axle or Barrel, "which is made faſt to it. This Axle, as it turns round, cauſes the "Rope E E to wind about it, and therefore the Axle and the Wheel "riſe, whilſt the Rope F, to which the Weight is faſtened, does alſo "wind itſelf upon the Axle the contrary Way; and this double "winding up of the Ropes makes both the Burthen and the Axle "and Wheel to riſe at the ſame time. Now it is evident, that all "this Riſe is perform'd without the Friction of any Part, and conſe- "quently,

" quently, the whole Power which draws the Rope N, is employed Lect. IV.
" without any Hindrance; which cannot be in other Engines.

" It may be objected that the Power which acts at N, must, be-
" sides their Weight, raise also the Axle and great Wheel, and that
" their Weight is one of those Obstacles which *Aristotle* says all En-
" gines are liable to; and that this Obstacle is equivalent to the
" Friction which is in other Organs. But it may be answered, that
" Friction is an Obstacle wholly unavoidable in all other Organs; but
" that it is easy to remedy the Obstacles of this, which is done by
" means of the heavy Body M, taken equal in Weight to the great
" wheel and Axle, which it sustains by means of the Rope II, which
" running over the Pulleys LL, is fixed to the Ring or Collar K that
" goes round the Axle F: For the Axle and the Wheel being coun-
" terpoised by this Weight, the Power which acts by drawing the
" long Rope N, acts for raising the Weight only. The Experiment
" which was made with this Engine, has confirmed the Truth of this
" Problem, by comparing its Effects with those of a Crane, in which
" the Proportion of the Bigness of the Axle to the Circumference of
" the Wheel, was the same as in my Machine: For it happened,
" that, in the Crane, a Weight of *One* hanging at a Rope going
" about the Wheel, drew up a Weight of *Seven*, when it had one
" half added to make it preponderate, or give Motion to the Power:
" And when the Weight to be raised, and the Weight which served
" as a Power, were proportionably encreased, there was also a Ne-
" cessity to encrease the additional Weight, which made the Power:
" And when the Weight to be raised, and the Weight which served
" as a Power, were proportionably encreased, there was also a Ne-
" cessity to encrease the additional Weight, which made the Power
" preponderate in the same Proportion: So that as it was required to
" add one half to the Power when the Weight was *Seven*, the Addi-
" tion to the Power became *One* for a Fourteen Pound Weight, *Two*
" for a Twenty-eight Pound, *Four* for a Fifty-six Pound, and so on;
" because the Resistance from Friction encreases nearly in the same
" Proportion that the Weights are encreased. But this did not happen
" in my Engine, in which *one Quarter* was always sufficient for the
" Draught (or to make the Power preponderate) not only when the
" Weight was Seven, but also when it was Fourteen Pounds, Twenty-
" eight Pounds, Fifty-six Pounds, *&c.* which evidently shews that
" this Engine acts without Friction."

Thus far Mons. *Perault:* But however plausible this Description may appear, a little Attention will shew, that if this new En-

Lect. IV. gine had no Friction, yet it is more inconvenient than an *Axis in Peritrochio*, with the same Proportions, and likewise that it has more
Pl. 18. F. 1. Friction than the same Machine in the common Use. A C E (*Fig.* 1.) is a common *Axis in Peritrochio*, which has the Wheel A E, five times bigger in Diameter than the Axle; so that A C, the Radius of the Wheel (which is the Distance of the Power) is to C B, the Radius of the Axle (the Distance of the Weight) as 5 to 1: Consequently *One* (for Example one Ounce, as in our Experiment) will keep *Five in Æquilibrio*. Now though the Friction of the Gudgeon at C is unavoidable, yet it may be diminished by diminishing the
† Ann. 4. Diameter of the Gudgeon †; provided it remains strong enough to sustain the Machine and its Burthen. Here one Penny-weight, or $\frac{1}{20}$ of the Power added to it, makes it preponderate, and give the Machine Motion with a due Velocity.

Now this very Engine made Use of in Monsr. *Perault's* Way, does so alter the Distances of the Weight and Power, that, instead of *One* for our Power, we must have *Two and a half* to keep the very same Weight *Five in Æquilibrio*, as may appear by a Sight
** Pl. 18. F. 2. of the second Figure **, where since in the Action of the Machine, when we pull the Rope P A, we make the Axle D B to wind itself up upon the Rope H D, it is evident that D is now become the Center of Motion D B (the whole Thickness of the Axle) the Distance of the Weight = 2; and the Distance of the Power is reduced to A D = 4. So that if two Men, having been employed in the common Way to raise Weights equal to the Strength of ten Men, an Engineer should alter the manner of working, and fit up the *Axis in Peritrochio* in Monsr. *Perault's* Way, instead of gaining an Advantage, he
†† Ann. 5. must call in three more Men to perform this Work ††. If it be answered, that what is lost in Strength will be gained in Time, it may not only be said, that one cannot always call in more Help on a sudden, but that even then, though we should not call this an Inconveniency, yet there will be still more Friction in this than in the common Method; for the Roller or Axle will find a Difficulty to wind on the Ropes, because they are not perfectly pliable, and the less so, the greater the Weight is that stretches them. This, together with the Friction of the Collar of the Rope of the Counterpoise to the Engine, makes the Hindrance greater than in the common Way: For it appears by my Experiments, that when the Power is become equal to 2^{r} to keep the Weight 5 *in Æquilibrio*, there must be added $\frac{1}{r}$ (here four Penny-weight) to put the Power in Motion.

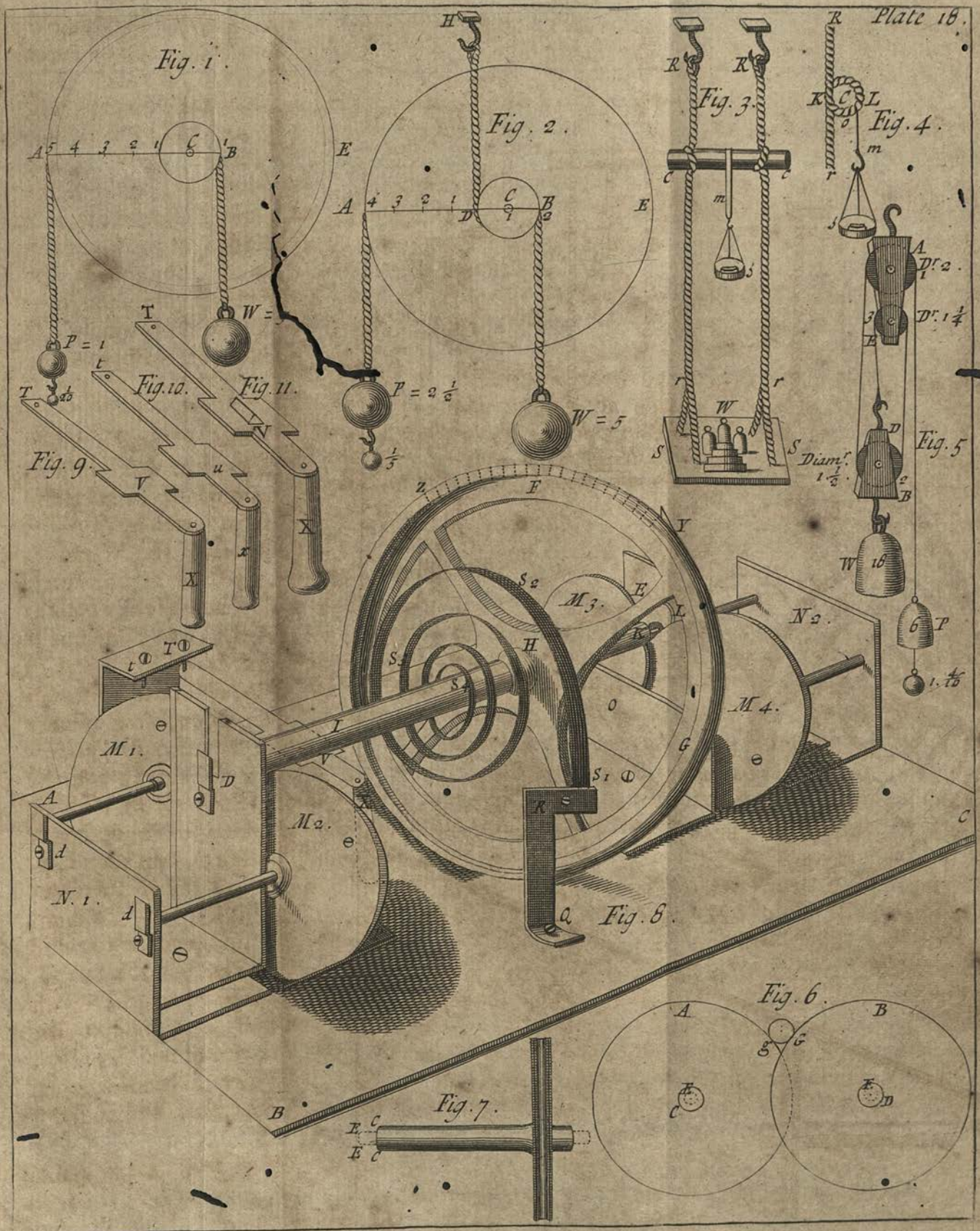
Plate 18.
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
P = 1
W = 5
P = 2 1/2
Diam.r 1 1/2
M 1.
M 2.
M 3.
M 4.
N 1.
N 2.

Lect. IV.

And, to shew that this Friction of the Ropes is not always the same as Monſ. *Perault* ſuppoſes it; when * P (or the Power) is made only one Ounce, and W (or the Weight) 2 Ounces, then to make the Power preponderate, only 2 Penny-weight and 18 Grains was ſufficient: But when P is $= 2\frac{1}{2}$, and W $= 5$, the additional Weight marked $\frac{1}{2}$ was 4 Penny-weight and 8 Grains.

* Pl. 18. F. 2.

It is plain from this, that Monſ. *Perault*'s Experiments were very inaccurately made, and therefore not to be depended upon.

I have been the more particular here, becauſe we are apt to be led into an Error by the Overſights of a Man of great Reputation, whom we don't eaſily ſuſpect of a Miſtake.

Tho' it be as difficult, at leaſt, to give a certain Account of the Forces required to bend Ropes of different Diameters (ſtretched by different Weights, in making them go round Bodies of different Bigneſſes) as to give an exact Theory of Friction; yet to conſider nothing of the Loſs of Motion occaſioned thereby, would be as prejudicial to the Practice of Mechanicks, as it would be to overlook the Friction of the Parts in Engines. Therefore, tho' the different Materials of which Ropes are made, their different Stiffneſs, according as they are more or leſs twiſted, and ſometimes the Temperature of the Weather (as to Moiſture and Dryneſs) at the Time that they are uſed, makes it very difficult to be exact in our Concluſions; yet we think it is of great Uſe to give the beſt Theory we can, and mention ſome of the Experiments, at a Medium, made upon Ropes pretty good in their Kind, and moderately twiſted; becauſe if any Part of a Rope of any Length of equal Thickneſs, and even Twiſt from End to End, be ſtretched by a known Weight round a Cylinder, Roller, or Pulley, and it be obſerved what Force will bend it about a Roller of a given Diameter, we may know what other Force will be required to bend it round any other Body, and when ſtretched with a different Weight; and, beſides, in new well made Ropes, the Difficulty of bending, *cæteris paribus*, is pretty near as the Diameters (not the *Solidities*) of the Ropes.

Experiments, *Plate* 18. *Fig.* 3.

To two immoveable Hooks R R, I fixed the two Ropes R *r*, R *r*, at the Diſtance of about 8 Inches from each other; and at the lower

Lect. IV. End of the Rope I hung the Scale S S, on which I placed the Weights W to ſtretch the Ropes. Then I took ſucceſſively three Cylinders like C C, each one Foot long (one of half an inch, another of an Inch, and the third of an Inch and an half Diameter) and having wrapped the two Ropes about one of the Cylinders, as may be ſeen in the Figure, by putting Weights in the little Scale *s* by means of the Ribbon *m* I brought down the Cylinder towards W, always taking care that the Parts of the Ropes did not rub againſt each other, and rolling the Cylinder up and down two or three times before I ſettled the Weight that I obſerved to bring down.

Pl. 18. F. 3.

N. B. *The Weight includes the Weight of the Cylinder and Scale.*

† Pl. 18. F. 4. In the 4th Figure † C repreſents the Section of the Cylinder or Roller, about which the Rope R *r* is wound, and K L its Diameter, and *m s* the little Scale and Ribbon as before.

The Weight W may be always looked upon as the Weight ſtretching the Rope; becauſe (though it ſtretches the two Ropes and conſequently each Rope is ſtretched but by half of it) the Cylinder C C * brought down by the Weight *s* bends or cauſes a folding in the two Ropes at C and C, which gives it the ſame Difficulty of Deſcent, as if only one Rope bearing the whole Weight was folded about it.

* Pl. 18. F. 3.

A TABLE

A TABLE of EXPERIMENTS, shewing what Forces were required to bend Ropes of different Diameters, stretched by different Weights, round Rollers of different Bignesses. Lect. IV.

The Quantity of the Weight W, *by which the Rope is stretched, expressed in* ℔ Averdupoids	*The Resistance of the Rope about a Cylinder of half an Inch Diameter, express'd in* ʒ Averdupoids.	*The Resistance of the Rope about a Cylinder or Roller of one Inch Diameter in* ʒ Averdupoids.	*The Resistance about a Roller of one Inch and half Diameter in* ʒ Averdupoids.	*Diameters of the Ropes of* 3 *Strands or Twists, express'd in tenth Parts of an Inch.*
60 ℔ —	225 ʒ	112 ½ ʒ	75 ʒ	— 0,5
	90 —	45 —	30 —	— 0,2
	45 —	22 ½ —	15 —	— 0,1
40 ℔ —	150 —	75 —	50 —	— 0,5
	60 —	30 —	20 —	— 0,2
	30 —	15 —	10 —	— 0,1
20 ℔ —	75 —	37 ½ —	25 —	— 0,5
	30 —	15 —	10 —	— 0,2
	15 —	7 ½ —	* —	— 0,1

The Experiment could not be made here, because the Cylinder of 1 ½ *Inch Diameter, which should have been used here, weighed above* 8 *Ounces; and the Weight required to bend the Rope, appears, by Analogy, to be but* 5 *Ounces.*

OBSERVATIONS *on the foregoing Table.*

THE Experiments mentioned here are taken at a Medium from a great many that I made, the Difficulty of bending the Ropes being sometimes a little less, and sometimes a little greater.

ONE may, from this Table, make a more extensive one, by putting in Numbers analogically for several other Thicknesses of Ropes, and larger Diameters of Cylinders or Rollers; but in Ropes of a Diameter bigger than half an Inch, a Roller of half an Inch is too little, and hardly ever used in Practice; nay, in such Cases, the bending of the Ropes is more difficult than in Proportion to the other Experiments; and in my Tryals, the Rope of 0,5 of an Inch Diameter generally required more than here set down.

Lect. IV. I FOUND that a woven Clock-line of 0,1 Inch Diameter required more Force to bend it than a twiſted Rope of the ſame Thickneſs, which ſeemed much ſtiffer; but if we conſider that the woven String flattens as it folds round a Cylinder; that has the ſame Effect as if the Cylinder was become leſs, one Fold of ſuch a Line making by its central Parts a leſs Circle than a twiſted Rope, which does not become flat.

MONS. *Amontons* made ſeveral Experiments of this kind, mentioned in the Memoirs of the *Royal Academy* of Sciences at *Paris*, for the Year 1699, and calculated a Table of the Force required to bend Ropes, which Table I cannot recommend, becauſe it is built upon a Miſtake: For he ſays, that the Difficulty of bending a Rope of the ſame Thickneſs, and loaded with the ſame Weight, decreaſes when the Diameter of the Roller encreaſes, but not ſo much as that Diameter encreaſes; but I have found by many repeated Experiments, that the Difficulty decreaſes directly as the Diameter of the Roller encreaſes; that is, *The Difficulty of bending a Rope round a Roller, is*, cæteris paribus, *inverſly as the Diameter of the Roller. N. B.* I believe Monſ. *Amontons*'s Miſtake aroſe from the Parts of the Rope rubbing againſt each other, which I always took care to avoid.

WHEN a Rope is carried under or over a Wheel or Sheeve of a Pulley, the Difficulty of bending is as great as if it went quite round a Roller, which will appear by obſerving the 4th *Fig.* of *Plate* 18: For while the Rope is folding about the Cylinder C in the Direction *r* K L, the under Part L *o* K unfolds of itſelf without any Difficulty.

I HAVE in this Lecture (Page 195, 196, 197) given Rules for the Friction in a compound Engine, and ſhewed the Application of thoſe Rules by an Example; but did not then take in the Difficulty of the folding of Ropes. But now I will give another Example wherein that alſo is conſidered, and ſhew how near the Theory agreed with the Experiment performed with a Machine made as nicely as poſſible, to make the Compariſon the juſter.

The Machine conſiſts of three Pulleys (two *upper* and one *lower*, or a *Tackle of three*) whoſe Diameters are exactly as follows, *viz.* two Inches, one Inch and a quarter, one Inch and a half; and

and all the Center-Pins of one quarter of an Inch Diameter exactly, and the Rope of one tenth of an Inch Diameter. The Weight is 18 Pound *Averdupois*, and conſequently the Power to keep it *in Æquilibrio* muſt be 6 Pounds, and a very little more muſt make the Power raiſe the Weight, if there was no Friction; but here no leſs than 20 Ounces are required, though the Machine be very exact. Lect. IV.

I HAVE ſhewn (in *Rule* 2. *Page* 195.) that two thirds of the Power are equal to the Friction of a Cylinder, whoſe Surface moves as faſt as the Power, and whoſe Gudgeons are equal in Diameter to the Cylinder. Now as the Diameter of the firſt Pulley 1 is eight times bigger than its Pin, its Friction muſt be 4 Pound divided by 8, or 8 Ounces; becauſe as the Surface of its Circumference moves with the ſame Velocity as the Power, its rubbing Surface on the Pin muſt move eight times ſlower. Pl. 18. F. 5.

THE ſecond Pulley 2, whoſe Surface moves as ſlow again as the Power, and whoſe Pin is ſix times leſs in Diameter than itſelf, muſt of Conſequence have its Friction only of $5\frac{1}{3}$ Ounces; becauſe two thirds of the Power, or 64 Ounces, muſt firſt be divided by 2, by reaſon of the Velocity of the Pulley's Surface being but half that of the Power; and then again by 6, becauſe the Pin being ſix times leſs, the Parts rubbing on the Pin muſt ſtill move ſix times ſlower: So that $\frac{64}{12} = 32$, and $\frac{32}{6} = 5\frac{2}{3}$ Ounces.

THE third Pulley 3, moving with a third of the Velocity of the Power, 64 Ounces muſt be divided by 3, and that Quotient again by 5, becauſe the Pin is here $\frac{1}{5}$ of the Diameter of the Pulley: So that the rubbing Parts of this Pulley have their Velocity one 5th of a 3d, or $\frac{1}{15}$, of the Velocity of the Power; and therefore $\frac{64}{15}$ will give 4,26, *&c.* Ounces. Now the Sum of all theſe Frictions (*viz.* 8 Ounces $+$ 5,3333 *&c.* Ounces, $+$ 4,26 *&c.* Ounces) makes 17,6 Ounces, which is the 5th and $\frac{4}{10}$ Part of the Power. This Addition to the Power will ſo encreaſe the Friction, as to require a Super-addition of the 5th and $\frac{4}{10}$ Part of that firſt Addition, and ſo on in this Series, 17,62 Ounces $+$ 3,2 Ounces (which is $\frac{1}{5}, \frac{7}{10}, \frac{6}{4}$) $+$ 0,59 Ounces (which is $\frac{3}{5}, \frac{2}{4}$) *&c.* in all 21,55 Ounces.

TO this muſt be added the Friction or Reſiſtance on account of the Difficulty of bending the Ropes, which by the laſt Table may be found in the following Manner.

SUPPOSE

Lect. IV. SUPPOSE the Part of the Rope, which is at 1 the Side of the firſt or upper Pulley to be fixed, then will the three Ropes D E, 3 B, and 2 A ſuſtain together the whole Weight W*, which (together with the Block) weighs 18 Pounds; ſo that we may conſider each Rope as ſtretched by ſix Pounds, and folding round different Cylinders of the reſpective Diameters of the Pulleys.

* Pl. 18. F. 5.

FOR the firſt Pulley, we look in the Table for a Rope of one tenth of an Inch, and find that when ſtretched with the Weight of 20 Pounds it requires 7,5 Ounces to bend it round a Roller of an Inch; therefore we muſt firſt make Uſe of this Analogy.

As 20 ℔ ſtretching a Rope of one tenth of an Inch in Diameter:
Is to $7\frac{1}{2}$ Ounces, the Force required to fold it on a Roller of one Inch Diameter::
So is 6 Pounds, when ſo much only ſtretches the Rope:
To 2,25 Ounces (that is, two Ounces and a quarter) the Force able to fold the ſame Rope, only ſtretched with 6 ℔, round a Cylinder of one Inch Diameter.

BUT as the firſt Pulley is not of one, but two Inches Diameter, we muſt diminiſh the Force neceſſary to fold the Rope in a reciprocal Proportion of thoſe Diameters, by the next Analogy, which will alſo ſerve for the two other Pulleys.

As the Diameter of the Pulley, where we want to know the folding Force:
To the Diameter of the Pulley where the Force is known::
So is the folding Force before found:
To the folding Force required.

That is, as 2 : 1 :: 2,25 : 1,125 Ounces for the 1ſt Pulley;
And, as 1,5 : 1 :: 2,25 : 1,5 Ounces for the ſecond Pulley;
And laſtly, as 1,25 : 1 :: 2,25 : 1,8 Ounces for the 3d Pulley.

By ſuch Analogies we may encreaſe the little Table above to any other Proportions, ſo as to ſhew by Inſpection the Force required to

* It is true, that in Motion, one Rope bears more of the Weight, and another leſs of it; but, upon the Whole, the different Difficulties of folding the Ropes taken together will amount to the very ſame. So that this is the beſt, becauſe the eaſieſt way of conſidering it.

to fold Ropes in most Cases; or most Cases may be deduced from this Table itself. Lect. IV.

BUT to go on with the Theory of the Friction of our Machine: These three last Frictions, or Resistances of Ropes added together, make 4,425 Ounces, which added to 21,41 Ounces the Friction above found, gives in all, 25,835 Ounces, a Friction greater by near six Ounces than the Experiment gave. But I have demonstrated in the Annotation to my third Lecture (*Page* 169.) that, *When a String or Rope runs over a single Pulley or Roller by the Descent of the preponderating Weight (the other Weight rising at the same time) the Pressure on the Axis of the Pulley is always equal to the Quadruple of the Product of the Weights multiplied into one another, and divided by the Sum of the same Weights.* And that Pressure being always less than the Sum of the two Weights, when they are unequal, so much as it is less must be taken out of the Account of all the Frictions, and the Experiment then will be extremely near the Theory. But to omit nothing in our Calculation, we will also examine what this Diminution of Pressure is.

THE Power (6 Pounds) together with what we have found necessary to be added to it on account of all the Frictions and Resistances of the Ropes, is to be look'd upon as the preponderating Weight in the Case of the Proposition abovementioned, that is, 6 Pounds and near 26 Ounces, or 122 Ounces: and 6 Pounds without any Addition is to be considered as the Weight overpoiz'd on the other Side of the first Pulley 1, which is an upper Pulley. Let the two Weights be multiplied together, that is, 6 Pounds or 96 Ounces × 122 Ounces, the Product of which is 11712 Ounces; which again multiplied by 4, or 11712 × 4, gives the Product 46848; then dividing the last Product by 218, (the Sum of the Ounces in both Weights) or $\frac{46848}{218}$, the Quotient will be 214,9, which subtracted from 218 the Sum of the Weights, will give 3,1 Ounces the diminished Pressure, or that Part of the Pressure which the Pin of the Pulley 1 is freed from when the Power preponderating runs down. Pl. 18. F. 5.

NOW as there is another upper Pulley over which the Rope also runs, there must be likewise taken off on that account 3,1 Ounces: So that if these two last Sums, or 6,2 Ounces be taken from 25,835

Lect. IV. Ounces, found by Theory equal to all the Frictions, there will remain 10,635 Ounces, the Addition necessary to make the Power over-balance the Weight with the least Augmentation possible; and that in the Experiment is 0,365, or a little more than one third of an Ounce; for 20 Ounces added to the Power 6 Pounds, makes it run down.

N. B. *Nothing was here allow'd as an Increase of Friction on account of the Weight added to bend the Ropes, which would still bring the Experiment nearer the Theory.*

In Practice we need not make this last Allowance, or consider this Diminution of Pressure, especially in Tackles of many Sheeves, because there are generally some Hindrances more than the common Friction; as for Example, when Sheeves rub against the Sides or Cheeks of the Blocks, or when their Hole wears bigger, which increases the Friction as much as if the Pin was become so much bigger.

Being willing to try how far the Theory of Friction and bending Ropes would agree with such Tackles of Pulleys as are commonly used in Building, and consequently be useful to direct our Practice, I made the two following Experiments.

Experiment I.

I took a Tackle of five Brass Sheeves in Iron Frames or Blocks; that is, three Sheeves in the upper Block, and two in the lower. Having made an *Æquilibrium* by hanging one hundred and a quarter at the lower Block, and a quarter of an hundred at the running Rope, I added 17 Pounds and a half before the Power could go down and raise the Weight.

Experiment II.

Two hundred and an half being balanced by half an hundred, the Addition of 28 Pounds made the Power raise the Weight.

N. B. *The Sheeves were 5 Inches Diameter, the Pins half an Inch, and the Rope three quarters of an Inch.*

In the first Experiment 17 lb and an half exceeds by $4\frac{1}{2}$ lb the Sum of the Frictions and Resistances deduced from the Theory.

But in the second Experiment, 28 lb exceeds the Sum of the Frictions, &c. not quite 2 lb. The Reason of this appeared to be, that the Rope at first was too big for the Cheeks that held the Sheeves; but in the second Experiment, where the Rope was more stretch'd, its Diameter became something diminished, and so brought off the Rope from rubbing so hard against the Cheeks. Lect. IV.

FROM knowing the Quantity of Friction in such large Tackles, we may know what to expect in Practice. For if one Man, who for a small time can exert the Force of 100 lb, thinks that he may draw up a Stone or a Roll of Sheet Lead, or any other such Weight to the Top of an House with a *Tackle of Five* (because this would seem feasible from mechanical Principles) will find himself mistaken on account of the Friction, which will not be surmounted without an additional Force of 50 lb.

I hope this Account which I have given of Friction and Hindrance to Motion in Mechanical Engines (however imperfect it is) may be of considerable use to direct such Persons as concern themselves with Engines and Manufactures. And to afford all the Help I can upon this Subject, I will give him some Considerations of the comparative Strength of Men and Horses, as well as the best Way of applying their Forces, being the Result of many Years Observations of my own, as well as what I have found in Authors who have treated of these Things.

AN Horse draws with the greatest Advantage, as we have already shewn from Mr. CAMUS, when the Line of Direction (being parallel to the Plane on which the Weight moves) is level with the Horse's Breast, and is able in such a Situation to draw 200 lb eight Hours a Day, and walking about two Mile and an half an Hour, which is about three Feet and an half in a Second. And if the same Horse is made to draw 240 lb, he can work but six Hours a Day, and cannot go quite so fast; and in both Cases, if he carries some Weight, he will draw better than if he carried none. We don't mean by this what an Horse can draw upon a Carriage; because in that case Friction is only to be overcome, so that a middling Horse well applied to a Cart, will often draw above 1000 lb; but so much as an Horse could draw up out of a Well over a single Pulley or Roller (made to have as little Friction as possible) is properly what an Horse can draw; and Horses, one with another,

Lect. IV. draw about 200 lb in ſuch a Caſe, as we ſaid before. To this may be referr'd the working of Horſes in all ſorts of Mills and Water-works, where we ought to know as near as we can, how much we make every Horſe draw, that we may judge of what the Effect will be when proper Allowance ſhall have been made for all the Frictions and Hindrances, before we cauſe any Machine or Mill to be erected.

When an Horſe draws in a Mill, Water-work, or Gin of any kind (in which the Horſe is made uſe of to draw round a Capſtern or *Axis in Peritrochio*) great Care ſhould be taken that the Horſe-walk be large enough in Diameter, otherwiſe the Horſe cannot exert all his Force as he goes round; for in a ſmall Circle or Horſe-Walk, the Tangent (in which the Horſe ſhould draw) deviates more from the Circle in which the Horſe is obliged to go, than it does in a great Circle. The Horſe-Walk ſhould not be leſs than 40 Feet in Diameter, when ever there is room for it; and the ſame Horſe loſes of his Force conſiderably in a ſmall Walk, becauſe he pulls in a Chord of the Circle, drawing the horizontal Beam behind him at acute Angles, ſo much, that in a Walk of 19 Feet Diameter, I have known an Horſe loſe 2 fifths of the Force that he exerted in a 40 Feet Walk. Moſt of the Mill-Wrights in *London* (and I believe in moſt great Cities) do not love to make large Horſe-Walks, even when they have room; becauſe, as there is generally want of room where they have been obliged to ſet up Works, they have accuſtomed themſelves to make their Geers for ſmall Horſe-Walks, and think it enough to give the ſame proportional Velocity to the Power and Weight, as is done in a larger Horſe-Walk, (becauſe if the Cog-wheel be ſo much leſs in the Diameter as the Horſe draws nearer to the Center, the Difficulty of drawing, were it not for the twiſting of the Horſe, would always be the ſame;) not conſidering the Strain put upon the Horſe: or when by Practice they have found how much a Horſe may eaſily draw, with all the Diſadvantages which the ſudden turning gives him, they won't take the Advantage which more room might give in removing that Difficulty, becauſe they don't care to go out of the Way which they have been accuſtom'd to. But ſuch Mill-Wrights, as have work'd at Coal-pits and Mines, know better, as they have been uſed to large Horſe-Walks in Coal-fields, *&c.*

I have

I HAVE often found that five Men are equal in Strength to one Horse *, and can with the same Ease push round the horizontal Beam in a 40 Feet Walk; but three of the same Men will push round a Beam in a 19 Feet Walk, which an Horse (otherwise equal to five Men) can but draw round. Lect. IV. * Ann. 6.

THE worst Way of applying the Force of a Horse is to make him carry or draw up Hill; for if the Hill be steep, three Men will do more than a Horse, each Man will climb up faster carrying 100 lb Weight, than a Horse that is loaded with 300 lb. This is owing to the Position of the Parts of a Man's Body, which are better adapted to climb than those of a Horse.

IT follows from this Observation, that those who have thought to gain great Advantage from the Weight of a Horse by applying it to an Engine to work the Forcers of Pumps, have not in the Execution found what they expected from a Calculation of the Weight of that Animal, because at every Step the Horse is really climbing up Hill, when his Weight is made use of; and therefore he goes the slower.

AS a Horse from the Structure of his Body can exert most Force in drawing horizontally in a strait Line, a Man can exert least Force that Way; as for Example, if a Man weighing 140 lb walking by a River or Canal Side, draws along a Boat or Barge by means of a Rope coming over his Shoulders, or any how fastened to his Body, he cannot draw above 27 lb, or about the seventh Part only of what a Horse can draw in that Case: for the whole Force that a Man exerts in that Action intirely depends upon his Weight, and not his whole Weight neither, only about $\frac{8}{14}$ Parts of his Weight, acting obliquely too, pushing him forwards as he stoops, produce the whole Force whereby the Man draws the Barge along, as has been demonstrated by Mr. *de la Hire*, in a *Memoire* which he presented to the *Royal Academy of Sciences* at *Paris*, in the Year 1699; of which I have given a Translation in my Notes †.

† Ann. 8.

HIS other Reasoning about the Application of the Force of a Man is just; but his *Data* not being true, some of his Conclusions, tho' truly drawn from his *Data*, are not true in Fact; therefore I have in the same Note given Remarks upon what he has said.

IN

Lect. IV. In drawing a Barge in the manner above-mentioned, a heavy Man (provided he be not unwieldy) will do more than another, unless he carried Weight proportionably, and the higher the Weight is carried, the better.

When a Man turns an horizontal Roller or Windlass by a Handle or Winch, he shall not have above 30 ℔ Weight acting against him, if he is to work ten Hours a Day, and raise the Weight about 3 Feet and an half in a Second, which is the common Velocity that a Horse draws with: I say 30 ℔, supposing the Semidiameter of the Windlass equal to the Distance from the Center to the Elbow of the Handle; for if there be a mechanical Advantage, as there usually is, by having the Diameter of the Axle, on which the Rope winds, four or five times less than the Diameter of the Circle describ'd by the Hand, then may the Weight (taking in also the Resistance on account of the Friction and Stiffness of the Rope) be four or five times greater than 30 ℔, that is, so much as it rises slower than the Hand moves.

In this Operation the Effect of a Man's Force varies in every Part of the Circle describ'd by the Handle. The greatest Force is when a Man pulls the Handle upwards from about the Height of his Knees, and the least Force when (the Handle being at top) a Man thrusts from him horizontally; then again, the Effect becomes greater as a Man lays on his Weight to push down the Handle: but that Action cannot be so great as when a Man pulls up, because he can lay on no more than the whole Weight of his Body, whereas in pulling he can exert his whole Srength. Lastly, a Man has but small Force to pull the Handle horizontally towards him when it is at lowest. Let us, as Monsr. *de la Hire* does, suppose a Man of moderate Strength to weigh 140 ℔, he may in the four principal Parts of pushing and pulling in the whole Circumference of Motion exert the following Forces, *viz.* in the strongest Point a Force equal to 160 ℔; in the weakest, a Force equal to 27 ℔; in the next strong Point, 130 ℔; and in the last or second weak Point, 30 ℔. Let us add all these Forces together, which will make 347, and divide them by 4, and we shall have 86 ℔ $\frac{3}{4}$; and this gives us the Weight that a Man might lift by a Winch, if he could exert his whole Force continually without stopping to take Breath; but as that cannot be, the Weight must return and over-power at the first

first weak Point, especially when the Handle moves slowly, as it must if a Man would exert his whole Strength all round. Besides, for raising such a Weight we must suppose the Man to act always along the Tangent of the Circle of the Motion, which does not happen in the Operation. Then there must be a sufficient Velocity given, * that the Force applied at the strong Points may not be spent before the Hand comes to the weak ones, so that it is difficult for a Man to continue that irregular Motion; and therefore when there are no other Advantages, the Resistance ought to be but 30 ℔: and even that could not be supported at the weakest Point, were it not for the Force remaining from the strong Point. Lect. IV. *Ann. 9.

If two Men work at the End of a Roller or Windlass to draw up Coals or Ore from a Mine, or Water out of a Well, they may more easily draw up 70 ℔ (still supposing the Weight and Power to have equal Velocities) than one Man can 30 ℔, provided the Elbow of one of the Handles be at right Angles to the other; for then one Man will act at the strong Point, when the other acts at the weak Point of his Revolution; by which Means the two Men will mutually and successively help one another. The common Way is to put on the Handles opposite to one another, which cannot give the Advantage above-mentioned; tho' there is some little Force gain'd even in that Position, because one Man pulling while the other thrusts, works at the strongest of the two weak Points, whilst the other works at the weakest, and so helps him a little.

There is indeed a Way to make a Man do a third Part more Work with a Windlass when the Motion is pretty quick, as about 4 or 5 Feet in a second †, and that is by the Application of a *Fly*, which is a Cross with Leaden Weights at its Ends, or rather (what is much better) a heavy Wheel at right Angles to the Axis of the Windlass or Roller. By this means, the Force of the Power, which the Man would lose, is kept in the Fly and equally distributed in all the Parts of the Revolution; so that for a little while a Man may act with the Force of 80 ℔, that is, overcome a continual Resistance of 80 ℔; and work a whole Day when the Resistance is but 40 ℔ ††. † Ann. 9.

When

†† The Fly may be applied to several Sorts of Engines, whether moved by Men, Horses, Wind or Water, or any animate or inanimate Power; and is of great use in those Parts of an Engine which have a quick circular Motion, and where the Power or the Resistance act-

Lect. IV. When a Man carries a Weight or a Burthen upon his Back, he exerts a great Force very effectually, many Muscles at once being employ'd in that Operation; the Muscles of his Neck, Back and Loins keep his Body and Head in the proper Position to sustain the Weight; those of his Shoulders and Arms help to keep it in its Place; and the Muscles of the Legs and Thighs raise the Weights of all the Body and Burthen as the Man walks along. In this Way of working, three Men do much more than an Horse, and two oftentimes do as much, nay even more, as may be observ'd in the daily Labour of the *London* Porters *. A Porter will carry 200 ℔, and walk at the rate of three Miles an Hour: A Coal-heaver or Porter that carries Coals, will carry 250 ℔, but then he does not go very far before he lays down his Burthen; tho' on the other hand he will often go up Stairs with that Weight. Chairmen do not act with all the very same Muscles as Porters, but as they have Straps brought down from their Shoulders to the Poles of the Chair, the Muscles of the Loins and Back are concern'd, and likewise the Extensors of the Legs and Thighs: Two of them will walk very fast with 300 ℔ (that is 150 ℔ each) at least at the rate of four Miles *per* Hour. Whereas a Carrier's Horse, that goes but about 2 Miles *per* Hour, carries only 224 ℔; or

* Ann. 10.

act unequally in the different Parts of a Revolution. This has made some People fancy, that the Fly adds a new Power; supposing that a Fly join'd to an Engine that is to move round does help to carry it about. But tho' it may be said in some Way to facilitate the Motion, by reason that it makes it more uniform and equal, yet upon the whole it causes a Loss of Power, and not Increase. For first, It requires a continual Supply of Power to put the Fly in Motion to a certain Degree of Velocity, and to keep and maintain it in that Velocity; for the Fly has no Motion of its own, but what it receives from the impress'd Force. Secondly, The rubbing and wearing of the Pivots or Gudgeons of the Axis do still hinder and lose the impress'd Motion. And thirdly, The Air thro' which the Weights at the Ends of the Fly do move, do also hinder the Motion thro' it, (tho' less when the Fly is circular;) and both these Impediments together, if the Fly be not still supplied with new Power, will make it stand still and be at Rest.

So that the Fly can of itself add no new Power to the Motion of the Engine to which it is applied, more than what is received from the first Mover that impress'd the Motion on it, but loses even some of the first Motion.

But the Reason how it becomes convenient and useful in many Engines (as we have shewn in the Windlass or horizontal *Axis in Petritrochio*) is this——That, whereas either the Powers exerted by the Engine are intermitted or unequal, and so the Motion is more difficult in one Part of the Revolution than another, or perhaps the Strength of the Man, or any other Power to be supplied, cannot be so well applied to one Part of the Revolution as it can to another: In these Cases the Fly becomes a Moderator, and makes the Motion of Revolution almost every where equal, tho' the Resistances are unequal, and the Forces impress'd are unequal; for that it has accumulated in itself a great Degree of Power, which it equally and gradually exerts, and as equally and gradually receives; whence making the Revolution in all Parts pretty near uniform, it becomes more pleasant, easy, and convenient to be acted and mov'd by the Impelling Force; which is the whole Benefit which is procur'd by this mechanical Engine, this Way applied. But I shall speak of the Fly and some other of its Uses in another Place.

or sometimes, when the Roads are very good, and the Horses strong, * 270 ℔. Lect. IV.

* Ann. 11.

Mr. *Richard Newsham* Engineer, of *Cloth-Fair* near *Smithfield*, has contriv'd his Engines to put out Fires in such a Manner, that Part of the Men that work them exert their Force by treading, which is more effectual than any other way that Men can work at such Engines, the whole Weight of the Body being successively thrown on the Forces of the Pumps; and even part of a Man's Strength may be added to the Weight by means of horizontal Pieces, to which he can apply his Hands when he is treading: Whereas, by applying the Hands to move Leavers or turn Winches, the Power must act very unequally. This is the Reason why with the same number of Men he has generally thrown Water farther, higher, and in greater Quantities, with the same-siz'd Engines, than other Engineers who have tried their Engines against his.

N. B. *His Engines have several Conveniencies peculiar to them, which makes them preferable to all others that I ever saw for extinguishing accidental Fires; but I reserve their Description for another part of my Book.*

The last and most effectual way of a Man's working, is the Action of *rowing*; wherein a Man acts with more Muscles at once for overcoming the Resistance, than in any other Position; and as he pulls backwards, the Weight of his Body assists by way of Leaver.

COROLLARY.

From the Consideration of the several Ways of a Man's acting in the Way of Labour, compar'd with the last, we may see how much People are mistaken who think to row a Galley, Boat or Barge, by vertical Oars fix'd on a horizontal Axis like a Mill-Wheel, the Men working this Machine by *heaving* at a *Capstane*; or turning Winches within the Vessel. For this will always be bringing the Men from easier and more effectual, to harder and less advantageous Work; as has been found by a great many more Engineers than will own it, and will be found by all that shall ever try it, be the Machine made in any Shape whatever, unless when the Men work in a rowing Posture.

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A PRODIGIOUS Force may be exerted by the Muſcles of the Legs and Thighs, to raiſe an immenſe Weight a ſmall Height; but as that Operation cannot be continued and brought to daily Labour, I refer the curious Reader to the Notes, where that is more fully explain'd *: Neither do I here take Notice of digging, hammering, cleaving of Wood, or any of the laborious Operations of Handicraft Trades; becauſe ſome Men are much more dextrous than others; and the ſame Man by long Uſe becomes ſo perfect in one way of working, that by an acquir'd Sleight he ſhall do twice the Work that an unexperienced Perſon can do, and yet not employ half ſo much Strength. But this is properly *Craft*, and not *Labour*, which laſt was all I meant to conſider.

* Ann. 7.

ANNOTATIONS *on Lecture* IV

Annotat. Lect. IV.

1. PAGE 192 —— [*The Friction is equal to about one third of the Weight*, &c.] There are ſome Caſes wherein the Friction does not come up to a third Part of the Weight of the Body which rubs; but as in moſt Caſes it does, I choſe to give that Proportion as the Foundation of the Calculation of the Friction of an Engine compos'd of many Parts before we erect it, eſpecially in regard to the Manufactures, becauſe the wearing of the Parts of Machines will in time alter their Figure and increaſe the Friction. And it will be better in Practice to find the Friction to be leſs than in the Theory, tho' that ſeldom happens, when calculated from a third of the Weight.

2. [Page 194. —— *It would require one third of the Weight of the Sledge*, &c.] In the Table of the Frictions of Sledges (*Page* 203, 204 and 205) quoted from Monſ. *Camus's* Experiments made on ſmall Models, there are more Caſes wherein the Friction is leſs, than where it is more, than one third of the Weight; but it is to be obſerved, that in all his Experiments the Sledge is in motion. But as I mentioned in my laſt Note, I rather choſe to continue the Proportion of a third on account of Accidents, beginning to draw from Reſt, and meeting with Rubs and Unevenneſs in the Streets, *&c.*

3. [Page 211. —— *We cannot expect that any Carriage to bear Weight can have ſo little Friction*, &c. — *the Manner of remedying Friction*, &c.] If the Axles in ſome Carriages being made of Iron, and in others only clouted (that is covered) with Iron, were to run in braſs Boxes fix'd in the Naves of the Wheels, they would go ſo much eaſier, and wear ſo much longer, without danger of firing, as to make full Amends for the extraordinary Expence. Where People are curious and don't value Coſt, as in Chaiſes and ſome Chariots, an Iron Axle being leſs in Diameter will have leſs Friction in proportion as it is ſmaller, and laſt very long if it turns in Boxes of caſt Braſs; but the Gudgeon in the Box muſt be ſufficiently long, which it may be without increaſing the Friction, as has been already prov'd, but will be more clearly evinc'd by Experiments made on a Machine which I ſhall deſcribe in this Annotation. If the Gudgeons are but two or three

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Inches long, as some have made them, thinking thereby to diminish the Friction, they will only wear two or three times faster than if they had been made four or six Inches long.

A Wheel of an Engine whose Axis is fix'd to, and turns with, the Wheel, may have the Friction of its Gudgeons diminished in any Proportion. As for Example, under the Iron Gudgeon *Gg* †, which we will suppose here of an Inch in Diameter, let there be two brass Rollers A, B of eight Inches Diameter each, whose Axes C, D, are horizontal and parallel to the Axis of the Wheel: The Rollers, which consequently are vertical (as may be seen in *Fig.* 7. ‡) are about an Inch thick, (or more if you will) and not in the same Plane, but one a little before the other, and parallel to it. Upon these Rollers is supported and turns the Gudgeon *Gg* of the great Wheel. *In this Case the Friction of the Gudgeon becomes eight times less than if it mov'd in the common Brasses*; for if we suppose the Gudgeon to move in the Direction *Gg*, it will not quit the Part *g* of the Wheel A on which it bears, to go to another bearing with its touching Part, as it happens when it turns and makes a Friction in the common way, but it brings along with it the Circumference of the Wheel or Roller A, turning it about in the Direction *Ag*, whilst its other touching Part G in its turning does also carry round the Roller B in the Direction G B; and these Rollers wou'd follow that Motion without any Friction, were it not for the bearing of their own Iron Axes C, D, in their Brasses. So that the Friction is transferred from the Gudgeon *Gg* to the Axes C, D, where the Velocity of the rubbing Parts being eight times slower than it wou'd have been at *Gg*, the Friction must be eight times less, as we have † shewn already. Now tho' these Rollers have four bearing Points, and the other two which support the other Gudgeon of the great Wheel's Axis have also four bearing Points, the Friction is no greater, than if there was but one bearing, because each of these Points sustains but an eighth part of the Weight. *Therefore the Friction is made eight times less by means of these Rollers: which was to be prov'd.*

† Pl. 18. F. 6.

‡ Pl. 18. F. 7.

† Page 193, 194.

SCHOLIUM.

If the Axes of the Rollers are made but of half the Diameter of the Gudgeon, as at E and F, then the Friction will be 16 times less; and yet they will be sufficiently strong: for as a Cylinder of half an Inch Diameter is equal in Strength to the fourth part of one of an Inch, the four Ends of the Axis of two Rollers at one End will be equal in Strength to the Gudgeon, and so at the other End.

COROLLARY.

Hence follows, that if the Ends of the Axes of the Rollers were each of them supported by two other such Rollers, the Friction would again be diminished 16 times more. Let us for Example suppose a Wheel to be six

Foot

Foot in Diameter, and to weigh 648 lib. the third of that Weight for Friction is 216 lib. which must be divided by 72, because the Diameter of the Gudgeon is so many times contained in the Diameter of the Wheel, and we shall have 3 lib. for the Friction of the Wheel upon common Brasses; but this Number must be divided by 16 times 16, that is, by 256, on account of the Rollers bearing on Rollers, which will thus reduce the Friction to $\frac{3}{256}$ lib. or little more than one 85th part of a Pound, or 3 Drams of 16 to the Ounce. Annotat. Lect. IV.

4. [Page 227. ——*the Friction on the Axle*, &c.] Since, we have said and already prov'd *, the Friction arises from the Weight that presses the Parts together, and not from the Number of Parts that touch; there is no Occasion for shortening the Axes of Wheels, either in Carriages or any other kind of Wheels, in order to diminish the Friction: for that will not only fail of the desir'd End, but make the rubbing Axis wear out much faster; and in Clock-work it is of bad Consequence, because when the Holes are counter-sunk to shorten the bearing, the Holes soon wear too big; whereas, if they were made only cylindrical, the Thickness of the Frame-Plate, the Friction wou'd be no more, and the Pivots wou'd wear much longer: and indeed it is now what all good Clock-makers practise; for if they counter-sink, it is but a small Depth to hold Oil. Experiments on the following Machine will make this more sensible and evident. * L. 4. pag. 192.

Plate 18. *Fig.* 8.

On the flat brass Plate A B C (which is here represented almost as big as the Machine) are fix'd two upright Plates D and E, with a Slit in one of them at D, and a Hole in the other between the Letters K and L, to receive the small Pivots at the Ends of the Axis D K of the Wheels Z F L G. But those Pivots, which are but about one 30th of an Inch in Diameter, do not bear on the Hole at K and the bottom of the Slit at D; for they are supported upon two circular Plates or Rollers at each End of the Axis, *viz.* The vertical Plates M 1, M 2, M 3, M 4, in the manner describ'd in the last Note, and represented at *G g*, *Fig.* 6. † and the Section of one of the Rollers or Plates is shewn by *Fig.* 7. So that when the Wheel turns round one way, all the rolling Plates turn the contrary way as freely as if the Pivot had been a Pinion, and the Wheels or Plates had been tooth'd, because the Axes of the Plates have very small Pivots which turn in very smooth Holes, that are made and polish'd in the four upright fix'd Plates or Cocks N 1, N 2, O and P; which last Cock has only the Corner of its Basis visible in the Figure, the Wheel M 2 hiding it in this Situation of the Machine. The little Cocks, such as D, *d*, *d*, serve for the Ends of the Pivots to bear against (both for the great Wheel and the four little Wheels, there being ten of them, of which only three are visible here) and by that Resistance against the Ends, the Shoulders as C C, *Fig.* 7. * will never rub at all. † Pl. 18. F. 6. 7. * Pl. 18. F. 7.

By

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By this means the great Wheel has so little Friction, that if we apply a Finger to its Circumference to put it briskly in motion, any Point of its Circumference will go more than the space of a Mile before the Wheel stops: for tho' one cannot count the Number of Revolutions of the great Wheel by looking at it, yet one may know their Number by looking at the Holes which are left in the small Wheels (one in each) for that purpose; for they being two Inches Diameter, turn but once whilst the great Wheel (whose Pivots give them Motion) turns sixty times.

The Friction of the great Wheel thus becoming so little, as to be in a manner insensible, it is fit for the Purpose intended. Then on the top of the Cock or upright Piece Q R, screw'd down fast by its Base or Foot Q, is fasten'd by a Screw at R one End of the Spiral Spring S 1, S 2, S 3, S 4, whose other End is fasten'd to the Axis of the great Wheel near S 4. Now if the Wheel be made to turn on its Pivots, by bringing the Point Z of its Circumference towards the Index Y, which points to Degrees on the Edge of the Wheel, (but not represented here except by the Dots mark'd from Z towards Y) as soon as we let it go, it will return towards Z, and make several Vibrations backwards and forwards, like the Balance of a Watch, for a long time: but if any thing bears against the Axis H I D, which is truly cylindrical and one 4th of an Inch Diameter, then there will be fewer Vibrations in proportion to that Friction. Now to shew that the Friction is proportional to the Weight that presses on the Axis, and not the Surface, the following Experiments are made.

Experiment I.

†Pl. 18. F. 9. Take the Piece of *Figure* 9 †, weighing half an Ounce (being made in the Form of a flat Cross, and after filing, ground smooth upon an Oil-stone on the flat Under-side of V, with a little Hole at the End T, and a small solid round Piece hanging down at the opposite End X) and lay it over the Axis of the Wheel betwixt I and D, so that the Weight of the hanging End X, drawing down the flat V upon the said Axis, makes the End T rise with its little Hole against the Point or lower End of the Screw T of the Cock *t* T, which keeps it in its place when the Axis turns round under it; then having brought Z to Y, or drawn back that Point against the Bent of the Spring about 90 Degrees, observe the Number of Vibrations that the Wheel makes before it stands still by the Friction of the Cross, in the Situation represented *Pl. 18. F. 8. by faint Lines, at T V X, *Fig.* 8 *. Suppose the Number of the Vibrations †Pl. 18. F. 10. be 50; then let the Cross of *Fig.* 10 †, be put on in the same manner and at the same time just by the other, and kept in its place by the bottom of the other Screw *t*; as it is exactly the same in Figure and Weight as the first Cross, and it will add as much more to the Friction of the Axis, the Weight pressing as well as the Surface rubbing being doubled; which appears by giving Motion to the Wheel as before, because then the Number of Vibrations will be but 25.

Expe-

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EXPERIMENT II.

Instead of the two Crosses abovementioned, put on the Cros of *Fig.* 11 *, which weighs one Ounce (that is, as much as the two others) but has the Surface under V exactly smooth'd and polish'd like the others: Then put the Wheel in motion as before, and the Vibrations will be but 25 in Number, tho' the Surface rubbing is but half, because the Weight is the same. And this is further prov'd by *Pl. 18. F. 11

EXPERIMENT III.

In which, the Piece X must be unscrew'd from its Place, and screw'd again in the same Hole on the other side of the Cros, to bring the little Flat of the Prism at V to bear upon the Axis of the great Wheel instead of the broad Part of the Cros. Then also will the Wheel lose its Motion after 25 Vibrations, tho' the bearing Surface be above twenty times less than when both the first Crosses were on; because the Weight is the same.

5. [Page 242.——*Should alter the manner of working*, &c.——*instead of gaining an Advantage, he must call in more Men to perform the Work.*

Some have endeavour'd to render this Engine more useful, by causing it to roll upon an inclin'd Plane instead of making it rise directly up in the manner describ'd, and condemn'd in my Account of it. I thought proper to shew here, what must be the Loss of the Power in proportion to the Inclination of the Plane.

I say therefore, *That in every Inclination of the Plane, if the Sine of the Angle of Inclination be taken in Parts of the* Radius *of the Axle, or Roller, the Power will be to the Weight*:: *as the* Radius *of the Roller + the Sine of Inclination*: *to the* Radius *of the Wheel, — the said Sine of Inclination; that is, in the* Fig. P (= 1): W (= 3):: d k: a k. † *Plate* 20. *Fig.* 10. †Pl. 20. F. 10.

In the present Experiment B E is an inclin'd Plane, on which the Roller C is to roll up, touching the said Plane at the Point *c*; A M is the Wheel behind that Plane; another such Plane, and equally inclin'd, being also supposed behind the Wheel, to support the other End of the Roller.

The Lines of Direction of the Power and Weight being *a* P and *d* W; through the Point of Contact or Center of Motion *c*, draw A D parallel to the Horizon, and perpendicular to *a* P and *d* W; through the Center of the Engine C, draw *a d* parallel to A D. Suppose the Angle B *c* A of the Plane's Inclination to be 30°, the right Sine will then be equal to half the Radius; therefore dividing C 2 (the Radius of the Roller) into two equal parts at *k*, if you draw *k c* and C *c*, the Angle *k c* C will be equal to B *c* A, and its Sine will be C *k*. Now since it is evidently the same thing to make use of *a d* for a Lever, whose Center of Motion is at *k* as of A D equal and parallel to it with its Center of Motion at *c*; it follows, that, in this Inclination of the Plane, the Distance of the Weight *d k* is greater than *d* C (the Distance of the Weight in the common Use of this Engine) by

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by the Addition of this Quantity C *k*, the Sine of the Angle of Inclination; and *k a*, the Distance of the Power, is less than C *a* (the Distance of the Power in the common way) by the Subtraction of the said Quantity or Sine C *k*: consequently that on an inclin'd Plane, the Power is to the Weight:: as D *c*: to *c* A. *Q. E. D.*

COROLLARY I.

Hence it follows, that the Radius of the Wheel, and the Radius of the Roller being given, the Loss of Power may be found in any Inclination of the Plane. Thus, as here the Power, which in the common way wou'd be but one 5th of the Weight, must be one 3d part of it; so if the Angle of the Plane's Inclination was but 11° 32′, the Power wou'd be one 4th of the Weight, *&c.*

COROLLARY II.

Hence follows also, that if the Plane B E be horizontal, no Force of the Power will be lost, because *c g*: *c f*:: C G: C F.

SCHOLIUM.

As the Friction of the winding of the Ropes, such as B *c* in the new Way, is greater than the Friction of the Pivot in the old Way (besides the Friction of the Collars of the Counterpoise to the Engine) so that Friction diminishes, as the Ropes bear less Weight, according to the Diminution of the Angle of the Plane; and when the Plane is horizontal, and without a Counterpoise, even then the winding up of the Ropes, and Pressure of the Roller against the Plane, is equal to the Friction in the common way.

N. B. *I made the Experiment with Pivots twelve times less in Diameter than the Roller, and fine pliable Silk instead of Ropes.*

6. Page 253.——*Five Men are equal in Strength to one Horse*, &c.] Such *English* Authors as have compared together the Strength of Men and Horses at a Capstane, have found their forces to be in that proportion, as Sir *Jonas Moore* and others, *&c.* But the *French* Authors always make an Horse equal to seven Men; which I believe to be according to their Observations. Nay, and I have observed the Labourers in *Holland* (one with another) to work with a Force pretty near in that Proportion. So that we may say, — That five *English* Labourers are equal to an Horse, and only seven *French* Men or as many *Dutch* Men. But here we do not at all consider Skill, or Weight, which make a Man of the same Strength do much more than another. But in *Turkey* the Porters will carry twice more than the strongest *English* Porters, as we shall more particularly consider in the next Note.

7. [Page

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7. [Page 258.—*A prodigious Force may be exerted*, &c.] About 30 Years ago one *Joyce*, a *Kentish* Man, famous for his great Strength (tho' not quite so strong as the King of *Poland*, by the Accounts we have of that Prince) shewed several Feats in *London* and the Country, which so much surprized the Spectators, that he was by most People called the second *Sampson*. But tho' the Postures which he had learned to put his Body into, and found out by Practice without any mechanical Theory, were such as would make a Man of common Strength do such Feats as would appear surprizing to every body that did not know the Advantage of those Positions of the Body; yet no body then attempted to draw against Horses, or raise great Weights, or to do any other thing in Imitation of him; because, as he was very strong in the Arms, and grasped those that try'd his Strength that way so hard, that they were obliged immediately to desire him to desist, his other Feats (wherein his manner of acting was chiefly owing to the mechanical Advantage gained by the Position of his Body) were intirely attributed to his extraordinary Strength.

But when he had been gone out of *England*, or had ceased to shew his Performances, for eight or ten Years; Men of ordinary Strength found out the Way of making such Advantage of the same Postures as *Joyce* had put himself into, as to pass for Men of more than common Strength, by drawing against Horses, breaking Ropes, lifting vast Weights, *&c.* (tho' they cou'd in none of the Postures really perform so much as *Joyce*; yet they did enough to amaze and amuse, and get a great deal of Money) so that every two or three Years we had a new *second Sampson*.

About 15 Years ago a *German* of middle Size, and but ordinary Strength, shew'd himself at the *Blue Posts* in the *Hay-Market*, and by the Contrivances abovementioned, pass'd for a Man of uncommon Strength, and got considerable Sums of Money by the daily Concourse of Spectators. After having seen him once, I guess'd at his manner of imposing upon the Multitude; and being resolved to be fully satisfied in the Matter, I took four very curious Persons with me to see him again, *viz.* the Lord Marquis of *Tullibardin*, Dr. *Alexander Stuart*, Dr. *Pringle*, and a mechanical Workman, who used to assist me in my Courses of Experiments. We placed ourselves in such manner round the Operator, as to be able to observe nicely all that he did, and found it so practicable, that we perform'd several of his Feats that Evening by ourselves, and afterwards I did the most of the rest as soon as I had a Frame made to sit in to draw, and another to stand in and lift great Weights, together with a proper Girdle and Hooks. I likewise shew'd some of the Experiments before the *Royal Society*; and ever since, at my Experimental Lectures, I explain the Reason of such Performances, and take any Person of ordinary Strength that has a mind to try, who can easily do all that the *German* above mentioned used to do, without any Danger or extraordinary straining, by making use of my Apparatus for that Purpose. I don't hear that any of these *Sampsons* have attempted since to impose upon People in the same manner in or near *London*.

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But now it will not be improper to mention what were the Feats of Strength which the *German* used to perform (for I never saw *Joyce*) and to shew, from the Make of the human Body, how easily any one may do the same.

1*st*, The strong Man I H L on the Frame A G B E F C D sat upon an
*Pl. 19. F. 1. horizontal Board, or rather inclining backwards as F G, * with his Feet against an upright immoveable Prop as D C F counterbrac'd at E, with a strong Girdle H round him a little below his Hips; to the Iron Rings of which Girdle was fasten'd a Rope by means of a Hook. The Rope went between his Legs and thro' a Slit of the Prop at L, and several Men at M N, or two Horses could not by their pulling, move him out of his Place.

N. B. *His Hands at* K *seem'd to pull, but were of no Advantage to him: nay, had he lifted the Rope ever so little with them, it would have been a Disadvantage: and tho' the Board on which he sat was (when I saw him) in the horizontal Position* F P, *it is much better and less dangerous to have it inclin'd as* F G, *and only an Hole at* L *for the Rope to run thro' instead of an opening from* L *up to* D, *as I shall shew in explaining this Operation.*

†Pl. 19. F. 3. 2*dly*, The same Man I H L † having fix'd the Rope above mentioned round a strong Post at R, and then pass'd it thro' a fix'd Iron Eye at L, fix'd it to his Girdle, and setting his Feet against the Post near the said Eye, rais'd himself from the Ground by the said Rope; which he broke by suddenly stretching out his Legs and fell backwards on a Feather Bed at B, laid on the Ground to catch him, that his Fall might not hurt him.

*Pl. 19. F. 4. 3*dly*, He lay down on the Ground in the Posture I H L *, with an Anvil K H on his Breast at H, upon which another Man M hammer'd with all his Force the Iron K with a Sledge Hammer; and sometimes two Smiths cut a great cold Iron Bar in two with Chizzels. Sometimes a great Stone, of which the half is seen at S, was laid upon his Belly and broke with a Blow of the great Hammer. But he had the Stone broke upon his Belly in the Posture of *Fig.* 5. which is much less dangerous when nothing is under the Back, than when a Man lies upon solid Ground, as we shall shew.

4*thly*, The pretended *Sampson* puts his Shoulders (not his Head, as he used to give out) upon one Chair and his Heels upon another, and supports one or two Men standing upon his Belly, raising them up and down as he breathes, making with his Back-bone and Thighs and Legs the Arch
†Pl. 19. F. 5. I H L †, whose Abutments are at I and L.

N. B. *The Stone of one and a half Foot long, one Foot broad, and five or six Inches thick, is laid on at* H *when it is to be broken by a Blow of a Hammer.*

*Pl. 19. F. 6. 5*thly*, He lies down on the Ground in the Posture I L H *, and the Man M standing upon his Knees, he draws his Heels towards his Breech, and so raises his Knees over C, and lifts up the Man gradually, till having brought his Legs perpendicularly under the Man, as in *Fig.* 7. he raises his

Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.

his own Body up, and putting his Arms about the Man's Legs, rises with him and sets him down on some low Table or Eminence about the Height of his Knees: and this he sometimes does with two Men; which is no difficult Performance. Annotat. Lect. IV.

6thly, He stands in the Frame ABCDEF*, and pretends to *raise up*, but does really *sustain*, a Cannon C laid on the Scale S *s*, the Ropes of the Scale being fix'd to a Rope or Chain L H, hanging at his Girdle H, his Assistant knocking away the Rollers R *r*, from under the Scale, when once he has fix'd himself so as to have his Ropes tight, and his Legs and Thighs quite streight. *Pl. 20. F. 2.

N. B. *It is very near as easy to break the Rope with the Eye* L *fix'd into the Ground or Floor by means of the Girdle* H, † Fig. 1. *as in the manner represented in the* * *third* Fig. *of* Plate 19. *But he never tried it that way*; *because it is so obvious, that several People would immediately have tried it too, and would find that there is no difficulty in breaking the Rope thus, as I have often done it; but by making the Fall backwards seem necessary in the Operation, few Persons care to try it this way.* †Pl. 20. F. 1. *Pl. 19. F. 3.

The † 4th *Figure* represents the Girdle made of strong double Horse Girt, with strong Iron Loops at G and R. The Hook is seen at *Fig.* 5; and the Position of the Iron Eye at *Fig.* 3. where you may observe that the Edge of the Eye, and not the open part, is towards the Post, so that the Rope does not easily slip thro' the Hole, but jams or stops in it, whereby the whole Strength of the Man's Efforts acts upon one part of the Rope, and so it is easily broke. †Pl. 20. F. 3, 4, and 5.

The Man likewise used to take a flat Piece of Iron of the Figure mark'd 7*, and twist it into a Screw. But his manner of doing it, made it very easy; for he first bent the Iron to a right Angle, as at *Fig.* 8. then wrapping his Handkerchief about the broad flat upper End of the Iron, he held that End in his left Hand, and with his right Hand applied to the other End twisted about the angular Point, as in *Fig.* 9. *Pl. 20. F. 7, 8, and 9.

N. B. *My Lord* TULLIBARDIN *took one of his Irons and did the same thing before him; and indeed what was harder, for he untwisted one of the Irons that the Man had twisted.*

In order to explain how the abovementioned Feats may be perform'd by Men of no extraordinary Strength, I have in the 6th Figure * drawn the lower part of a Skeleton, containing so many of the Bones of the human Body as are concerned in these Operations, making the Figure pretty large, to shew the better how the Girdle is to be applied. *Pl. 20. F. 6.

The Bones mark'd, I S A P A I (*a*) which compose the Cavity call'd the *Pelvis*, contain a bony Circle or double Arch of such Strength, that it wou'd require an immense Force to break them by an external Pressure directed

(*a*) These Bones are thus distinguish'd by Anatomists. S, the *Os Sacrum*; I I, the *Ilium*; AA, the *Os Ischium*; whose strongest part has on each Side an hemispherical Concave, in which the round Head of the Thigh-Bone is receiv'd and turns round, being held by a strong Ligament in its middle: those Parts of the Bone that join together before betwixt A A and above P are call'd the *Os Pubis* or *Ossa Pubis*.

Annotat. Lect. IV. rected towards the Center of the Circle, or the middle of the *Pelvis*. It is also to be observ'd, that those Parts of this bony Circumference, which receive the Heads of the Thigh-bone above, at, and below A, call'd the *Ischium* or *Coxendix*, are the strongest of all, so that a very great Force may push the Heads of the Thigh-bones upwards (or, which is the same thing, the upper Parts of the *Coxendix* downwards) or towards each other in a lateral Direction from A to A, without doing any Hurt to the human Body.

Now if the Girdle above described be put round the Body in the manner represented in the Figure, and be drawn downwards at G, by a great Weight W, it will press on the *Os Sacrum* behind, and the *Ilium*; then it will by its Pressure on T T, the great *Trochanters* of the Thigh-bones, drive the round Heads the faster into their Sockets, so as to make them less liable to slip out and strain the Ligament by a Push directed upwards. So that the semicircular Part of the Girdle, T C S C T presses together the bony Arch denoted by the same Letters, which according to the nature of Arches, is the stronger for that Pressure. The Abutments of the Arch cannot come nearer together by reason of the Resistance of the strong Bones A P A, neither can they fly outwards, because the Girdle keeps them together. Then the Thighs and Legs T D B are two strong Columns, capable of sustaining four or five thousand Pounds at least, provided they stand quite upright. The Muscles here are put to no Strain, being no farther concerned than to balance each other; that is, the antagonist Muscles, *Extensors* and *Flexors* only keep the Bones in their Place, which makes them resist like one entire Bone form'd into an Arch.

*Pl. 20. F. 2. This shews how easily the Man of *Fig.* 2. * may sustain a Cannon of two or three thousand Pound Weight. The same Solution will also serve for †Pl. 19. F. 1. the Resistance of the Man of *Fig.* 1. *Plate* 19. † whom five Men (nay ten Men or two Horses) cannot pull out of his Situation when he sits so as to have his Legs and Thighs in the horizontal Line P F, or in a Line inclining downwards towards A: for then, tho' there is a difference in the sitting Posture from the standing Posture before describ'd, yet by reason of the Mobility of the Heads of the Thigh-bones in the *Acetabula* or Cavities of the *Coxendix*, the Arch is the same and as strong as before, its Abutments being equally supported by the Legs and Thighs. It is only the bending of the Back-bone above the Girdle to bring up the Body which makes the Difference of Position in the Man, tho' not sensibly in the resisting Parts. The Impossibility of overcoming the Resistance of the Man that sits with the Girdle about him, without crushing his Legs and Thighs end-wise into one another, depends upon what has been said *Ann. 5. L. 3. in the * fifth Note of the third Lecture (*Page* 149) where we have shewn that a Power acts ineffectually upon a Leaver, when it draws it against the Center of Motion. This will be further explain'd by the 2d *Fig.* of *Plate* *Pl. 19. F. 2. 19, * where the Leaver H L, whose Center of Motion is at L, represents the strong Man's Legs and Thighs, the Power of the Men or Horses pulling at M, being applied at H, and drawing in the Direction H L. The same will happen when the Leaver is in the Position h L; but if the Man should

sit

sit with his Breech higher than his Feet, so as to have his Legs and Thighs in the Position of the Leaver *h* L, M *l h*, the Line of Direction of the Power wou'd make with the Leaver the Angle *l h* L, whose Sine being *l* L, would reduce the Action of the Power to the same thing as if the Weight of the Man was suspended at *h* the long *Brachium* of a bended Leaver *h* L *l*, and the Power should draw at the Point *l*, by the short *Brachium* *l* L. Then if the Power was to the Weight in a *Ratio* something greater than that of *h l* to *l* L, the Man wou'd be pull'd upwards in an Arc whose Center is at L, his Resistance decreasing continually; because then only his Weight would act against the Power, by the help of a Leaver which in its Motion would continually increase the *acting Distance* of the Power, and diminish that of the Weight. Now as this may happen to a Man sitting upon a horizontal Board, if his Girdle be a little too high, or he be pull'd on the sudden, before he is rightly fix'd and his Legs and Thighs are in their due Position; I would always advise that the Board should recline in the manner F G, * to prevent such a Surprize, which can hardly happen then, because the Point H must rise quite above the Line L P before the Power can gain any Advantage. Nay, for greater Security, instead of the Slit D L (or L *l* † *Fig.* 2.) I use only an Hole at L, for the Rope to pass thro', and always be between the Legs and Feet. Annotat. Lect. IV. *Pl. 19. F. 1. †Pl. 19. F. 2.

I have observ'd the pretended *Strong Man* sometimes to have a short strong Stick of about a Foot long tied to the Rope at K, that in case of a Surprize, that Stick might stop against the Props D and C, so as to prevent his being drawn any farther forward in such a Case; and then he held the Stick in his Hands, pretending to pull with his Hands to make the Trick appear the more strange.

But in breaking the Rope the Muscles must act in extending the Legs; and that we may the better explain that Action, we must consider a Man breaking the Rope as represented in the first *Fig.* of *Plate* 20. * that way being more simple than when it is broken in manner of *Fig.* 3. *Plate* 19. †. *Pl. 20. F. 1. †Pl. 19. F. 3.

The Rope being fasten'd to a Post at P, or any other fix'd Point, is brought thro' an Iron Eye L to the Hook of the Girdle H of the Man H I, and so fix'd to it by a Loop, or otherwise, as to be quite tight, whilst the Man's Knees are so bended as to want about an Inch of having his Legs and Thighs quite upright. Then if the Man on the sudden stretches his Legs and sets himself upright, he will with Ease break the very same Rope which held two Horses exerting their whole Strength when they draw against him; such as a Cart Rope, or a Rope of near three quarters of an Inch diameter, which may be broken by a Man of middling Strength, by the Action of the ten Muscles (*a*) that extend the Legs, five belonging to each Leg.

If

(*a*) The five Muscles that extend each Leg are describ'd by the Anatomists. Their Names are, 1. *Membranosus*, arising from the upper Part of the Spine of the *Os Ilium*, and inserted a little below the Knee into the outer and the foreside of the *Tibia* and *Fibula*. 2. The *Rectus* springing from the lower Part of the Spine of *Os Ilium*, and inserted also a little below the Knee into the foreside of the

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If the Rope is strong enough to bear 1800 lib. but will break by hanging a little more Weight to it; two Horses, or ten Men, cannot break it by fair pulling or drawing against the sitting Man of *Fig.* 1. *Plate* 19. *

*Pl. 19. F. 1.

For an Horse in common hard Work of six Hours a Day can only pull 240 lib. † he cannot draw more than double that Weight when he is whip'd and exerts himself; so that two Horses, or ten Men, equivalent to them, cannot with a Jerk draw above 1000 lib. whereas the Rope has been suppos'd strong enough to sustain 1800 lib. and yet it may be broken by an ordinary Man in the Posture of *Fig.* 1. *Plate* 20. *. Neither need we wonder that the Muscles *Extensors* of the Legs should exert so much Force, when we consider their Bigness and Length; especially if we compare them with the four Muscles that pull up the lower Jaw-bone (which tho' they all four do not weigh a Pound) yet enable some Men to crack an Apricot or a Peach-stone, which could not be broken without an immense Weight. ‡ See the 6th Note of our third Lecture, from *Page* 158 to *Page* 164; or more fully to satisfy one's Curiosity, one may consult *Borelli*'s Book *de Motu Animalium*, where he has shewn the particular Strength of the Muscles.

† Page 251.

*Pl. 20. F. 1.

‡ L. III. Ann. 6.

The manner of breaking the Rope, as represented in * *Fig.* 3. of *Plate* 19. tho' more troublesome, is also more effectual for breaking the Rope, than that which we have been describing; for the same Man may break a Rope in this Position, which he cannot break in the other. * To understand this we must observe, that the Man takes the Rope so short, that when he climbs up against the Post, if the Eye L (thro' which the Rope passes) be between his Toes, his Heels being lower at T, when his Knees are streight, the length of his Legs and Thighs T H is greater than the length of the Rope and Girdle from L to H; so that we may consider in the Man and Rope the Triangle *l h t* drawn below the Man in the Figure; the Side *l* h, representing the Length of the Rope and Diameter of the Girdle; the Base *l t*, the Man's Feet; and the longest Side *t* h the extended Limbs or the Legs and Thighs when streight. Now in the Rotation of the long Sides of this Triangle, when the Side *l* h comes to be horizontal at *l* r (moving in the Arc h r *r* round the Center *l*,) the Side *t* h will be in the Situation *t h* (as it moves round the Center *t* in the Arc h *h s*) and consequently either the Rope must stretch from r to *h*, or the Point h, by the bending of the Knees must be brought nearer and come to r; or else the Rope must break, which is what will happen, especially when we consider that the lower the Body (with the Limbs stiff) comes down round the Heels for the Center of Motion, the greater will the Distance r *h* be, as we may see a little lower at *r s*; so that if the Rope did stretch a little at first, it must break at last, and the Man fall down upon the Feather-bed, or

*Pl. 19. F. 3.

the *Tibia*. 3. The *Vastus Externus*, springing from the Root of the greater *Rotator*, and inserted a little below the *Patella*, near the same Place with the former. 4. The *Vastus internus*, which arises from the Root of the lesser *Rotator*, and likewise is inserted a little below the *Patella*. 5. The 5th is the *Crureus* springing from the forepart of the Thigh-bone, between the two *Rotators*, and ending in the same Place with the former.

Fig. 1.

Fig. 2.

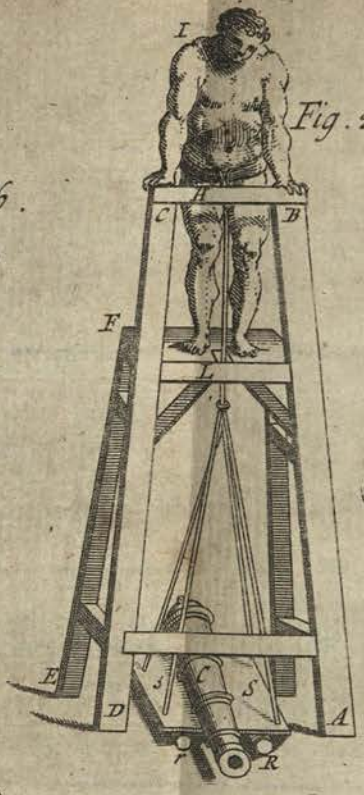


Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

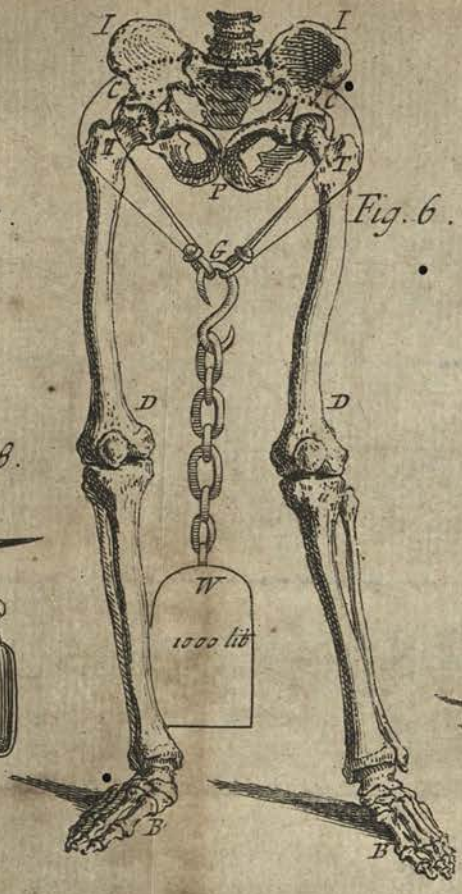


Fig. 7.

Fig. 8.

Fig. 9.

Fig. 10.

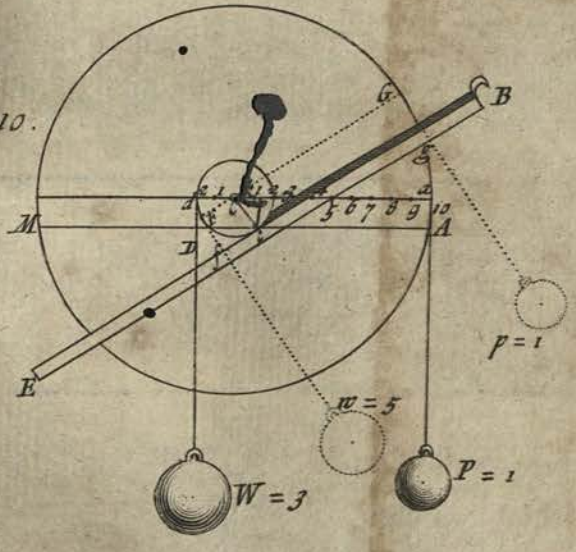


Fig. 11.

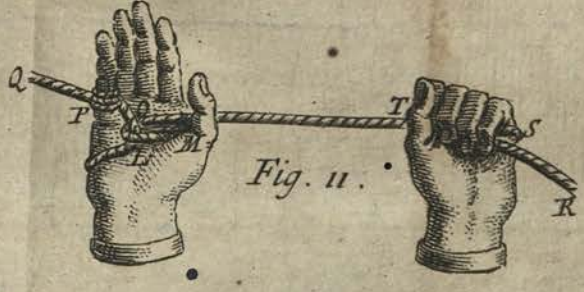
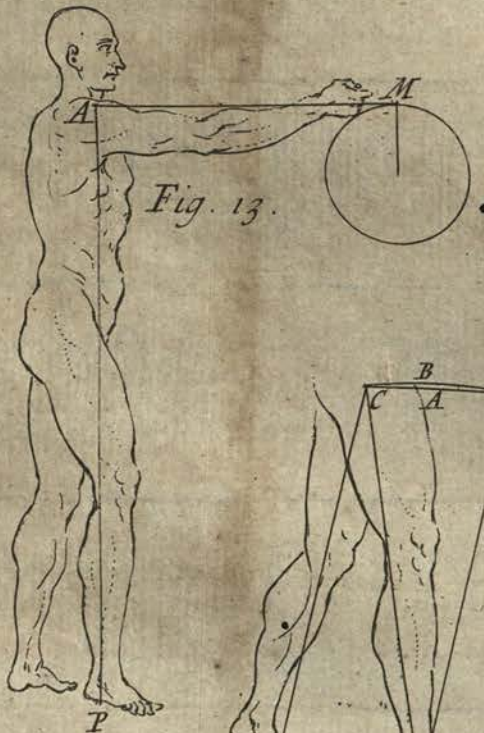


Fig. 12.

Fig. 13.



or other ſoft Body to receive him, at B. For if the Man finds that the Action of the Muſcles Extenſors of the Legs does not break the Rope, he can in this Poſition eaſily add the whole Weight of his Body with a Swing and a Jerk, as he throws himſelf backwards. Annotat. Lect. IV.

The Poſture of *Fig.* 4. *Plate* 19.* (where the ſtrong Man having an Anvil on his Breaſt or Belly, ſuffers another Man or two to ſtrike with a Sledge Hammer and forge a Piece of Iron, or cut a Bar cold with Chizzels) tho' it ſeems ſurprizing to ſome People, has nothing in it to be really wonder'd at; for ſuſtaining the Anvil is the whole Matter, and the heavier the Anvil is, the leſs are the Blows felt: And if the Anvil was but two or three times heavier than the Hammer, the ſtrong Man would be kill'd by a few Blows. This would be eaſily underſtood by calling to mind what we have ſaid in the ſecond Lecture; for the more Matter the Anvil has, the more *Inertia* and the leſs liable it is to be ſtruck out of its Place; becauſe when it has by the Blow receiv'd the whole *Momentum* of the Hammer, its Velocity will be ſo much leſs than that of the Hammer as it has more Matter than the Hammer. Neither are we in that Caſe to attribute to the Anvil a Velocity leſs than the Hammer in a reciprocal Proportion of their Maſſes or Quantities of Matter; for that would happen only if the Anvil was to hang freely, in the Air (for example) by a Rope, and it was ſtruck horizontally by the Hammer; but the Reſiſtance of the Ribs which make an Arch under it, will ſtill diminiſh that Velocity: So that if the Hammer ſtriking the Anvil when hung freely, could make it move an Inch out of its Place, by putting a Reſiſtance behind it equal to the Weight of the Anvil, it would move but half an Inch, and but a quarter of an Inch if that Reſiſtance was double, *&c.* Thus is the Velocity given by the Hammer diſtributed to all the Parts of a great Stone, when it's laid upon the Man's Breaſt to be broken; but when the Blow is given, the Man feels leſs of the Weight of the Stone than he did before, becauſe in the Reaction of the Stone, all the Parts of it round about the Hammer riſe towards the Blow; and if the Tenacity of the Parts of the Stone, is not ſtronger than the Force with which it moves towards the Hammer, the Stone muſt break; which it does when the Blow is ſtrong, and ſtruck upon the Center of Gravity of the Stone. *Pl. 19. F. 4.

N. B. *That the Parts of Bodies ſtruck move towards the Blow, is a Conſequence of a Law of Nature, the Explanation of which here follows.*

The common Experiment of laying a Stick with its Ends upon two drinking Glaſſes full of Water, and ſtriking the Stick downwards in the Middle, with an Iron Bar, ſo as to break the Stick without breaking the Glaſſes or ſpilling the Water, plainly proves the Fact; and the Obſervation of what happens in the Experiment, ſhews it to depend upon the firſt *Law of Nature* (Page 293.) For if the Glaſſes A, B, * have upon their Brims *Pl. 16. F. 8.

Annotat. Lect. IV. Brims the Stick A B, whoſe Center of Gravity at C, is ſtruck by a Bar, whoſe Section is ſeen at C, the Stick, by the firſt *Law*, endeavouring to remain in its Place of Reſt, has only its Tenacity to reſiſt the Blow of the Bar, which breaks the Stick by the Time it is got to *c*; and now the two Parts of the Stick muſt be in the Poſition *c a* and *c b*, and ſtill riſe, turning towards *C*; for M, *m*, the Centers of Gravity of the two Parts of the Stick endeavouring to remain in their reſpective Places, become the Centers of Motion of thoſe two Sticks; whence it happens, that as by the Continuance of the Stroke, they are both puſh'd downwards at *c*, their Ends *a* and *b* muſt by a circular Motion be carried upwards and towards C.

This is confirmed by making the Experiment the contrary way; for if you ſtrike the Stick upwards in the Direction *c* C, you will break the Glaſſes, if the Blow be ſtrong; and if the Blow be not very quick, yet you ſpill the Water.

There is alſo another Experiment, which makes this very evident.

*Pl. 16. F. 9. Let the Circle or Hoop of Lead A B C * be hung up by the String S G, and then ſtruck by the Bar above-mention'd at the Point B, in an horizontal Direction; the Effect will be, that the Ring will change its Figure from the Circle A B C D E F to the irregular Oval A *b* C *h* D *e* F *g*; ſo that not only the Parts of the Ring at A and C come towards the Blow, while the Bar B (repreſented by its ſquare Section) moves from B to *b*, but alſo the fartheſt Part of the Ring E comes forward to *e*, the Points A, C, F, D, becoming Centers of Motion for the Arcs (or Quadrants) B G, B H, E G and E H, to be put into the Poſitions *b g*, *b h*, *e g*, and *e h*, in which Caſe they muſt alter their Figure, becauſe they are not ſeparated from one another at the Points B, H, E, G, *&c.*

I ſhould be too tedious to be as particular in my Explanation of the other Feats of Strength as I have been in theſe above-mention'd. Therefore I will only lightly conſider the following ones; eſpecially, ſince the Principles already explain'd in the Lectures paſt, and the Conſiderations juſt mention'd, will enable any one eaſily to diſcover the Reaſon of all ſuch Performances.

†Pl. 19. F. 5. In the 5th *Fig.* of *Plate* 19. † the Man I H L (the Chairs I, L, being made faſt) makes ſo ſtrong an Arch with his Back-bone and the Bones of his Legs and Thighs, as to be able not only to ſuſtain one Man, but three or four, if they had room to ſtand; or, in their ſtead, a great Stone to be broken with one Blow.

*Pl. 19. F. 6, 7. In the 6th and 7th *Figure* of the ſame *Plate* *, a Man or two are rais'd in the Direction C M, by the Knees of the ſtrong Man I H L lying upon his

his Back. Now we must observe that the five Muscles *(a)* which bend the Legs (tho' weaker than the Extensors, because they are not to carry the Body in our common Motions) act with their greatest Force at the Beginning of this Operation, as all Muscles do when from their full Extent they begin to contract; and to relieve them in their Action, as the Heels come forward from the Point L, they stop against the Ground, and keep the Body M in the Place which it is risen to: So that the Action of these Flexors is re-iterated, and they have time to be recruited with fresh Spirits (or whatever Fluid is the first Cause of their Inflation;) and when they are so far contracted as to act more weakly, the Pressure of the Weight affects them less and less, the Bones supporting more of the Weight as they become more perpendicular, and consequently the Muscles have less occasion to act. See *Fig.* 7. * The rest of the Performance, *viz.* of setting the Man M upon a Table, is very easy and obvious, the strong Man having now only his own Body to lift up, which he helps by putting his Hands round the Man's Feet or Hams, and in raising himself up, rather pushes him off of his own Knees than lifts him up upon a Table placed at N at Arms ends, as he pretends. † Annotat. Lect. IV. *Pl. 19. F. 7. †Pl. 19. F. 7.

In breaking the Rope one thing is to be observ'd, which will much facilitate the Performance; and that is to place the Iron Eye L *, thro' which the Rope goes, in such a Situation, that a Plane going thro' its Ring shall be parallel, or nearly parallel to the two Parts of the Rope; because then the Rope will in a manner be jamm'd in it, and not slipping through it, the whole Force of the Man's Action will be exerted on that part of the Rope which is in the Eye, which will make it break more easily than if more Parts of the Rope were acted upon. So that the Eye, tho' made round and smooth, may be said in some measure to *cut the Rope*. And it is after this manner that one may break a Whip-cord, nay, a small Jack-line with one's Hand without hurting it; only by bringing one part of the Rope to cut the other; that is, placing it so round one's left Hand, that by a sudden Jerk, the whole Force exerted shall act upon one Point of the Rope. See the 11th *Figure* of *Plate* 20 †, where the Cord, to be broken at the Point L in the left Hand, is mark'd according to its Course, by the Letters R T S L M N O P Q, folding once about the right Hand, then going under the Thumb into the middle of the left Hand; where crossing under another Part, it is brought back under the Thumb again to M, then round the back of the Hand to N, so thro' the Loop at L *Pl. 20. F. 3. † Pl. 20. F. 11.

(a) The five Muscles that bend the Legs are these: 1. The *Longissimus*, or *Fascialis*, arising from the inner Knob of the *Os Ilium*, and a little above the Knee ending in a Tendon; which is inserted under the Knee, into the fore and inner Side of the *Tibia*. 2. The *Gracilis* springing from the jointing of the *Os Pubis*, and inserted by a strong Tendon a little lower than the former, in the inner Side of the *Tibia*. 3. The *Seminer-vosus*, arising from the Knob of the *Ischium*, turning into a round Tendon under the Ham, and inserted also into the inner Side of the *Tibia* towards the back-side, running as far as its Middle. 4. The *Semimembranosus*, proceeding from the same Knob, and ending by a broader Tendon than the third, in the hinder Part of the *Tibia*. 5. The fifth is call'd *Biceps*, likewise beginning at the *Ischium*, and at last inserted into the outer Side of the upper Appendix of the *Fibula*.

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L to O, and three times round the little Finger at P and Q; which last is only that the Loop N O may not give way. Before the Hands are jerk'd from one another, the left Hand must be shut, but the Thumb must be held loose, lest pressing against the Fore-finger it should hinder the Part T L of the Rope from carrying the Force fully to the Point L; but the little Finger, and that next to it, must be held hard, to keep the Loop N O firm in its Place.

The making use of the Muscles that extend the Legs for lifting great Weights is no uncommon Practice among some of our working Men, tho' it is not observ'd, because it is done without an *Apparatus*. We see Hackney Coach-men often get out of their Boxes, and with their Rumps easily lift up the Coach behind to make way for one another, or to avoid some great Rub, or some Hole, or any other Inconveniency; and this they do with so much ease, that if they have four Persons in the Coach, and three or four Trunks behind, they never think it worth while to desire any Person to get out, or to take off any of the Weight. The Coal-Porters at the *Custom-House* Key (commonly call'd *Fellowship Porters*) carry one hundred and three quarters Weight of Coals, running all the Way, tho' at every Turn they go up two Ladders, and often the length of St. *Dunstan*'s Hill, which is a Street pretty steep and ill pav'd, and perhaps climb up a Stair-case or two before they shoot their Coals: and this most of them will do above sixty times a Day. But their manner of doing it, is to stoop so as to let the Sack bear chiefly on their Rumps, holding one Hand behind them to keep together the Mouth of the Sack, that they may with more Expedition shoot out their Coals, whilst their other Hand secures the Sack from slipping down from above; and this Posture very much eases the Action of the Muscles of the Loins, the Extensors of the Legs being then chiefly concern'd.

Since I began to write this, I have been credibly inform'd that the Porters in *Turkey* carry seven or eight, nay sometimes nine hundred Weight upon the lower End of their Back, or rather their Rump, only resting on a Stick before them, whilst they receive their Loads, to support their Body and save the Muscles of their Loins; but we may easily guess that other Persons must be very careful in putting on and taking off the Burthen.

I believe the Strength of the *Testudo* made by the *Roman* Soldiers when they stood close together with their Shields over their Heads, must be owing to some such sort of Posture of the Body: Otherwise they could never have been able to bear the Weight of Chariots driving over them, as some Historians have inform'd us. In that Case every Man, except those of the first Row (*a*), cover'd the Man who stood before with his Shield at the same Time bearing upon his Rump that stood before him: and when they

(*a*) Those of the first Row held their Shields inclin'd before them as they stoop'd. Sometimes the first Row kneel'd, and the second Row bore on the Shoulders of the first as they stoop'd and cover'd them with their Shields, &c.

they stood against any Shock, their Muscles had no other Labour but to keep their Knees stiff; the bony Arch already describ'd † being sufficient to support a much greater Weight. Annotat. Lect. IV.

† Pl. 20. F. 6.

There are several Cases, wherein it would be of singular use to apply the Force of one or more Men, by means of the Girdle and Hook and Chain, in the manner above mentioned; as for example, when the Resistance is very great, but the Bodies that resist are to be remov'd but a little Way: If we lift very heavy Goods a small Height, to remove any Thing from under them; if we would draw a Bolt or Staple, and find we can't do it even with an Iron Crow, the Hand pulling it upwards at the End, then the Hook of the Girdle being applied at the End of the Crow, the Force exerted by stretching the Legs would be ten fold of what the Hands were able to do, without more help at the same Place.

There may also be many Occasions on board a Ship. I'll instance but one. Let F G * be the Tackle for raising or lowering the Main-top-Mast, part of which is represented by *m* 1, *m* 2; the Block G is fix'd below, and as the Block F comes down, it pulls along with it the top Rope F B C, *m* 1 running over the Block B (fix'd at A) and round the Block C in the Heel of the Top-mast, so as to draw up the lower End *m* 1 of the said Main-top-mast, which when hoisted up to its due Height, is made fast by the Iron Pin or Fidd I, which is thrust thro' it, and then its own Weight and the Hole D of the Cap will keep it in its Place. We'll suppose that the Force requir'd thus to raise the Mast must be that of six Men pulling upon Deck at the fall of the Tackle, that is, at the running Rope F G K at K on the other Side of the Main-mast L l. Now in order to let down this Mast on the sudden, as in case of hard Weather, it is necessary the same Tackle and Power must be made use of, tho' it be but to lift it a very little Way, that a Man may be able to get out the Fidd I, before the said Mast can be let down and slip to N on the Side of the Main-mast. I say, that if the *Hands* are so employ'd otherwise, that instead of six Men there be only one Man at the Rope K; if he has a strong Girdle to which he fastens it (or makes a Bow in the Rope itself, to fix it round the lower Part of his Back, *&c.*) he may exert much more Force in the Direction G K than the six Men in the common way of pulling; and if he draws to him (sitting on the Ground, and pushing his Feet against the first firm Obstacle that he finds, as against O P) only two Inches of the Rope G K, he will raise up the Main-top-mast the third Part of an Inch, which will be sufficient for the Iron Fidd I to be drawn out.

* Pl. 21. F. 1.

N. B. *If more Force should be requir'd for this Operation, as in large Ships, several Men at once might make use of Ropes about their Middles instead of Girdles, and fasten them all to different Parts of the Fall of the Tackle; and for fix'd Points they might set their Feet against the wooden Steps of a Stern Ladder taken down and lying on the Deck, fasten'd at one End to one of the Ring-Bolts: For tho' in this case each Man could not apply so much Force as the single Man before suppos'd,*

Annotat. Lect. IV.

because as they all must sit a little on one Side of the Rope G K, *their Pull will be something oblique; yet five Men, in this case, will very easily do the Work of Fifteen.*

8. [Page 253.——*The whole Force whereby a Man draws*, &c.——*I have given a Translation in my Notes*, &c.]

To illustrate and confirm what I have said in the Lecture, I shall here give Part of *Mons. De la Hire*'s Memoire given in to the Royal Academy of Sciences, in the Year 1699, entitled, *An Examination of the Force of Men to move Weights, whether by lifting, carrying, or drawing; consider'd as well absolutely as when compar'd with that of other Animals which carry and draw, as Horses*, &c. In which all his Reasoning is just, tho' some of his *Data* being wrong, lead us also to a wrong Conclusion; but I shall set all right by my Observations upon it.

" I suppose first that a Man of a middle Stature, who is pretty strong, " weighs 140 lib. of our Weight. (*) I consider first, that such a Man as " I have suppos'd, having both Knees on the Ground, can rise bearing only " on his Toes, still keeping his Knees together; and as this Action is per- " form'd by means of the Muscles of the Legs and Thighs, it is evident " by the Supposition I made of his Weight, that the Muscles of the Legs " and Thighs have a sufficient Force to raise 140 lib. (*a*).

" But a Man bending a little in the Hams can raise himself up though " loaded with a Weight of 150 lib. together with the Weight of his Body, " which he raises at the same Time (*b*); so that the Force of the Muscles " of the Legs and Thighs can raise a Weight of 290 lib. that is 150 lib. " the Burthen carried, and 140 lib. the Weight of the Body when the Rise " is but 2 or 3 Inches.

" Such a Man, as we have suppos'd, and shall all along suppose him, can " also lift from the Ground a Weight of 100 lib. plac'd between his Legs, " taking hold of it with his Hands as with two Hooks, and raising him- " self up. (*c*) Whence it follows that the Muscles of the Loins alone have " Force enough to raise 170 lib. namely, the 100 lib. of the Weight, and " 70 lib. half the Weight of the Man; because he is to raise not only the " 100 lib. Weight, but also the whole upper Part of his Body above the " Middle, because we have supposed him to stoop to take up the Weight.

" As for the Strength of the Arms for drawing or lifting a Weight, one " may suppose it of 160 lib. which depends upon the Muscles of the Shoul- " ders and Arms. For if a Man with both Hands takes hold of some " fix'd Body placed over his Head, he may easily enough, by the Strength " of his Arms alone, draw up his Body, and even 20 lib. more, just as if " he was burthen'd with a Weight of 20 lib. It is easy to make the Experi- " ment; for if a Weight of 160 lib. be fastened to a Rope and thrown " over a Pulley, and a Man who weighs only 140 lib. pulls at the other End " of

(*) The *French* lib. is betwixt one 11th and one 12th Part greater than our Pound *Averdupoids*.

" of the Rope, it is plain that he will never be able to raiſe the 160 lib. Annotat.
" Weight; the utmoſt he can do, being only to hang with all the Weight Lect. IV.
" of his Body by the Rope: for the Weight at the other end being greater
" than the Weight of the Man, will keep him ſuſpended; the Pulley being
" only a continued Balance with equal Arms: but if the Man has faſtened
" to him a Weight of 20 lib. then he will make an *Æquilibrium* with the
" Weight on the other Side; and if ever ſo little be added to the 20 lib.
" Weight, he will raiſe the oppoſite Weight, the Muſcles of his Shoulders
" and Arms being of ſufficient Force to ſuſtain the whole aggregate
" Weight.

" Tho' the Muſcles of each part of the Body can exert great Forces
" to raiſe Weights, the Force of a Man is not to be reckon'd as the Sum
" of the different Forces of all his Muſcles taken together, even tho' the
" Spirits which ſwell the Muſcles, and by ſhortening them draw the Ten-
" dons at their Ends, could as eaſily be diſtributed to all the Parts as to any
" one particular Part, becauſe each Part commonly ſerves to ſupport that
" which is immediately next to it. As for Example; the Muſcles of the
" Arms and Shoulders by their Contraction can lift a Weight of 160 lib.
" but if the Body be inclin'd, the Arms will not be able to ſuſtain that
" Weight, unleſs the Muſcles of the Loins are at the ſame time ſtrong
" enough to ſuſtain the upper part of the Body, together with the Weight
" which it is loaded with; and if the Hams were alſo bent, then the Muſ-
" cles of the Legs and Thighs muſt ſtill exert a greater Strength, becauſe
" they muſt ſuſtain the Weight of 160 lib. and alſo at the ſame time the
" whole Weight of the Body. Whence it happens, that in this Diſpoſi-
" tion of the whole Body, the Force is diſtributed by the Diſtribution of
" the Spirits into all the Parts; for which reaſon a Man will not be able to
" raiſe 160 lb. from the Ground.

" Not but there are Men, whoſe Spirits flow ſo abundantly and ſo ſwift-
" ly into their Muſcles, that they exert three or four times more Strength
" than others do; and this ſeems to me to be the natural Reaſon of the
" ſurprizing Strength that we ſee in ſome Men, who carry and raiſe Weights
" which two or three ordinary Men can hardly ſuſtain, tho' theſe Men be
" ſometimes but of a moderate Stature, and rather appear weak than
" ſtrong. There was a Man in this Country a little while ago, who wou'd
" carry a very large Anvil, and of whom were reported ſeveral wonderful
" Feats of Strength: but I ſaw another at *Venice*, who was but a Lad, and
" did not ſeem able to carry above 40 or 50 lib. with all poſſible Ad-
" vantages; yet this young Fellow ſtanding upon a Table, raiſed from the
" Earth and ſuſtained off of the Ground an Aſs, by means of a broad Girt,
" which going under the Creature's Belly, was hung on to two Hooks that
" were faſten'd to a Plat of ſmall Cords, coming down in Treſſes from the
" Hair on each ſide the Lad's Head, which were in no great Quantity.
" And all this great Force depended only upon the Muſcles of the Shoul-
" ders and thoſe of the Loins (*d*); for he ſtoop'd at firſt whilſt the Hooks
" were faſten'd to the Girt, and then raiſed himſelf, and lifted up the Aſs

" from

Annotat. Lect. IV. " from the Ground, bearing with his Hands upon his Knees. He rais'd " also in the same manner other Weights that seem'd heavier, and used to " say he did it with more ease, because the Ass kick'd and struggled when " first lifted off of the Ground. (*e*)

" Now to examine the Force of a Man to carry a Burthen upon his Shoul- " ders: I say that Weight may be of 150 lib. and a Man may walk with " that Load easily enough upon an horizontal Plane, provided he does not " take great Steps; but he can by no means go up a Mountain or a Stair- " case with the same Weight. For the Action of walking, when we bear a Pl. 20. F. 12. " Burthen, must be consider'd as the circular Motion of C, the common " Center of Gravity of the Body and the Weight together about the Foot F, " that advances as the Center of the Arc of Motion, the Effort of the " Muscles of the other Leg which act against the fix'd Point D, serving " only to drive the Center of Gravity forward; and if the Arc C E, " which that Center describes, be small, the Effort of the hind Leg need " not be great, to make the Centre describe it; because then it pushes up " the Weight of the Body, and the Burthen only the Height of A B the " vers'd Sine of half the Arc, which is inconsiderable in this Case in re- " spect to the whole Arc, which is the Length that the Burthen advances.

" So that a Man carrying a Burthen, can walk so much the easier as his " Steps are shorter, because the vers'd Sine will be so much the less; for if he " shou'd take large Steps, the hind Leg must push up the Body and Burthen " the Height of the vers'd Sine of so much a greater Arc as the Steps are " greater; that is, the vers'd Sine of the Arc which measures half the " Distance that the Burthen advances forward.

" It is also plain, that a Man thus loaded can by no means go up a Stair- " case, or a very steep Hill with that Burthen; because according to what " we have already explained, the Action of the Muscles of the Legs being " only able to raise a Weight of 150 lib. to the Height of two or three " Inches, he wou'd not be able to raise it five Inches, which is the Height of " common Stairs or Steps, (*f*) nor go up a Mountain, unless he took such " short steps as to rise only two or three Inches perpendicularly every time.

" What remains now is to consider the Force of a Man for drawing or " thrusting horizontally. (*g*) But to make this the more clear and intelligi- " ble, I shall consider that Force as apply'd to the Handle of a Roller or " Windlass whose Axle is horizontal, on which Axle the Rope which su- " stains the Weight winds up, supposing the Distance from the Center of the " Roller to the Elbow of the Handle to be equal to the half Diameter of " the Roller, that the Force applied may be estimated without any Aug- " mentation on account of the Machine; neither do I consider here the " Friction of the Pivot, or Difficulty of folding the Rope.

" *First* then, it is plain, that if the Elbow of the Handle be in an hori- " zontal Situation, and about the Height of a Man's Knees, a Man that lifts " it by drawing up, may exert a Force capable of raising 150 lib. hanging " at the end of the Rope if he takes all possible Advantages, according to " what I have already explain'd. But if he is to depress the Handle, his

" Strength

Strength for that purpose cannot be more than equal to 140 lib. which Annotat.
" (as has been supposed) is the Weight of his Body; for he can lay on no Lect. IV.
" more Weight, than that, except he was loaded with some Weight, and
" then the Effect might be so much greater.

" *Secondly*, If the Elbow of the Handle be plac'd vertically, and it be at
" the same Height as a Man's Shoulders, it is certain that a Man can exert
" no Force to cause it to turn, by pushing or drawing with his Hands, if
" both Feet being set together the Body be upright, as represented in * *Fig.* *Pl. 20. F. 13.
" 13. by the Line A P, the Line A M representing the Line of the Arms
" being horizontal, and making a Right Angle with A P; because in that
" Position, neither the Force of the whole Body, nor of any part of it, nor
" its Weight, can exert any Force for drawing or pushing; which is known
" by Mechanicks: for I look upon the Breadth taken up by the Feet but
" as a Point at P. But if the Handle be higher or lower than the Level of
" the Shoulders, then the Line A M, which goes from the Shoulders to
" the Hands, and the Line A P going from the Shoulders to the Feet,
" will make an obtuse or an acute Angle, and then a Man may exert some
" Force for drawing or pushing the Handle; but that Force depends
" only upon the Weight of the Body, as may be easily perceiv'd and de-
" monstrated; and that Weight or Force must be consider'd as collected
" in one Point, which is the Center of Gravity of the Body, and about
" the Height of the Navel: I say, we must have regard only to the Weight
" of the Body, in order to determine the *Æquilibrium*; for the Action of
" the Muscles of the Legs and Thighs serves only to preserve that *Æquili-*
" *brium* as we walk.

" Let the Handle in *Fig.* 2. * be placed at D the same height as the *Pl. 21. F. 2.
" Shoulders A, and the Center of Gravity of the Body be at C, the Body
" being very much inclin'd towards the Handle; but the End of the Feet
" must be at P. *First*, We must consider that Point P as the *Fulcrum* or
" Center of Motion of a Leaver or strait Rod P C H, which going thro'
" G, the Center of Gravity of the whole Body, meets the Line of the
" Arms M A at the Point H. *Secondly*, We must consider the Point C of
" the Leaver loaded with the whole Weight of the Body equal to 140 lib.
" which Weight endeavours to descend perpendicularly according to its
" natural Direction, the end H of the Leaver being sustain'd in the horizon-
" tal Direction M A H; whence it will be easy to determine by mechanical
" Principles, what Effect the Weight of the Body at C, acting in its
" natural Direction, can have on the Handle according to the horizontal
" Direction H D.

" For, first, let us suppose P H to be divided into 140 Parts, of which
" P C contains 80. Now since the Weight of the whole Body acting at
" C is but 140 lib. when apply'd at H it will act but with the Force of 80
" lib. just as if a Weight of 80 lib. was to be suspended at that Point, and
" act in its natural Direction; which, according to what we have supposed,
" must be perpendicular to M A. If therefore from the Center of Mo-
" tion P, the Sine P F be drawn perpendicular to M A F, the Weight

" of

Annotat. Lect. IV. " of 80 lib. at H, acting in its natural Direction: will be to the Action of " the same Weight upon the Handle according to the horizontal Direction " M A H:: in the *Ratio* of P F to H F. And this does very much diminish " the Action of the 80 lib. in a moderate Inclination of the Body A C B. " And if we suppose for Example, that the Line P C H makes an Angle " P H F of 70 Degrees with the Line M A F, the Line of the Body A C B " will then be inclined to the Horizon in an Angle of more than 60 " Degrees, which is the greatest Inclination of Body with which a Man can " walk. Then

" The Sine of 70 Degrees, which is P F :
" Will be to the Sine of its Complement, which is H F::
" As 3 :
" To 1, nearly;

" And consequently the Action of the 80 lib. supposed to act at H accord- " ing to the natural Direction, will be to the Action exerted in the hori- " zontal Direction, but as 80 to its third, which is something less than " 27 lib.

" Those who have not made the Experiment of pushing horizontally with " the Arms, or drawing a Rope horizontally as a Man walks with the Body " inclin'd forward, whether the Rope be fasten'd towards the Shoulders or " towards the middle of the Body, (for the Effect will not differ, provided " the Inclination of the Body is the same; because the Sine of Inclination " and its Complement are always in the same *Ratio*) can hardly be persuaded " that the whole Force of a Man is reduc'd to so little in pulling horizon- " tally, as not to exceed 27 lib.

" Not but that a Man may lean or incline his Body so as to sustain a much " greater Weight than 27 lib. for if the Line P H should make an Angle of " 45 Degrees with H F, it is certain that the Weight of the Body would " sustain 80 lib. But as he must then have his Body in the Position of the " Line A B, much more inclin'd to the Horizon than 45 Degrees, he would " be so far from being able to walk, that he would not be able to stand in " such a Posture.

" The same Demonstration may also serve to shew that a Man has much " more Force to draw if he goes backwards than in going forwards. For *Pl. 21. F. 3. " in such a Situation of the Body, the Line P C H, **Fig.* 3. which goes " from the Feet at P, thro' the Center of Gravity C, and on which the " Increase of Force depends, will always be much more inclin'd to the " Horizon than the Line of the Body A C B, quite contrary to what it was " in the former Position. But then this way of drawing cannot be put in " practice, except for pulling a Rope, when a Man continues in the same " Place; otherwise we should naturally come into that Posture: for Na- " ture and Experience have taught us to take all possible Advantages in " common Operations.

" For the same Reason, Watermen and those that row at Sea always pull " their Oars backwards; for that way they can exert more Force than in " pushing forwards like the *Gondoliers* at *Venice*, who work in that manner,

" for

" for no other Reason than to see well before them, which is more necessary to them than exerting a great Force, by reason of the many short Turns they must make in the Canals, and the Care they must take to avoid one another." Annotat. Lect. IV.

What remains is to compare the Force of Men with that of Horses for drawing, *&c.* but having consider'd that in my Lecture, I need not give the Translation of the rest of *Monsr. de la Hire*'s Paper; only proceed to make my Observations upon it.

(a) *The Muscles of the Legs and Thighs are much stronger than* M. De la Hire *supposes, as appears by what has been said concerning the Feats of Strength; and the Action of rising backward from one's Knees is very short of the utmost that a Man can do in that Posture: For a Man may carry some considerable Weight, and yet rise from his Knees, tho' the Arc describ'd by the Center of Gravity be then an Arc of a good many Degrees.*

(b) *It is usual in* London *for Men to raise themselves up with, and carry* 250 *lib. which is almost as much more as* M. De la Hire *has suppos'd; therefore all the Consequences drawn from this Supposition must fall short.*

(c) *Working Men generally lift* 150 *lib. with their Hands, and some* 200 *lib. but here the Excess of the Force of the Muscles of the Loins is not so much greater than* M. De la Hire's *Supposition, as the Excess of the Force of the Muscles of the Legs.*

(d) *What he attributes here to the Muscles of the Loins was really perform'd by the Extensors of the Legs; for the young Man's stooping with his Hands upon his Knees was not with his Body forwards and his Knees stiff, but his Body upright and his Knees bent, so as to bring the two Cords with which he lifted to be in the same Plane with his Ankles and the Heads of his Thigh-Bones; by which means the Line of Direction of the Man and the whole Weight came between the strongest Part of his two Feet, which are the Supports: Then as he extended his Legs he rais'd himself without changing the Line of Direction. That this must have been the manner I am pretty well assured of, by not only observing those that perform such Feats, but having often try'd it myself. As for the Muscles of the Loins, they are incapable of that Strain, being above six times weaker than the Extensors of the Legs, at least I found them so in myself.*

About the Year 1716, *having the Honour of shewing a great many Experiments to his late* Majesty King GEORGE the First, *his* Majesty *was desirous to know, whether there was any Fallacy in those Feats of Strength that had been shewn half a Year before by a Man, who seemed by his Make to be no stronger than other Men: Upon this I had a Frame of Wood made to stand in (such as is represented by the* 2d * Fig. *of* Plate 20.) *and with a Girdle and Chain lifted an Iron Cylinder made use of to roll the Garden, sustaining it easily when once it was up. Some Noblemen and Gentlemen who were present, try'd the Experiment afterwards, and lifted the Roller; some with more Ease, and some with more Difficulty than I had done. This Roller weighed* 1900 *lib. as the Gardener told us. Afterwards I try'd to lift* 300 *lib. with my Hands* (viz. two

* Pl. 20. F. 2.

Annotat. Lect IV. *two Pails with* 150 *lib. of Quickſilver in each) which I did indeed raiſe from the Ground, but ſtrain'd my Back ſo as to feel it three or four Days; which ſhews that, in the ſame Perſon, the Muſcles of the Loins (which exerted their Force in this laſt Experiment) are more than ſix times weaker than the Extenſors of the Legs; for I felt no Inconveniency from raiſing the Iron Roller.*

(e) *The Reaſon why the Aſs kicking and moving made his Weight more inconvenient than a heavier Burthen, was, that by ſuch a Motion the Line of Direction vacillated, and as it went forwards and backwards, the Muſcles of the Loins were forc'd to act to bring it into its Place again.*

(f) *That* M. de la Hire *has taken the Strength of the Muſcles that extend the Leg and ſtretch the Foot, too little, appears from the Practice of all thoſe that carry Grain in Sacks and Meal, who eaſily go up Stairs with* 200, *and often above* 250 *lib. Weight; and the Men who carry Coals from Carts into Houſes, go up Stairs with* 250 *Weight. They can't indeed go down Stairs with ſo great a Weight as they can go up.*

(g) *All that follows in this Diſſertation concerning a Man's puſhing or drawing may be depended upon; becauſe it is mathematically deduc'd from the Weight ſuppos'd, and cannot but be true in a Man weighing* 140 *lib.*

9. [Page 255. —— *There muſt be a ſufficient Velocity given*, &c.] If there ſhould be no greater Velocity given to the Hand which is to turn the Winch, than I have ſuppos'd, there could not be Motion enough communicated from the advantageous to the diſadvantageous Part of the Revolution, as would enable a Man to raiſe 30 lib. with the ſame Velocity as the Hand mov'd: Therefore we muſt ſo contrive Matters as to increaſe the Velocity of the Hand moving, at leaſt one ſixth Part; but then we muſt lay only 25 lib. upon the Hand moving ſo much the ſwifter, which yet will perform as much Work in a Day as if the other Way had been practicable. Nay, in a great many Caſes it will not be amiſs to give the Hand one third more Velocity, and load it but with 20 lib. eſpecially if a Fly be made uſe of; and the bigger the Circle is which the Fly deſcribes, the better will the Force be diſtributed. This Work thus perform'd may be continued ten Hours a Day, and very little fatigue the Labourer.

10. [Page 256. —— *Daily Labour of the* London *Porters*, &c.] At the *Cuſtom-Houſe* Key, and at ſeveral Wharfs, one may obſerve what great Loads are carry'd by Porters employ'd to carry Goods to and from the Veſſels on the Water; ſuch indeed are the Loads which ſome carry, that an Horſe would ſoon be kill'd by carrying the ſame Weight. They that work for the *Cheeſemongers* at ſo much *per Ton*, generally carry 300 Weight of Cheeſe at every Turn, and work all Day long.

11. [Page 257. —— *the Horſes ſtrong*, &c.] If too great a Load be laid upon an Horſe's Back there is danger of breaking it; which is the Reaſon that People do not generally lay on very great Loads. The *Fell-mongers* and *Skinners* are ſaid to load their Horſes more than any other People; but they

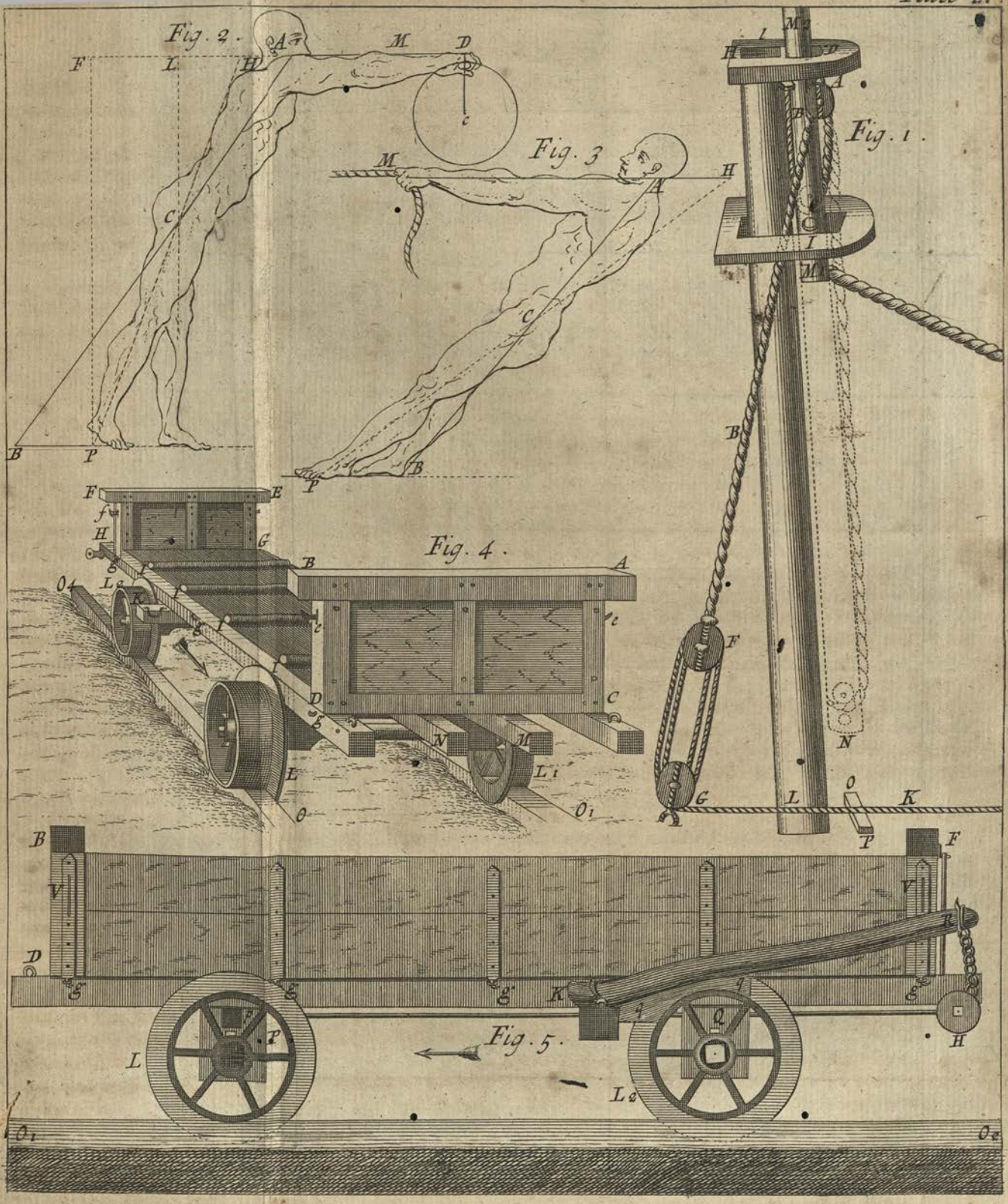
Fig. 2.
Fig. 3
Fig. 1.
Fig. 4.
Fig. 5.

they lay ſeveral Skins and Hides over the Horſes Shoulders and Hips, that the Back may not be too much ſtrain'd: I am inform'd that they put on ſometimes 4 or 500 Weight; but then the Horſes go extremely ſlow. The moſt that can be done with an Horſe is to make him draw: And thoſe Horſes that do moſt Work are ſuch as draw great Loads in Carts on very high Wheels (the Horſes themſelves being very tall) up St. *Dunſtan*'s Hill in the *Eaſt*, where a Carter does ſometimes lay on 2000 lbs Weight, and makes one Horſe draw it up the Hill; but at every difficult Place, the Man ſets his Shoulder to the Cart in ſuch a manner, as conſiderably to help the Horſe, who would be unable to draw up the Load without that Help: And ſo ſenſible is the Horſe of it, that he does not exert his whole Strength, till he finds that his Maſter helps him. Annotat. Lect. IV.

I promis'd in the 15th *Note* to *Lect.* III. to give an Account of Mr. *Allen*'s Carriages at *Bath*, and having juſt receiv'd an Account from a Friend well ſkill'd in Mechanicks and Drawing, who took the Meaſures and Draughts upon the Place, think proper to communicate it here, as having conſider'd the Nature of Carriages in this Lecture.

A DESCRIPTION *of the Carriages made uſe of by* Ralph Allen, *Eſq; to carry Stone from his Quarries, ſituated on the Top of a Hill, to the Water-ſide of the River* Avon, *near the City of* Bath. By CHARLES DE LABELYE.

Theſe Quarries are at the Diſtance of a Mile and a half from the River, and about 500 Feet above the Level of its Surface, which makes a Slope ſo ſteep, that the ſmall Price the Stone is ſold at, would hardly defray the Charges of bringing it down without ſome proper Contrivance, ſuch as the following, which is a great Improvement on ſome Carriages and *Waggon-ways* made uſe of at the Coal Mines near *Newcaſtle*.

1. *Fig.* 4. *Plate* 21. * repreſents this Carriage in Perſpective, as ſeen from a Diſtance of 12 Feet from the left of the Fore-wheels, the Height of the Eye being about 6 Feet. The *Geometrical Plan* was laid down from a Scale of 20 to an Inch; that is, every 20th of an Inch, anſwers to one Inch of the Machine. * Pl. 21. F. 4.

2. *Fig.* 5. * repreſents the Elevation of one of the Sides of this Carriage, when both the Fore-wheel and Hind-wheel of that Side are lock'd from a Scale of 20 to an Inch. * Pl. 21. F. 5.

3. *Fig.* the 6th. † repreſents the Elevation of the hind Part of the Carriages, with all the Iron-work employ'd in the locking of the Wheels, and it ſhews alſo the Profil of the Wheels, and Axle-tree, together with the Sections of the Side-planks and of the Frame on which it moves, from a Scale of 10 to an Inch. † Pl. 21. F. 6.

4. From the Conſideration of theſe three Figures, it will appear, that this Carriage conſiſts of a ſtrong Floor of (Oaken) Planks, three and a

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half Feet wide, and about 13 Feet long, ſtrengthen'd above by ſeveral Ribs to defend it from the Stones that lie upon it, and fix'd upon four Beams of the ſame Wood, about four Inches ſquare and 14 Feet long.

5. At ſix Inches from the Ends are fix'd the Fore-ſide and Back-ſide ſtrongly faſten'd to the Beams, and to the Floor, by ſeveral Screws and Nuts. See *Fig.* 4, and 6. †

†Pl. 21. F. 4, and 6.

6. To theſe two Ends, when Occaſion requires, may be faſten'd two Sides made of Planks 13 Feet long, which fit into the Side of the outward Beams by means of *Hooks and Rings*, and are kept up by means of *Latches*, to be ſeen in *Fig.* 4, 5, 6 †. Theſe Sides are alſo further ſtrengthen'd by a Chain going acroſs in the Middle of the Carriage.

†Pl. 21. F. 4, 5, and 6.

7. At right Angles under theſe Beams, at a proper Diſtance (ſee *Fig.* 5.) are faſten'd two ſtrong Timbers, by means of large Screws and Nuts.

8. In theſe Timbers well ſtrengthen'd and plated with Iron, where the great Streſs lies, are placed two ſemicylindrical Pieces of Braſs at each End, to ſerve as a *Collar* for the Axle-trees of the Wheels, which being well greaſed, turn with very little Friction.

9. There is likewiſe under theſe four Beams (already mention'd) another Piece of Timber of about 6 Inches by 4 well faſten'd to it, at right Angles, and at ſuch a Diſtance, as is ſeen in *Fig.* 5.

This Piece ſerves as a fixed Point to place a Leaver, which locks (or keeps from turning) the Hind-wheel, by preſſing upon it.

†Pl. 22. F. 1, and 2.

10. The Axle-tree is about three Inches Diameter. See † *Plate* 22. *Fig.* 1. One of its Ends is ſquare, the other round; and on theſe two Ends, the Wheels are plac'd in an alternate Poſition; that is, the right handed Fore-wheel is on a Square, and the left on a round Part of the Axle-tree, whilſt the right-handed Hind-wheel is on a round End, and the left on a ſquare End of the Axle-tree; thereby any one of the Wheels may be lock'd ſeparately: for when the Wheel placed on the round End is lock'd, the other, together with the Axle-tree, revolves within it; and when the Wheel which is faſtened on the Square is lock'd, the other revolves notwithſtanding, as uſual upon the Axle-tree, which is then unmoveable.

11. Theſe Wheels are made of caſt Iron about 20 Inches Diameter, and have a *Flanch* 6 Inches broad next to the Carriage, which hinders them from running off from the Oaken Frame on which they move. Their Plan and Profil are ſeen in *Fig.* † 5. of *Plate* 21, and *Fig.* 1. of *Plate* 22. and their Sections either thro' their Spokes (or Radii) or between any two of them, as * *Fig.* 3d and 4th of *Plate* 22.

†Pl. 21. F. 5. & Pl. 22. F. 1. * Pl. 22. F. 3, and 5.

12. The manner of locking and unlocking of the Wheels is as follows: When either of the Hind-wheels is to be lock'd, a ſtrong Leaver (which they call the Jigg Pole) is placed on that End of the Timber (deſcribed in Paragraph 9.) next to the Wheel to be lock'd; and after it is paſſed thro' the Iron Loop to ſecure it the better, a Chain coming from the Roller to be ſeen in † *Fig.* 5, and *Fig.* 1. of *Plate* 22. is clapt over the Extremity, and by means of a ſhort Iron Bar and the *Ratchet* and *Click*, ſeen in *Fig.* 5 and 6. *Plate* 22. * one of the Drivers or Perſons that attend the Carriage in a very little Time ſtops the Wheel either partly or intirely.

†Pl. 21. F. 5. & Pl. 22. F. 1. * Pl. 22. F. 5, and 6.

To

To unlock which, 'tis only lifting up the End of the *Click* or *Catch* made long on purpoſe; for then the Leaver preſſing no longer, the Chain is ſlacken'd, taken off, aad the Jigg Pole laid in the Cart, till another Occaſion ſerves. They have two Jigg Poles, one for each Hind-wheel. Annotat. Lect IV.

13. The Fore-wheels are lock'd, by means of a thick ſquare Iron Bolt, ſeen in *Fig.* 5, of *Plate* 21. † coming out in the Direction of the Axle-tree, between the Spokes or Radii of the Fore-wheels. Theſe Bolts, are protruded forwards to lock the Wheels, and drawn back to unlock them ſeparately, by means of a Contrivance, part of which is ſeen in *Fig.* 5. of *Pl.* 21. and 1*ſt* of *Pl.* 22 †. Towards the Middle of the Back-ſide are two Iron Rods, (ſee one in *Fig.* 6. *Pl.* 22.*) turning ſeparately on the Square of an Axle. When either of theſe are brought by the Hand from a vertical Poſition, in which they are drawn, to an horizontal one, the Iron Rod, of which it takes hold by its lower End, is puſhed forward four or five Inches, and by means of the Contrivance in † *Fig.* 1, 2, 3, 4, of *Pl.* 23. ſhoots the ſquare Bolt between the Spokes or Radii. † Pl. 21. F. 5. † Pl. 21. F. 5. Pl. 22. F. 1. *Pl. 22. F. 6. † Pl. 23. F. 1, 2, 3, 4.

When the Wheel muſt be unlock'd, this Bolt is drawn back into its former Poſition, by bringing the Iron Rod into its former vertical Situation, as in *Fig.* 1. of *Pl.* 22 †. As theſe Carriages are loaded with a conſiderable (often with upwards of four Tons) Weight of Stone, when they come down the Hill, all the Contrivances explain'd above would be uſeleſs, and that great Weight would ſink the Carriage too deep into the Ground, without Pieces of Oak laid all along the Way which theſe Carriages then paſs over. Theſe are ſufficiently ſeen in *Fig.* 4, 5, of *Pl.* 21.* and *Fig.* 1. of *Pl.* 22. † Pl. 22. F. 1. * Pl. 21. F. 4, 5. and Pl. 22. F. 1.

Altho' theſe Carriages are very heavy even when empty, yet by means of the Frame on which they move, and the little Friction the Axle-trees feel revolving in the Braſs Collars, two Horſes not only draw them up the Hill very eaſily when empty, but draw them along on the Plain when loaded, at a very good Rate. As ſoon as the Carriages come to the Brow of the Hill, the Horſes are taken off, and one or more of the Wheels lock'd, by the Driver, who ſtands behind to moderate the Motion as he thinks proper.

When the Carriages are come to the Water-ſide, and have been unloaded, they change the Horſes from End to End, ſo that the Part of the Carriage which went before deſcending, becomes the Hind-part in aſcending the Hill, which avoids the Trouble of turning with theſe Wheels.

Theſe Carriages are loaded at the Quarries, and unloaded at the Water-ſide, by means of a very good and curiouſly contriv'd Crane, fully deſcrib'd by Dr. *J. T. Deſaguliers*, in his *Courſe of Experimental Philoſophy*, Lect. III. Ann. 15.

N. B. *Mr.* Allen, *to whom I am oblig'd for a full and thorough Survey of theſe Carriages, told me, that one of them, when compleatly finiſhed, and ready to be uſed, ſtands him in upwards of* 30 *Pounds: Which I thought proper to mention as a very reaſonable Price, conſidering the good and Workmanlike Manner in which every Thing is performed.*

Tho'

Annotat. Lect. IV. Tho' Mr. *Labelye*'s Description is very intelligible, and his Draughts extreamly well done, yet, to make every thing still plainer, I have added Letters to his Draughts, and the following References.

References to the Figure of Mr. Allen's *Carriages and their several Parts.*

Plate 21. *Figure* 4.

ABCDIIIIHFEG. The Body and Bottom of the Carriage without the Sides, which are put on upon occasion in the Place BDHF, and fastened by means of the Hooks *g g g*, and the Latches *e f*.

IIII. The cross Pieces on the Bottom to strengthen it.

MN. Strong Pieces of Timber under the Bottom.

K. A cross Piece under the Bottom, having an Iron Loop at Top to receive the End of a Leaver, that presses on the Hind-wheel L 2, to stop it from turning round when the Motion is too rapid.

L, L 1, L 2. Three of the four Wheels, the fourth being out of sight in this Position of the Carriage, whose Circumferences have a Flanch on the Inside, that the other Part may bear on the Timber or Waggon-way.

H. An Iron Roller for the Chain to wind on, to hold down the stopping Leaver, as it presses on the Hind-wheel.

O, O 4, O 1, *&c.* The Waggon-way, or parallel Timbers laid with a Descent for the Carriages to run down by their own Weight.

Figure 5.

FHDB. The left Side of one of the Carriages fix'd by means of the Hooks at *g g g g*, and the Latches *e f*, shewn in the fourth Figure.

L. A Fore-wheel with a round Hole in the Nave to receive a round End of the Axis that goes thro' the Piece of Timber P, from another Part of which the Bolt *p* is shot between the Spokes to stop the Wheel from turning round, when the Motion is to be retarded.

H *p*. An Iron Rod push'd forwards from behind to bolt or lock the Fore-wheel above-mention'd.

L 2. A Hind-wheel fix'd upon, and turning round with the Axis coming thro' the Piece of Timber Q, the End of which Axis is made square for that Purpose.

P K. A Leaver, whose End goes thro' an Iron Eye, on the Timber K, having there its Center of Motion, with the Compass-piece *q q*, to press on the upper Part of the Wheel L 2, to stop it upon occasion, from turning with the Axis.

H. A Roller on which is wound the Chain H R, which pulls down the End of the Leaver at R, and keeps it in its Place, to press hard upon the Wheel at *q q*.

O 2, O 1. The Waggon-way which supports the strong Part of the Circumference of the Wheel, while the Flanch or large Circumference of each

Fig. 1.

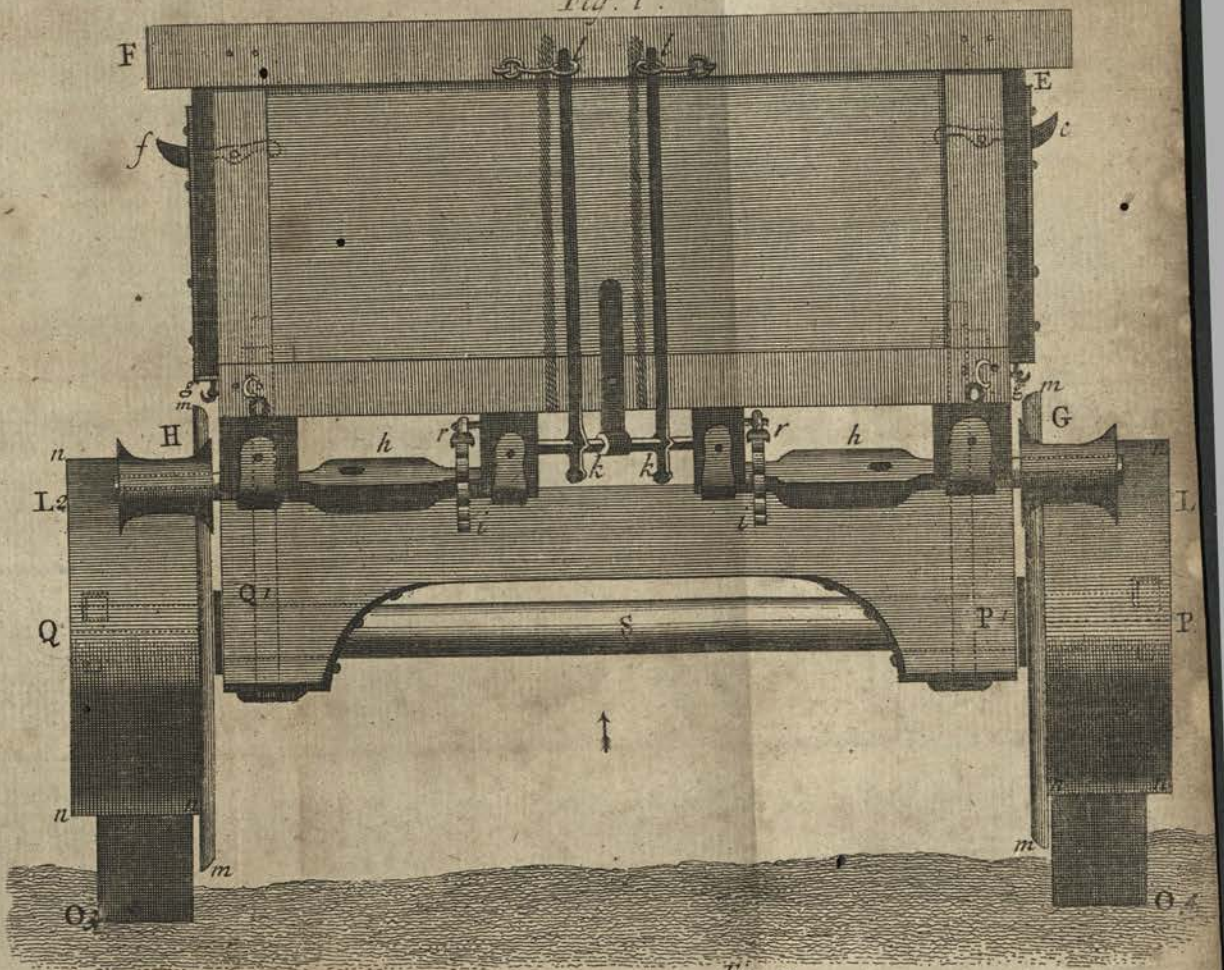
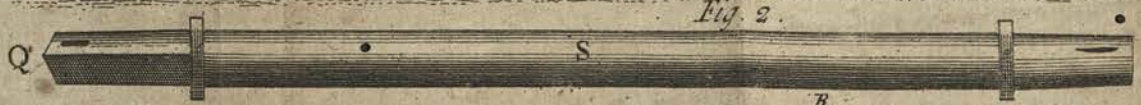


Fig. 2.



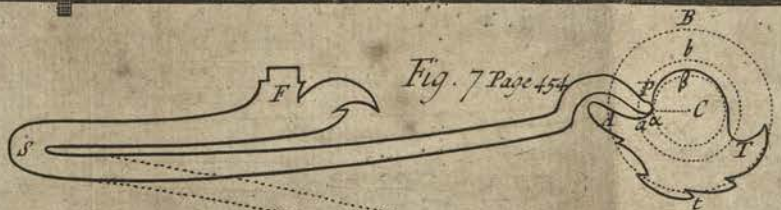


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Fig. 5.

Fig. 3.

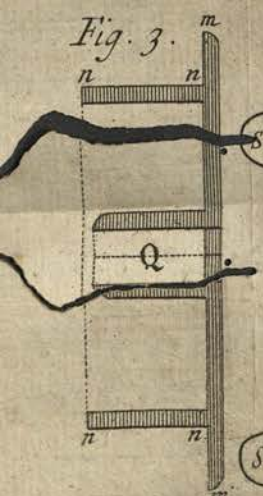


Fig. 8. Page 454

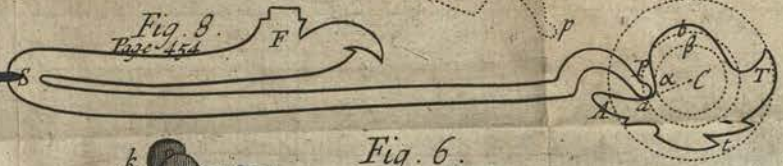


Fig. 6.

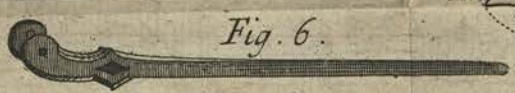
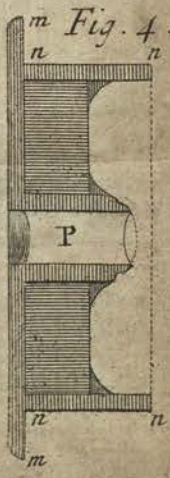


Fig. 4.



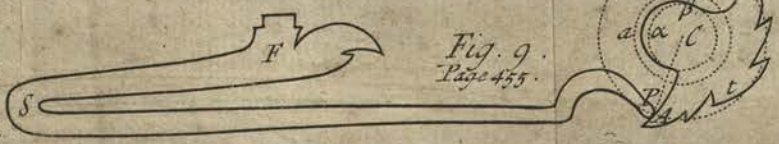


Fig. 9. Page 455.

each Wheel falls on the Inside of the Timber, that the Carriage may not jump or run out of the Way. Annotat. Lect. IV.

Plate 22. *Figure* 1.

This Figure drawn by a larger Scale (*viz.* of 10 Inches to an Inch) shews the Elevation of the Hind-part of one of the Carriages, with the Profil of the Wheels, *&c.*

FHGE. The End of the Carriage behind, bearing upon four Timbers, whose Ends are seen here.

f g, *e g*. The Ends of the Sides hook'd on at *g*, *g*, and latch'd at *f* and *e*.

H *h r i*, and G *h r i*. A Roller with its Winch, and Ratchet and Latch, to receive the Chain that draws down the Leaver or Jigg pole. The Chain goes on the Part H, or G, the Holes to turn the Axis of the Roller with an Iron Hand spike are seen at *h h*, the Ratchet at *i i*, and its Catch at *r r*.

l k, *l k*. Two perpendicular Iron Bars, whose lower Ends *k k* shoot forward each an horizontal Bar (not represented here, but shewn in the last describ'd Figure at H *p*) to bolt either of the Fore-wheels singly, or both at once.

L 2, L 4. The Profils of the two Hind-wheels, with their Flanches *m m*, *m m*, and bearing Parts *n n n*, *n n n*; the left Wheel receives the square End of the Axis Q, in a square Hole, so as to turn with it, and the right Wheel has a round Hole to receive the End of the Axle at P, which is round also; so that this Wheel can turn round without turning the Axle P S Q along with it.

O 3, O 4. Shews the Section of the Timbers or Waggon way; where may be seen the Manner of the Wheels bearing on the Timbers at *n n*, *n n*, whilst the Flanches *m m*, *m m*, come down to keep the Carriage in its place.

Q 1, P 1. The cross Timbers thro' the Bottoms of which the Iron Axles of the Carriages pass, which are fix'd up under the Carriages by Pins and Nuts here represented by prick'd Lines.

Figure 2.

Represents one of the Iron Axle-trees S, whose End Q at the right Hand is square, the other End at the left Hand being round, with an Hole for the Linch Pin at each End.

Figure 3

Shews the Section of the Hind-wheel on the left Side, or the Fore wheel on the right Side, with a square Hole Q, the Rim of Circumference of the Wheel *n n*, and the Flanch *m m*.

Figure 4.

Shews the right Wheel behind or left Wheel before, mark'd with the same Letters except P, which shews the round Hole to receive the round End of the Axis.

Figure

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Figure 5.

Shews the Catch *r*, and the Ratchet *i*.

Figure 6.

Shews one of the perpendicular Rods of the sixth Figure, whose Handle is at *l*, and the Bottom or open End is to join to one of the horizontal Bars, which shoots a Bolt in between the Spokes of one of the Fore-wheels.

Pl. 23. F. 1.

Plate 23. *Figure* 1.

This Figure represents the upper Part of the Timber under the Carriage between the Fore-wheels, thro' the lower Part of whose Ends the Axle-tree passes, and in the Substance of which is let in the Machinery for stopping the Fore-wheels, where the two Bolts A B and C D are seen, which may be separately shot out to the right or left Hand thro' a square Staple P, or Q; and resting on one of the cross Pins *e f*, E F or G H, *g h*, by means of the horizontal Bars I K already describ'd, one of which is to be seen in the 5th Figure of Plate 21, mark'd H *p*, and the Ends of both of them are seen in this Figure. The Bolt A B on one Side is represented as shot out between the Spokes of the Wheel, and the other is in its usual Place, where it does not touch the Wheel. N B. *Pulling back the Bar* L I *unbolts* B A, *by means of the Elbow* L N B, *and pushing forwards the Bar* K M, *bolts* C D, *by means of the Elbow* M N C.

Pl. 23. F. 2.

Figure 2.

The second Figure shews an End of the Timber or the Section of the Machinery to move the Bolts as cut across one of the Bolts, and at right Angles to the Axis at A, where is seen the Hole P, over the Axis, and the prick'd Lines E F shews the Pin or Shaft of the Screw on which the Bolt slides as it comes out of its Staple.

Pl. 23. F. 3.

Figure 3.

The third Figure represents the rectangular Elbow-Piece, such as B L N, or C N M of the last Figure, where is seen the Center of Motion round the Pin N, the End L receiving the protruding End of the horizontal Bar, and the End B receives the End of the Bolt to throw it forward as at B, Fig. 1. or draw it back as at C, in the same Figure.

Pl. 23. F. 4.

Figure 4.

The fourth Figure represents the Iron-work about the End of the Timber at I I I, with the Holes for the cross Pins, the Bolt B A and Staple P.

Before I begin my next Lecture, it may not be improper to give an Account of a Man of very great natural Strength, who lives now here in London, *and shews several surprizing Effects of his Strength. I should indeed have given an Account of him in the 7th Annotation; but I was unwilling to do it upon common Report, till I had seen him myself, which I only did since the last Sheet was printed off.*

Thomas

Thomas Topham, born in *London*, and now about 31 Years of Age, five Feet ten Inches high, with Muscles very hard and prominent, was brought up a *Carpenter*, which Trade he practised till within these six or seven Years that he has shewed Feats of Strength; but he is intirely ignorant of any Art to make his Strength appear more surprizing: Nay, sometimes he does Things which become more difficult by his disadvantageous Situation; attempting and often doing, what he hears other strong Men have done, without making use of the same Advantages. Annotat. Lect. IV.

About six Years ago he pulled against an Horse, sitting upon the Ground with his Feet against two Stumps driven into the Ground, but without the Advantages represented by the *First Figure*, *Plate* 19; for the Horse pulling against him drew upwards at a considerable Angle, such as is represented in the 2d Figure of that Plate, when h N is the Line of Traction, which makes the Angle of the Traction to be N h L: and in this Case his Strength was no farther employed than to keep his Legs and Thighs strait, so as to make them act like the long Arm of a bended Leaver, represented by L h *, on whose End h the Trunk of his Body rested as a Weight, against which the Horse drew, applying his Power at right Angles to the End *l* of the short Arm of the said Leaver, the Center of Motion being at L at the Bottom of the Stumps *l*, *o* (for to draw obliquely by a Rope fastened at h is the same as to draw by an Arm of a Leaver at *l* L, because *l* L is a Line drawn perpendicularly from the Center of Motion to the Line of Direction † h N) and the Horse not being strong enough to raise the Man's Weight with such Disadvantage, he thought he was in the right Posture for drawing against an Horse; but when in the same Posture he attempted to draw against two Horses, he was pulled out of his Place by being lifted up, and had one of his Knees struck against the Stumps, which shattered it so, that even to this Day, the *Patella* or Knee-pan is so loose, that the Ligaments of it seem either to be broken or quite relaxed, which has taken away most of the Strength of that Leg. *Pl. 19. F. 2. † Ann. 5. to Lect. III. p 147.

But if he had sat upon such a Frame as is represented in the first Figure *, and described in Page 266, 268, and 269, he might (considering his Strength) have kept his Situation against the pulling of four strong Horses without the least Inconvenience. *Pl. 19. F. 1.

The Feats which I saw him perform a few Days ago, were the following:

1. By the Strength of his Fingers (only rubb'd in Coal-ashes to keep them from slipping) he roll'd up a very strong and large Pewter-dish.

2. He broke seven or eight short and strong Pieces of Tobacco-pipe with the Force of his middle Finger, having laid them on the first and third Finger.

3. Having thrust in under his Garter the Bowl of a strong Tobacco-pipe, his Legs being bent, he broke it to Pieces by the Tendons of his Hams, without altering the bending of his Leg.

4. He broke such another Bowl between his first and second Finger, by pressing his Fingers together side-ways.

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5. He lifted a Table six Feet long, which had half a hundred Weight hanging at the End of it, with his Teeth, and held it in an horizontal Position for a considerable time. *It is true the Feet of the Table rested against his Knees; but as the Length of the Table was much greater than its Height, that Performance requir'd a great Strength to be exerted by the Muscles of his Loins, those of his Neck, the* Masseter *and* Temporal (*Muscles of the Jaws*) *besides a good Set of Teeth.*

6. He took an Iron Kitchin-poker, about a Yard long, and three Inches in Circumference, and holding it in his right Hand, he struck upon his bare left Arm, between the Elbow and the Wrist till he bent the Poker nearly to a right Angle.

7. He took such another Poker, and holding the Ends of it in his Hands, and the Middle against the Back of his Neck, he brought both Ends of it together before him; and, what was yet more difficult, he pull'd it almost streight again: because the Muscles which separate the Arms horizontally from each other, are not so strong as those that bring them together.

8. He broke a Rope of about two Inches in Circumference which was in part wound about a Cylinder of 4 Inches Diameter, having fasten'd the other End of it to Straps that went over his Shoulders; but he exerted more Force to do this than any other of his Feats, from his awkwardness in going about it: for the Rope yielded and stretch'd as he stood upon the Cylinder, so that when the Extensors of the Legs and Thighs had done their Office in bringing his Legs and Thighs strait, he was forced to raise his Heels from their Bearings, and use other Muscles that are weaker. But if the Rope had been so fix'd, that the Part to be broken had been short (in the Manner explain'd in the 7th Annotation of this Lecture) it would have been broken with four times less Difficulty.

9. I have seen him lift a rolling Stone of about 800 lib. with his Hands only, standing in a Frame above it, and taking hold of a Chain that was fastened to it. By this, I reckon he may be almost as strong again as those who are generally reckon'd the strongest Men, they generally lifting no more than 400 lib. in that manner. The weakest Men, who are in Health and not too fat, lift about 125 lib. having about half the Strength of the strongest. *N. B.* This sort of Comparison is chiefly in relation to the Muscles of the Loins; because in doing this one must stoop forwards a little. We must also add the Weight of the Body to the Weight lifted. So that if the weakest Man's Body weighs 150 lib. that added to 125 lib. makes the whole Weight lifted by him to be 275 lib. Then if the stronger Man's Body weighs also 150 lib. the whole Weight lifted by him will be 550 lib. that is, 400 lib. and the 150 lib. which his Body weighs. *Topham* weighs about 200 lib. which added to the 800 lib. that he lifts, makes 1000 lib. But he ought to lift 900 lib. besides the Weight of his Body, to be as strong again as a Man of 150 lib. Weight who can lift 400 lib.

Now as all Men are not proportionably strong in every Part, but some are strongest in the Arms, some in the Legs, and others in the Back, according to the Work and Exercise which they use, we can't judge of a Man's

Man's Strength by lifting only; but a Method may be found to compare together the Strength of different Men in the ſame Parts, and that too without ſtraining the Perſons who try the Experiment. Annot at. Lect. I V.

Here follows the Manner of doing it, which was communicated to me by Richard Graham, *Eſq*; F. R. S. *To which I have made ſome ſmall Additions. But the firſt Hint was from Mr.* Geo. Graham.

Plate 23. Figure 5.

A B C D is a ſtrong Frame of Wood with an Hole thro' the upright Piece D C, at D, big enough to receive an Iron cylindrick Bar of an Inch Diameter or ſomething bigger; a ſtrong Iron Plate being fix'd on each Side, that the Iron may not gull the Hole. This Bar has a Square upon it, whoſe Side is about an Inch and one eighth, to receive the two ſeparate and unequal Arms of a bended Leaver D F, and D E; and then a ſtrong Nut *d* is ſcrew'd over them at D, to keep all tight. The Arm D E, which as a Stilyard is to carry a great Weight W, is kept from falling below an horizontal Poſition by an Iron Pin at K, which ſtops the ſhort Arm D E from inclining towards G; but both Arms are moveable round the Axis D, towards *e*, or N. The Arm D F has a round Croſs-Bar at Top, about ſix Inches long, as may be ſeen in its ſeparate Repreſentation *d f*. To the upright Timber A B, is made faſt the Iron L N, with a Croſs alſo at top (ſee *n l*) and Holes for Iron Pins to faſten it in its Place. There is alſo another ſtrong Piece of Iron H G I, faſtened to the Timber that carries the Leaver by a ſtrong wooden Screw at I, and the Pin K going thro' its Wings and the Timber. See its ſeparate Figure at *h g* I. S is a Collar to put on, when you don't uſe the upright Arm of the Leaver. M is the Center of Gravity of the Stilyard D E.

1. To try one's Strength by Means of the Machine; with the left Hand take hold of the round Part of the Croſs at N, and of the round Part of the Croſs at F, with the right Hand, and bring your right Hand towards the left, in the Direction of F N, which will move D E, and raiſe the Weight W. When you can juſt raiſe it up ſo as make F D quit the Pin at K, the Force of the Arms will thus be found. Multiply the Weight W, (ſuppoſe an half hundred, or 56 lib.) by its Diſtance from the Center W D, (which we will ſuppoſe here 15 Inches) which will give 840 for the *Momentum* of W, on the Stilyard: Then add to it the *Momentum* of the Stilyard itſelf, which you will have by finding what Weight can draw up the Stilyard by its Center of Gravity, namely, the Weight *w* (*Fig.* 6.) which draws it up by a String going over the Pulley C; and multiplying that Weight by M D, its Diſtance from the Center (which we will here ſuppoſe 10 Inches) you will have 60, which being added to 840, make the Sum 900, and that Sum, divided by F D the Diſtance of the Power, will give 90 lib. for the Force of the Man's Arms who applies his Hands at F and N. If another Man raiſes double the Weight at W + ſo much Weight as will anſwer to double the Weight of the Stilyard at M, he is twice as

Annotat. Lect. IV.

strong. The Removal of the Weight W towards E will also serve to shew how much the Force is greater, instead of adding Weight at W †.

†Pl. 23. F. 5. The same Way may be found what Force the Arms can exert in pulling from each other, by applying one Hand at F, and the other at H. And to try the Force with which *Topham* bent the Poker bearing behind his Neck; a Man must put a Strap round his Neck, which must be fastened to F: then the Head being placed on the Side N, of N L, with both his Hands he must take hold of the Cross at N, and push forwards with his Hands, all the while pulling backwards with his Neck, to bring F towards N.

*Pl. 23. F. 6. 2. The sixth Figure is another wooden Frame with the upright Timbers A B, C O, a little farther asunder. * So that a Man may stand upon the strong Plank F G, and with the Girdle and Chain thro' the Hole H, pull up the Stilyard D E by the Hook I; which Stilyard in this Case has not the upright Piece F D, but instead of a bended Leaver becomes a Leaver of the third kind, where D I is the Distance of the Power, and D W the Distance of the Weight; and therefore $\frac{W \times WD + w \times MD}{DI}$ will be the absolute Weight that is lifted, or Force of the Muscles extending the Legs. But here the Weight of the Trunk of the Body must be added, and considered according as some Men are heavier than others. N. B. *The Notch* K *hinders the Leaver from falling below the horizontal Situation*; *and* w *drawing over the Pulley* C, *shews how the Leaver acts at its Center of Gravity: So that when* w *is hung on, the Leaver is to be look'd upon as an Instrument without Weight.*

†Pl. 23. F. 7. 3. The seventh Figure † has the Stilyard at D E with a wooden Cylinder of an Inch and an half, or two Inches in Diameter, upon its Axis continued behind at D F, which Cylinder is drawn separately at *d f*, and its Iron Axis at *g h*, and the Nut at i.

†Pl. 23. F. 8. 4. The eighth Figure † represents a Machine to try the Strength of the Fingers; in which, when you thrust the Fore-finger under G, and the third Finger under H, which are fix'd Points, the middle Finger may push upon N to lift a Weight at W, in the same manner that *Topham* breaks the Pieces of Tobacco pipes. If the Hand being held with the Palm upwards, the two first and two last Fingers be thrust in under G and H, and the Thumb presses upon N, that will shew in what manner the Tops of Pewter Pots and Silver Tumblers have been squeez'd together by Men very strong in the Fingers; and thus will be shewn the Strength that any Man can exert for such an Action, *&c.* N. B. *The 9th Figure is only Part of the 8th Figure drawn larger.*

LECTURE

LECTURE V.

Concerning Sir Iſaac Newton's *three Laws of Motion.*

1. THERE are ſeveral Organs or Inſtruments, which may be call'd Mechanical, or enumerated among the Inſtruments, commonly, but erroneouſly called *mechanical Powers*; which I purpoſely omitted (or only mention'd without explaining their manner of acting) in the third Lecture; becauſe the Knowledge of the Laws of Motion is neceſſary for underſtanding upon what Principle they act: And ſuch are the *battering Ram*, the *Hammer* or *Mall*, the *Fly*, the *circular Pendulum*, the *Sling*, and the *Bow* or *Spring*. Lect. V.

I SHALL therefore in the firſt Place conſider thoſe Laws, and draw ſeveral Corollaries Conſequences from them, which I ſhall illuſtrate by Experiments, and apply them among other Things to explain the Uſe of thoſe Inſtruments.

LAW I.

Every Body perſeveres in a State of Reſt, or of uniform Motion in a right Line, unleſs it be compell'd to change that State by Forces impreſs'd thereon.

2. THERE is in all Matter (whatever kind of Body it be ſhaped into) an Inactivity, whereby it reſiſts any Force that endeavours to make it change its State, in proportion to its Quantity of Matter; and this is call'd the *Vis Inertiæ*, or Force of Inactivity. For it is as impoſſible for a Body to go into Motion of itſelf from Reſt, as to change its Shape from one Figure to another. This is evident to Senſe, and I believe no Body doubts of it; but the ſecond Part of this Law does not appear ſo evident without a little Attention.

We ſee plainly that there muſt be ſome extrinſecal Agent or Power not eſſential or belonging of Neceſſity to the Body, to put it into Motion: but we don't ſo readily perceive that a Body in Motion would continue to move for ever without the Action of an extrinſecal Agent; becauſe we ſee Bodies here on Earth gradually loſe their Motions, and for want of attending to all the Cauſes that deſtroy the Motion of Bodies, we often imagine that Motion languiſhes, and at laſt quite periſhes of itſelf. But if we conſider what external Cauſes retard and deſtroy Motion, we ſhall ſoon perceive that if thoſe Cauſes were remov'd, a Body once put into Motion in any Direction would continue in that Motion and Direction for ever.

A Stone thrown forwards with the Hand goes on with the Motion that it has received from the Hand, and would continue in that State of Motion for ever, if there was neither Air nor Gravity. When we conſider the Reſiſtance of the Air, it is evident, that the Body in going forwards muſt remove the Parts of the Air to make itſelf way; and as it muſt communicate Motion to thoſe Parts which it removes, ſo much Motion as it communicates, ſo much it muſt loſe; ſo that after ſome Time, and having gone through a certain Space, it muſt hang in the Air immovable, that is, if nothing but the Air acted upon it. But beſides the Air, Gravity (which is a Force puſhing downwards) alters its Direction, and brings it gradually to the Ground. This Endeavour, which every Body has to continue in its State of Motion, does not ſeem to ſome to be properly call'd a Force of Inactivity (*Vis Inertiæ*;) but when we conſider that the Body is purely paſſive, neither augmenting nor throwing off any of its Motion of itſelf, we ſhall find it to be intirely inactive even in that State. Thus when we ſtand by a River-ſide, and obſerve a live Fiſh carried down the Stream, he is wholly inactive in that Motion, and continues in that Motion all the while he is inactive: it muſt be an Action exerted (equal to the Force of the Water) that will make him come to Reſt, and appear ſo in reſpect of the Spectator on the River-ſide.

Suppose a Man exerts a certain Force to roll a Bowl on a Bowling-green neither mow'd nor roll'd, and can only throw it 20 Yards with that Force; when the Green is mow'd, he ſhall with the ſame Force throw it farther, 30 Yards for Example; then if the Green be roll'd as well as mow'd, the ſame Force may throw the Bowl (for Example) 40 Yards: and ſtill if more of the Obſtacles are taken away, the

the Bowl will yet go farther. Whence one may eaſily conclude, that if the Plane on which the Bowl rolls could be made perfectly ſmooth and mathematically true, the Bowl truly ſpherical, and the Reſiſtance of the Air wholly taken away, the Bowl would for ever roll (or rather ſlide) on that Plane if it was infinitely extended. Lect. V.

3. We have ſhewn by a Quotation from Sir *Iſaac Newton* (L. 3. N° 85.) that if a Body be acted upon by two Forces whoſe Directions make any Angle, the Body will move in the Diagonal of a Parallelogram, two contiguous Sides of which repreſent (by their Lengths) the reſpective Quantity of thoſe Forces, and (by their Inclination) their Directions: and alſo that the Body will go thro' the whole Diagonal by the Action of the ſaid two Forces *, in the ſame Time that it would have gone thro' either of the contiguous Sides, if only one of the Forces had acted upon it. * Ann. 1.

This is not ſo readily conceived by thoſe who are not accuſtom'd to mathematical Reaſoning; becauſe, while they obſerve the Action of one of the Forces, they do not attend to the other: But it will appear very evident, by conſidering ſuch Caſes wherein the Space in which the Body moves is carried * in a different Direction from that which the Body appears more immediately to be moved in. As for Example, let us ſuppoſe T S († *Pl.* 23. *Fig.* 10.) to be a *Tragſchuit* or *Dutch* travelling Boat (I mean the Plan of it) H *b g* G, the Canal in which the Boat goes in the Direction T *s*, and A and B two Perſons ſitting over-againſt one another in the Boat. Now let us ſuppoſe the Perſon at A toſſing any Body, as for Example, a Ball, to the Perſon at B, and ſo reciprocally: All the People in the Boat will conſider the Ball ſo toſs'd as only moving in the Line A B, whether the Boat ſtands ſtill or goes on along the Canal, tho' there be but one Force acting upon the Ball in the Direction A B when the Boat ſtands ſtill, whereby it really moves in that Line; but when the Boat goes along, another Force in the Direction A *a* does alſo at the ſame Time act upon the Body, which by that compound Action is really carried in the Line A *b*, tho' the Perſons ſitting by fancy the Ball to go ſtill in the Line A B, becauſe they are carried along too, and forget that the Force which draws the whole Boat does alſo carry the Ball along, which is a Part of it, as well as themſelves. Now this will be evident to the Sight of a Man that ſtands upon the Bank at C; for when the Boat ſtands ſtill, he looking towards D on the oppoſite Bank, ſees the Ball move (as it really does) in the Line A B. * Ann. 2. † Pl. 23. F. 10.

Lect. V. A B. But when the Boat moves and goes from the Poſition T S to the Poſition *t s*, in the ſame Time that the Ball moves acroſs from the Perſon at A to the Perſon at B, it is plain (and will be ſeen by a Man on the Bank at C) that the Ball moves in the Diagonal Line A *b*, becauſe the Perſon who was at B does not receive it till he is carried to *b* by the Motion of the Boat; which his Oppoſite (being at the ſame Time carried to *a*) does not attend to, tho' the Thing be ſo viſible to the Man at C. Yet if the Man at C ſtood upon a Plank at A C faſten'd to the Boat, being then carried along from C to *c*, he would not diſtinguiſh between the Line A B and the Line A *b*, but (when he came to *c*) ſuppoſe that the Body had mov'd in the Line *a b*, which he would miſtake for A B; unleſs looking over the Canal, he ſhould conſider that he no longer ſaw the Point D over-againſt him, but the Point *d*. Thus, in a Ship, if a Man ſtanding on the Deck near the Maſt toſſes up a Stone to another who ſtands on the Round Top, or receives a Stone dropp'd from thence, the Stone will move parallel to the Maſt (ſuppoſing the Maſt upright) whether the Ship be at Anchor or under Sail; tho' in the firſt Caſe the Stone moves perpendicular to the Horizon, and in the laſt Caſe oblique to it, as it riſes or falls in the Diagonal of a Parallelogram, whoſe contiguous Sides repreſent the two Forces acting upon the Body, the one perpendicular, and the other parallel to the Horizon; juſt as in the laſt Inſtance, the two Forces A B and A *a* acted upon the Ball horizontally projected in the Boat.

N. B. *Whether the Boat goes faſter or ſlower (in which Caſe* A b *becomes longer or ſhorter) the Ball will perform its diagonal Motion in the ſame Time; for ſince the Force* A B *continues the ſame, all that is done by the lateral Force* A a *is only to make the Ball come to a different Point of the Line* B n, *as* m *or* b *or* n, *inſtead of* B; *while the Force* A B *in the ſame Time carries it croſs the Boat, or from ſome Part of the Line* A a *to ſome Part of the Line* B b.

I HAVE been the more prolix in this Explication, becauſe I have met with a great many Perſons of good Senſe, who for want of Attention have not been able to conceive this compound Motion, on which all Mechanicks are founded. For this Reaſon I have ſubjoin'd Variety of Experiments to the ſame Purpoſes; becauſe if one don't, another may, convince.

EXPE-

EXPERIMENT I. *Pl.* 23. *Fig.* 11. Lect. V.

THE Machine of *Plate* 23. *Fig.* 11. consists of a Brass Plate ABCD, upon which another Plate IKLM slides forwards and backwards in the Direction T S or S T, between the Rulers A D and B C, under the Edge of two other Rulers E F and G H, of which E F is a little Rack with Teeth to receive the Teeth of the Wheel N, which is fasten'd to the Plate I M K L by the Cock N O, and turns round its Axis N, as the said Plate is push'd forwards and backwards. The Wheel in its Motion along E F carries another Rack P Q at right Angles to the former, either to the right or to the left, according as the Axis of the Wheel at N is push'd in the Direction T S or S T. This last Rack has an Arm S R with a little Socket at R to receive a black-lead Pencil, which according to the Motion of the End R will draw a black Line upon the Paper on which the Machine is to be laid. Pl. 23. F. 11.

THERE must be drawn upon the Paper a Square *e f g h*, one of whose Sides is equal to the Distance between the Edges of the Rulers A D and B C. Then having push'd up the Wheel N as far as it will go, lay the Machine upon the Paper, so that its End A B may be parallel to *g h* one Side of the Square above-mention'd, at such a Distance as to have the Point of the Pencil fall upon the Point *e*, and the Edge of the Rack P Q over the Line *a b*: then with one Hand pressing down the Plate A B C D of the Machine at T, to keep it immoveable upon the Paper, pull down the Cock N O in the Direction S T a Length equal to *e g*; and the Pencil, instead of describing the Line *e g*, will describe the Diagonal Line *e h*, because as the Wheel in its Descent is turn'd round by the Rack E F, it throws the Rack P Q to the left, just the Length of *e f*, by means of the Teeth in the said Rack, and the Slit in it which lets it go laterally along the Pins I M by the Action of the Wheel, while the said Pins bring it perpendicularly down from the Line *a b* to the Line *d c*. And to shew that the Diagonal is describ'd in the same time that the Side *e g* would have been described if there had been no lateral Motion; take off the Wheel, and then the Pencil's Point will describe the Line *e g*, while the Cock is drawn down the same Length as before, or push'd up again. This shews that it is the same thing for a Body at *e* to be acted upon by two Forces in the Directions *e f* and *e g*, as to be carried from the Line *e g* to the Line *f h*,

Lect. V. *f h*, at the ſame time that it is brought down from the Line *e f* to the Line *g h*.

Experiment II. *Pl.* 23. *Fig.* 12.

Plate 23. Fig. 12. The 12th Figure of Plate 23 is a ſquare Frame of Wood, of which the Part C D B E is to be ſet up on its Side B E, whilſt the other Part of it *c d e b* is drawn out of its Groove into the Poſition *d δ ε e*; by which means the Ball A is made to deſcribe the Diagonal A *a* aſcending, and the ſame Diagonal deſcending is deſcrib'd when the ſliding Part is puſh'd back into the Frame in the Direction *δ d*. There is a Wire A *e* fix'd to the ſliding Part for the Ball to move up and down on, and a String *b c* which goes thro' an Hole in the Pieces *d c* and D C, ſo as to have its End faſten'd at C; by which means the Ball muſt riſe when *d δ ε e* is drawn out laterally; and it falls back again by its own Weight when *d δ ε e* is puſh'd in again: and when *d δ ε e* is left in its containing Frame, the Ball will riſe and fall by pulling or letting go the String C *c b*. If the Wire A *c* be conſider'd as a Rope parallel to the Maſt of a Ship, what we have ſaid of a Body let fall from the round Top of the Maſt, and moving parallel to the Maſt, (whether the Ship be at Anchor or under Sail) by one, or two Forces acting upon it, will be illuſtrated.

But as I have heard ſome object, that this, and the laſt mention'd Machine, did indeed, by the Contrivance of them, make a Body move in a Diagonal; but they did not prove that Nature acted that way: I have made the thing evident to Sight by the following

Experiment III. *Pl.* 23. *Fig.* 13.

Pl. 23. F. 13. * The round Table G E F D ſtands upon a Pivot P, and has an Hole on each Side of its Center half way from the Circumference, at A and B. At right Angles to the Line that goes thro' A C B is drawn a Diameter D E, and from ſeveral Points of that Diameter equally diſtant from the Center on each Side, are drawn the Rhombs or equilateral Parallelograms, D A E B, *d* A *e* B, and *δ* A *ε* B. The Ball D has two Strings faſten'd to it at the ſame Point, and thoſe Strings being carried thro' the Hole A B, have hanging at their End two equal Weights W and X, which by the joint Force

of

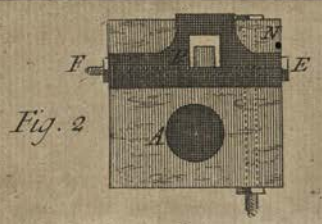


Fig. 2

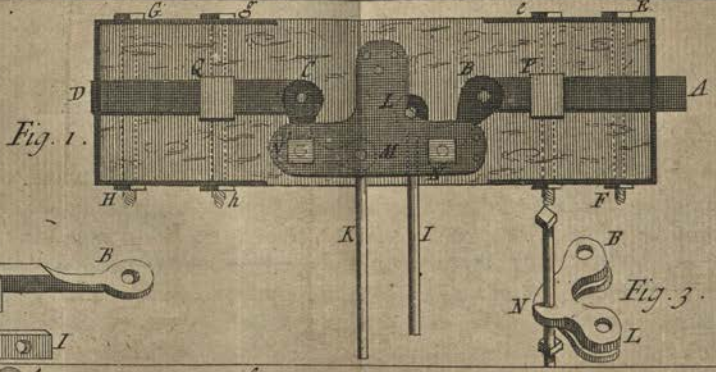


Fig. 1.

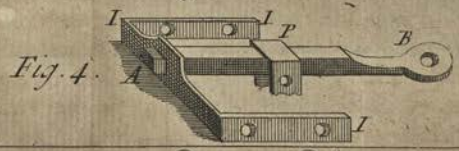


Fig. 4.

Fig. 3.

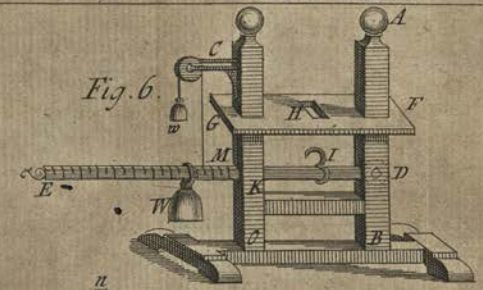


Fig. 6.

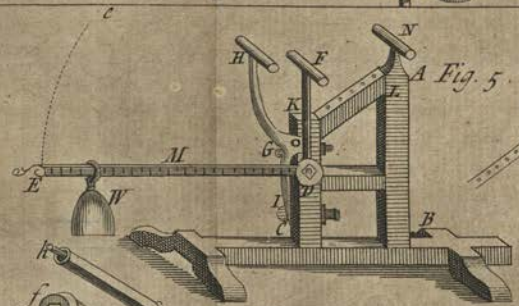
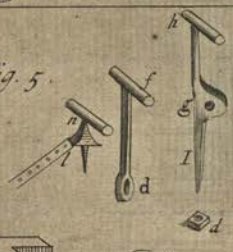


Fig. 5.



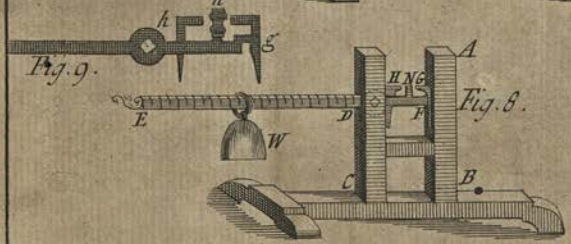
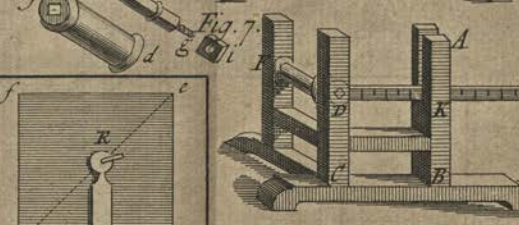


Fig. 9.

Fig. 8.



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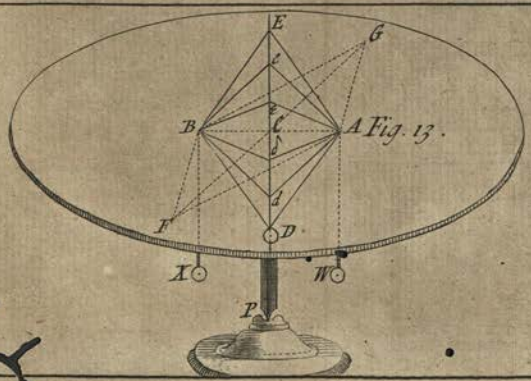

Fig. 13.

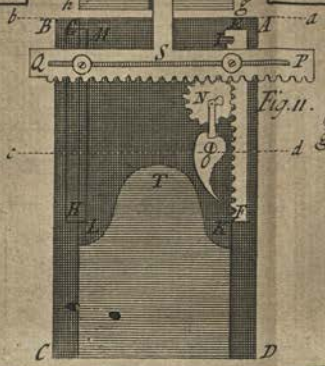

Fig. 11.

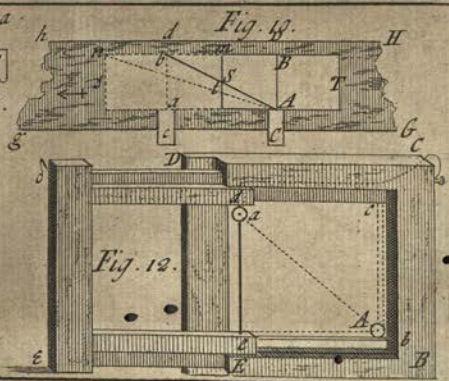

Fig. 12.

of their Deſcent will bring the Ball D to reſt upon C the Center of the Table, whilſt the Parts of the Strings on the Table are in the Line A C B. If the Ball D be pull'd back to *δ*, *d*, or D (whereby the Weights W and X will be rais'd up, and the Strings will lie upon and have their Directions in the contiguous Sides of one of the Parallelograms) as ſoon as you let it go again, it will run in the Line D E, which is the common Diagonal of all the Parallelograms; as may be beſt perceived by holding one's Finger or a Wire at the Center C, which the Ball will ſtrike in its Motion. *Here it is plain, that the natural Gravity of the Bodies* W *and* X *is the only Force (divided into two equal Parts) which acts upon the Ball* D, *and that whether the Directions of thoſe Forces be at acute, right, or obtuſe Angles, the Ball is ſtill carried in the Diagonal of a Parallelogram of equal Sides, repreſenting equal Forces. But if one of the Weights as* W, *be taken as heavy again as the other, the Ball from* D, *d*, or *δ*, *will run in a Direction between* A *and* C; *and not run over the Center till you let it go from* F, *in which Caſe the two Strings will make two contiguous Sides of the Parallelogram* F A G B, *whoſe Proportion to each other is alſo as Two and One; the Diagonal being* F C G, *which paſſes thro' the Center.*

It was objected to *Galilæo*, when he aſſerted the Motion of the Earth, that if the Earth did turn round from *Weſt* to *Eaſt*, a Cannon-Ball ſhot upright, would not fall down in, or near the Place from which it was ſhot (as we ſee by Experience it does) but move to the *Weſt* according to the Space of Earth that had been carried towards the *Eaſt*, during the Riſe and Fall of the Ball; which he anſwer'd, by ſaying, that the Ball is acted upon by two Forces. One, that of the Powder, which throws it up; and the Other, that of the Earth, which carries it to the *Eaſt*, making it in its Riſe deſcribe a Diagonal Line towards the *Eaſt*, and from the upper End of that Diagonal, another Diagonal Line alſo towards the *Eaſt* in its Fall. He alledg'd the Caſe of the Ship that we have already mention'd. But nothing will make the Thing ſo plain as the following

Lect. V.

EXPERIMENT IV. *Pl.* 24. *Fig.* 1.

Pl. 24. F. 1. TO an horizontal Piece of Wood G H, fix'd about 10 Feet above the Floor of my Room, I ſcrew'd the two Hooks S *s*, and by means of four Strings hung on the ſaid Hooks, the Braſs Plate or flat Pendulum ABCD, which (by reaſon of the Diſtance of the Strings at their Points of Suſpenſion S *s*) perform'd its Vibrations in a circular Arc over the graduated upright wooden Plane E F which ſtood under it, the Center of the Pendulum never moving out of the Plane of the Wood as it vibrated in the Direction E F, or back again, at the Diſtance of about nine Feet from the Piece G H. Over the two Pullies I, K, ſcrew'd into the upper Side of the Piece G H, ran the String LKIW, whereby the Weight W might be let down upon the Pendulum ABCD, or lifted off from it at pleaſure; there being a Hole thro' G H, juſt under the Circumference of the Groove of the Pulley I.

LET the Pendulum (the Weight W being upon it) be drawn out of the Perpendicular to E, and then, when it is looſed, it will vibrate towards F, and ſo back again towards E, *&c.* for a conſiderable Time. During that Motion, if by pulling the String at L the Weight W be lifted off of the ſwinging Plate, and then let down again, it will always fall upon the ſame Part of ABCD, tho' ABCD be mov'd a great way out of its Place during the Time that the Weight W is off of it; and that, at what ever Part of the Vibration W is lifted off, and at whatever Part of it, it is receiv'd again. For tho' the Force of the Hand pulling the String is the Cauſe of the Riſe of W, and Gravity the Cauſe of its Deſcent; yet it has alſo another Force acting upon it, which moves it in the Arc of Vibration, as Part of the Pendulum compounded of A B C D and W. Juſt ſo as any Part of the Earth, on which a Cannon is ſet upright, is carried from *Weſt* to *Eaſt*, by the Motion of the Earth round its Axis; the Cannon Ball ſhot up, beſides the Motion receiv'd from the Exploſion of the Powder, has another Force impreſs'd upon it by the Earth's Motion, as it is a Part of the Earth, and is carried from *Weſt* to *Eaſt* in the Air, as well as the Cannon is upon the Earth, ſo as to fall down again into, or near, the Mouth of the Cannon.

A CANNON-BALL or any other Projectile never deſcribes a right Line in its Motion, but when it is thrown perpendicularly up or down,

down, which right Line of Ascent or Descent is the Diagonal of a Parallelogram, as we have already shewn; but *when the Body projected is thrown horizontally or obliquely, it describes a Curve call'd a parabolick Line.* Now this Curve is made up of an infinite Number of small Diagonal Lines, that continually change as the Directions of the Forces alter in the Body's compound Motion, which is easily explained from what has been said of the Action of two Forces, and the Consideration of the Acceleration of Motion in the Fall of Bodies; which (as it is a Fact and Matter of Observation) must be now taken for granted; tho' we shan't explain the Cause of it, but in the Explication of the second Law of Nature. Lect. V.

LET a Body at A (* *Pl.* 24. *Fig.* 2.) as for Example, a Cannon-Ball, be shot forward in an horizontal Direction A F, so as to move with a determinate Velocity; for Example, 4 Rods in a Second of Time, or from A to B. It is evident by the first Law, that it must the next Second go on in the same Direction thro' the same Space, *viz.* from B to C, and so to D, then to E, then to F, *&c.* It is also plain, that if there be another Force whose Direction is at right Angles to the Line A F, as in the Line A L, which is perpendicular to the Surface of the Earth, (or to take the thing more generally, a Force pressing from the Line A F towards the Line L*f*.) the Action of this last Force upon the Body setting out at A will neither accelerate nor retard its Motion from the Line A L to the Line B M, nor from the Line B M to the Line C N, nor from the Line C N to the Line D O, nor from the Line D O to the Line E P, nor from the Line E P, to the Line F*f*; but only cause the Body to go thro' different Points of those Lines instead of the Points B, C, D, E, F. Now I say, that *b*, *c*, *d*, *e*, and *f*, will be the Points of the Lines BM, CN, DO, EP and F*f*, which the Body will go thro'. The Force pressing downwards in the Direction A L is the Force of Gravity, by whose Action it is observ'd that Bodies fall with an accelerated Motion, in such Manner that in the first Second of Time a Body falls 1 Rod (or 16½ Feet,) the 2d Second, 3 Rods; the 3d Second, 5 Rods; the 4th, 9 Rods, *&c.* which Spaces are here express'd by A G, G H, H I, I K, and K L. And if the falling Body, during the Time of its Fall, be acted upon by another Force in an horizontal Direction, as in the Direction A F, (or rather from the Line A L towards the Line F*f*) that Force will not at all disturb the uniform Acceleration of the Motion of the *Pl. 22. Fig. 2.

Lect. V. the descending Body, but only make it fall to some other Point of the Line L*f* (or of the Surface of the Earth) than the Point L, as for Example to the Point *f*.

The several Points of the Curve in which the Body moves will be thus found. The Body at A is acted upon by the two Forces A B and A G; the Lines G *b* and B *b*, respectively equal and parallel to A B and A G, compleat the Parallelogram A *b*: consequently the Body must move in the Diagonal A *b*. The Body at *b* is acted upon horizontally by the Force *b k* equal to A B (or brought down from B C) and at the same Time by the Force *b h* increas'd by Acceleration from 1 to 3, and transferr'd from G H to *h h*; for as without the horizontal Force, it would in the 2d Second of Time have carried the Body from G to H (as it is not alter'd in the Quantity of its Effect by the horizontal Force) it will therefore be able from *b* to carry down the Body to *h*; but the horizontal Force *b k* acting at the same Time, does in the same Time carry the Body from the Line *b k* to the Line *h c*, as the Force *b h* carries it from the Line *b k* to the Line *h c*; and consequently at the End of the 2d Second of Time the Body will be at *c*, where the two above-mentioned Lines meet, having describ'd the Diagonal *b c* by the joint Action of the Forces *b k* and *b h*. The Body at *c* is acted upon by the horizontal Force *c δ*, (=CD) and the perpendicular Force *c i* (so increas'd as to be = H I) at the same Time; therefore the Body will go in the Diagonal *c d*. For the same Reason, when the Body is at *d* it will go in the Diagonal *d e* of the Parallelogram *d ε e κ*, by the joint Action of the Forces *d ε* and *d κ*; and so from *e* it will go to *f* in a new Diagonal, *viz.* that of the Parallelogram *e φ f l*, by the Action of the two Forces *e φ* and *e l*, &c.

SCHOLIUM.

If we had divided the Time of the whole Motion into more, and therefore less, Parts; we should have had more Diagonals from A, the Beginning of the Fall of the projected Body, to *f* the End of it, which would have made its way A *b c d e f* more curve by having more Changes of Direction in it. Now as the Motion of Bodies downwards by the Force of Gravity is continually increasing (or the Line A G continually becoming longer) and not increasing by Fits, as we have suppos'd it, to make the Action of the two Forces the better

conceiv'd,

conceiv'd, there is a continual Change of Direction in the Motion of the Body forwards and downwards (or the Diagonals become infinitely small) so that it describes a Parabola in going from A to *f*. Lect. V.

THIS may be illustrated by the following

EXPERIMENT V.

THE Machine represented by the 3d, 4th, and 5th Figure of *Pl.* 24. was contriv'd by Dr. 's *Gravesande* (Professor of Mathematicks and Astronomy at *Leyden*) to make this parabolick Motion of Projectiles evident to Sight. A B *b* G F is a solid Piece of Wood 15 Inches high and two Inches thick, standing up upon its End F G, and set upon the Board or flat Piece F E, which has a little shallow Pit at E to be fill'd with soft Clay; so that when the Ball B falling from B along the curve Surface B *b*, (which is made very smooth, or faced with Brass or planish'd Tin) and describing in the Air the parabolick Line *b* ϰ *δ* E, it may make a Mark to shew exactly the Place where it fell. The Piece G (*Fig.* 4.) is sometimes to be put on upon G E of *Fig.* 3. where the prick'd Lines represent it at G 3 *δ*, having also a Pit to fill with Clay to receive the Mark of the falling Ball at *δ*. The Piece K (*Fig.* 5.) is to be put on over it upon Occasion, where it is represented by the prick'd Lines 1 3 ϰ, to intercept the falling Ball, and shew by the Clay in its Pit where the Point ϰ, of the Line *b* ϰ, falls. There is also an upright Board B D E to be taken off and on upon Occasion, on which are fasten'd the 3 Rings *r*, *r*, *r*, thro' which the Ball will move when it describes a Parabola from *b* to E; the Pieces of *Fig.* 4. and *Fig.* 5. being then remov'd. Pl. 24. Fig. 3, 4, 5.

THE Height B C, or the Distance between the Line A B D and the Line C *b c d e*, is of six Inches; and the Height *b* G is of nine Inches, being divided into three unequal Parts, of which the uppermost contains 1 Inch, the next 2 Inches, and the lowest 5 Inches. Now if the Time, in which the Ball falls from *b* to G, be divided into three equal Parts (which we will here call Instants) the Ball will in the first of them fall the first Space mark'd 1, that is, one Inch; and in the next Instant the Space 3, or 3 Inches; and in the 3d Instant the Space 5, or 5 Inches. N. B. *This happens on account of the accelerated Motion, the Cause of which we shall explain in considering the second Law of Motion.* When we let the Ball fall from

Lect. V.

B, and it rolls down along the curve Surface B *b*, it has acquir'd such a Velocity at *b* as is able to throw it forwards (double the Distance B C or Height from which it fell) by an uniform Motion, *viz.* from *b* to *e*, in the horizontal Line *b c d e*, if it was not forc'd out of it downwards by the Action of Gravity; but Gravity neither accelerates nor retards its Motion forwards, only makes it come to the Point E instead of the Point *e*, in the same perpendicular Line, and in the same Time. And if we consider two other Points of the Parabola *b* ϰ δ E, *viz.* ϰ and δ, we shall find, that the Ball which in the first Instant of Time must have gone to *c* by the horizontal Motion or projectile Force, is brought down by Gravity to ϰ; and instead of being carried to *d* the second Instant, it is brought to δ in the Parabola, which Point is directly under *d*, and in the same horizontal Line as the End of the 4th Inch or 2d Space in the Line of Fall *b* G. To prove it experimentally; put on the two Pieces G and K of *Fig.* 4 and 5. then letting the Ball fall from B, it will run down the Curve B *b*, and then in the Air go from *b* to ϰ, where it will make an Impression on the Clay. Take off the Piece off *Fig.* 5. and let the Ball roll down again from B, and it will fall on the Point δ, as will also be seen by its Impression on the Clay at δ; then take off the Piece of *Fig.* 4. and the Ball will go to E: and lastly, when you put on the Board B D E with its Rings *r*, *r*, *r*, the Ball falling from B will describe the Parabola *b* E passing thro' all the Rings, tho' there were as many more, provided the Parabola pass thro' their Centers. N. B. *The Reason why the Ball falling in the Curve* B *b acquires a Velocity to carry it forwards uniformly twice the Length or Height* B C, *in a Time equal to the Time of the Fall, will be also shewn in explaining the next Law.*

If a Cannon-Ball or any other Body be shot or projected obliquely upwards, it will likewise by its Rise and Fall describe a Parabola.

Constuction. *Pl.* 24. *Fig.* 6.

Pl. 24. F. 6.

Draw a Line A *i* for the Amplitude of a Parabola, and upon its middle Point 3 raise the Perpendicular 3 E, upon which take the Height 3 *e* for the Axis of the Parabola. From *e* to E upon the Axis continued set off *e* E = *e* 3, and draw the Line A E, which will be a Tangent to the intended Parabola in the Point A. Divide the Line A E into any Number of equal Parts; as for Exampl four.

four, which will be mark'd by the Points B, C, D, E: continue on the Line AE beyond F, so as to set on upon it four more equal Parts as EF, *&c.* not express'd in this Figure for want of Room, but supposed to be over the Points G, H, I. Then from the Points B, C, D, F, *&c.* let fall the Perpendiculars B*k*, C*m*, D*o*, F*s*, G*u*, H*x*, and lastly I*i*, which will fall upon the Point *i* of the Line A*i* (or Amplitude of the Parabola.) Divide the Axis *e* 3 into 16 equal Parts, and mark the Points *q*, *z*, 2, at the End of the first, fourth, and ninth Divisions; so that the four equal Parts, *eq*, *qz*, *z* 2, and 2 3 may respectively contain one, three, five, and seven of the equal Parts. Thro' the Points *q*, *z*, and 2, draw the Lines *df*, *cg*, and *b h* parallel to the Line A*i*, which Lines will meet the Perpendiculars in the Points *b*, *c*, *d*, *f*, *g*, and *h*, and be Ordinates to the Axis *e* 3. Upon the Line *e* E or Axis continued mark *ep* = *eq*, and *e*Z = *ez*, and likewise *e*K = *e* 2: from the Points K, Z, and *p* draw K*b*, Z*c*, *p d*, and on the other side *pf*, Z*g*, and K*h*. If a curve Line be drawn thro' the Points A, *b*, *c*, *d*, *e*, *f*, *g*, *h*, and *i*, it will be a Parabola, or the Line which a Projectile shot from the Point A in the Direction AB will describe; provided it has a Velocity given to it able to carry it the Length AB, in the same Time that by the Force of Gravity it would fall the Length B*b*. Produce to, or beyond *t*, *w*, and *y*, the Lines *pf*, Z*g*, and K*h*, which (as well as *p d*, Z*c*, and K*b*) are Tangents to the Parabola in the Points *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*. Then thro' those Points draw the Lines *b a*, *c k*, *d m*, *e o*, *f q*, *g s*, *h u*, and *ix*, severally parallel to those Tangents, which will be terminated by the Perpendiculars at the Points *a*, *k*, *m*, *o*, *q*, *s*, *u*, and *x*, and compleat the Parallelograms AB*b a*, *b l c k*, *c n d m*, *d p e o*, *e r f q*, *f t g s*, *g w h u*, *h y i x*. A*a*, B*b*, *bk*, *lc*, *cm*, *nd*, *do*, *pe*, *eq*, *rf*, *fs*, *tg*, *gu*, *wh*, *hk*, *yi*, are all equal; which may be easily demonstrated from the Construction, and the Nature of the Parabola; but we rather chuse to demonstrate it from Gravity, in the following Explication. Lect. V.

Demonstration.

Let a Ball be projected in the Direction ABCD, *&c.* (*Pl.* 24. *F.* 6.) By the first Law it would go thro' the equal Spaces AB, BC, CD, DE, EF, *&c.* in equal Times or Instants, and go on continually in the Line AF, *&c.* if Gravity did not act upon it to bring it down; but for the Reasons before alledged during the first Instant, Gravity will have brought it down to *b* (or, which is the same thing, it will move in the Line A*b* Diagonal of the Paral- Pl. 24. F. 6.

Lect. V. lelogram A B *b a*, two contiguous Sides of which B A and A *a* represent the two Forces that act upon the Body;) and at the End of the second Instant, instead of going forward according to the new Direction A *b* in the Line *b l*, it will be brought down to *c*, moving in the Line *b c* Diagonal of the Parallelogram *b l c k*, where *b l* represents the *now* projectile Force, and *b k* = B *b* = *l c* the Force of Gravity, which always pressing downwards with the same Force, must be represented by the equal Perpendiculars A *a*, *b k*, *c m*, *d o*, *e q*, *f s*, &c. whilst the projectile Force in its different Directions is represented by the unequal and decreasing Lines A B, *b l*, *c n*, and *d p*, in the Ascent of the Body; but in its Descent, by the unequal increasing Lines *e r*, *f t*, *g w*, and *h y*, all Tangents to the Parabola. To go on, the projectile Body will successively describe the Diagonals *c d*, *d e*, *e f*, *f g*, *g h*, and *h i*, of the Parallelograms, two contiguous Sides of which represent the Quantities and Directions of the Forces *c n* and *c m*, *d p* and *d o*, *e r* and *e q*, *f t* and *f s*, *g w* and *g u*, *h y* and *h x*.

N. B. *We have call'd Diagonals the Curves intercepted between the Perpendiculars at the Points* A, b, c, d, e, f, *&c. or, which is the same thing, made the Diagonals curved; because those Diagonals are really such, as we have hinted in the Explication of the Motion of a Body, projected horizontally.*

SCHOLIUM.

The projected Body moves with a Velocity continually diminish'd till it comes to the Vertex of the Parabola at *e*, then goes down the other half of it with a Motion continually accelerated. For since the Spaces AB, BC, CD, DE, EF, *&c.* have been taken equal in the Line of the first Direction of the Projectile; by the first Law, the Projectile must go thro' them in equal Times, if none but one Force (*viz.* the projectile Force) acted upon the Projectile. But as Gravity acts upon the Body at the same Time, tho' it cannot hinder it from successively reaching the Perpendiculars (which are equidistant and parallel to each other) in the same Space of Time that it would have done if there had been no Gravity, it will bring the Body, at the End of every Instant, to other Points of the Perpendiculars, which are still nearer and nearer to each other, till the Body comes to *e*: as for Example, the Body at the End of the first Instant, instead of being at B, is brought down by Gravity, to *b* falling one Space equal to *e q*; at the End of the second Instant, instead of

of being at C, it is brought down to *c*, having fallen from the first Direction or Line of the projectile Force thro' a Space equal to *e z*, or four Times *e q* = B*b*, the Space fallen thro' the first Instant; at the End of the third Instant, instead of being at D, it will be brought down to *d* thro' a Space equal to *e* 2, containing 9 Times *e q*, or B *b*, the Space fallen thro' the first Instant: at the End of the fourth Instant, instead of being at E, it will be brought down to *e*, thro' a Space equal to *e* 3, containing sixteen of the Spaces of the first Fall B*b* or *e q*. Now since the Spaces which the Body goes thro' in its Ascent, in equal Times (that is, the Lines A*b*, *b c*, *c d*, and *d e*) are not only less than the Spaces A B, B C, C D, D E, but also less than each other, because their Change of Direction makes them continually cut the parallel Perpendiculars at less oblique Angles; it is evident that *the projected Body continually diminishes its Velocity till it comes to its utmost Height at* e, the Vertex of the Parabola. And this will also appear, if we consider the Motion of the Projectile here, as we did that of the horizontal Projectile; *viz.* by examining how several Parts of the Line describ'd by the Projectile are like so many Diagonals describ'd by the Action of two Forces, one of which changes its Quantity and Direction by extremely small Intervals, tho' here we take great Intervals to make the Thing more plain. Lect. V.

When therefore the Body projected sets out at A in the Direction A B, A B represents the projectile Force, and A *a* the Force of Gravity; as the Lines B *b*, and *b a* compleat the Parallelogram by Construction, the Body will go in the Diagonal A *b* †. The Body being at *b* will, by the first Law, endeavour to continue to move in the Line *b l*, being the Diagonal continued, that is, in the new Direction which it has now acquir'd, and with the same Velocity which it now has, (which is less than what it had at A, because the Diagonal A*b* is shorter than the side A B; therefore *b l* now represents the projectile Force, and *b k* the Force of Gravity, which we take equal to A *a*; because we have no regard to the Body's being in Motion, but consider it as setting out from the Point *b*, by the Action of the two Forces *b l* and *b k*: The Body thus acted upon will move in the Diagonal *b c* of the compleated Parallelogram *b l c k*, with a Velocity as much less than it would have had in the Line *b l*, as the Diagonal *b c* is shorter than *b l*. The Body at *c* will, by the first Law, endeavour to go on in the Line *c n* with the Velocity which it now has, but the Action of Gravity

† Pl. 24. F. 6.

Lect. V. represented by $c m$ ($= b k = A a$) bringing it down out of that Line $c n$, will make it go in the Diagonal $c d$ with a Velocity lessen'd in proportion as $c d$ is shorter than $c n$. Lastly, the Body at d acted upon by the Forces $d p$ and $d o$ will go in the Diagonal $d e$ with chang'd Direction and diminish'd Velocity, as we have shewn before.

As the Body comes down from its utmost Height at e, it continually increases its Velocity. For, first, if we consider it when at e; the projectile Force is $e r$, Gravity $e q$, and $e f$ (the Diagonal of the Parallelogram $e r f q$) the Line describ'd by the Body. Now as the Line $e r$ representing the Direction and Velocity of the Body at e, is shorter than the Tangent $f t$ representing its Direction and Velocity when at f, (because $e r$ is perpendicular and $f t$ oblique to equidistant Parallels) the Diagonal $f g$ must be longer than the Diagonal $e f$, therefore the Body's Velocity will be so much the greater. Thus will the Diagonal $g h$ of the Parallelogram $g w h u$, the Space gone thro' by the Body in the next Instant, be greater than $f g$, and consequently its Velocity increas'd in that Proportion. And lastly, $h i$ the last Space being the Diagonal of the Parallelogram $h y i x$, will be still greater than the last, and consequently the Velocity also greater.

COROLLARY I.

HENCE may be known the different Velocity of a Projectile in any Point of the Parabola which it describes, whether in its Ascent or Descent: and *it will always be to the Velocity in any other Point, directly as the Lengths of the Parts of the Tangents to the Parabola, at those Points which are intercepted between the same or equidistant Parallels.* As for Example, the Velocity at A : is to the Velocity at b : : as B C : to $b l$; for since (by Construction) B C is equal to A B, and A B represents the Velocity of the Body at A, as $b l$ represents the Velocity of the Body at b; the Velocities at the said Points A and b will be respectively as B C to $b l$. Thus if we compare the Velocities at A and at d, they will be to one another as D E to $d p$, for the same Reason. So likewise in the Descent the same will hold good; as for Example, the Velocity at e : is to the Velocity at f : : as r 10 : to $f t$; or $e r : f t$, 10 7 : t 5, or 7 8 : 56. So the Velocity at f : is to the Velocity at h : : as 56 : to $h y$; or $f t$, or t 5 : to $h y$.

COROL-

COROLLARY II.

HENCE follows also, that at equal Heights above the horizontal Line or Amplitude A *i*, the Body will have the same Velocity; because it will then be in correspondent Points of the ascending and descending Parabola, where the Tangents, having equal Obliquities to the Parallels to the Axis, will have equal Parts cut off by those Parallels wherever they are equidistant.

COROLLARY III.

IT also follows, that however unequal the Spaces be that a Projectile describes in equal Times, the horizontal Distances (that is, its Advances forward) will all be equal in equal Times; but we have already prov'd this another way. N. B. *I don't mean that the same Quantity of projectile Force will make a Body go forward equally fast; for that will vary according to the Angle that its Direction makes with the Horizon: but that if the Amplitude of the Parabola (or the whole horizontal Distance which the Body goes thro', whilst it describes any one Parabola) be divided into equal Parts by Perpendiculars; the Body will go from one Perpendicular to another in the same Time.*

4. IN considering the Motion of a Projectile, we have made the Line A *i* a streight Line, which would be strictly so if the Earth was flat; and indeed so much of it as a projected Body goes over, must be taken for such: but if the Force of the Powder, or whatever throws the Body forward, was much greater; or the Force of Gravity (that is, the Attraction of the Earth) much less; then we must consider the Line A *i* as a Curve or part of the Earth's Circumference.

AS for Example; Let ABCDE* represent the Surface of the Earth, and A F an high Mountain, such as *the Peek of Tennerif.* Now if a Cannon was fir'd in the Direction F L, the Ball would go forward in a Curve, for the Reasons before alledg'd, perhaps as far as B, where it would come to the Ground. If the Force of the Powder was proportionably greater, it would go as far as C before it came to the Ground. Suppose the Force of the Powder yet greater, and the Ball would go to D: suppose it still greater, and the

*Pl. 24. F. 7.

Lect. V. the Ball would go to E, not falling to the Ground till it had gone over two thirds of the Circumference of the Earth. Lastly, one might suppose the Force of the Powder great enough for the Ball not to come to the Ground at all, but to come to the Point F from whence it set out at first, and (the Cannon being removed out of the way) to go continually round the Earth, at the Distance of about 3 Miles from the Surface of the Earth (that is, at the Height of the *Peek of Tenneriff*) Gravity only keeping it from going off from the Earth in a Tangent Line. Were the Force of the Powder greater than in the last Supposition, the Ball would go farther and farther from the Earth in a spiral Line.

If the Point F was remov'd sixty Times farther from the Center of the Earth, or to the Height of 240,000 Miles, that is, to the Distance of the Moon; then the Force of Gravity (or the attracting Force, call'd in that case *the accelerating Force*, of the Earth) would be 3600 Times weaker than at the Surface of the Earth; because, as we recede from the Center of the Earth, Gravity decreases as the Squares of the Distances increase: and the Moon being 60 Times farther from the Center of the Earth, than the Earth's Surface, the Square of that Distance 60 is 3600. In such a Case there would be no Occasion for a greater projectile Force than that of our common Powder to make a Cannon-Ball circulate round the Earth, at the Distance of 240,000 Miles. The Moon itself may be consider'd as such a Projectile; for having once received an Impulse in a Line parallel to a Tangent of the Earth, while it is endeavouring (by the First Law) to keep its first (rectilineal) Direction, Gravity is continually impelling it towards the Center of the Earth, and by that Impulse turns it out of the right-lin'd Direction of the projectile Force, and carries it continually round the Earth. Neither can Gravity bring the Moon to the Earth, because the projectile Force still subsisting, always endeavours to throw off the Moon along the Tangent of its Orbit; and Gravity only alters the Direction continually, whereby the Moon endeavours every Moment to fly off along a new Tangent. Thus in every Point of the Orbit, these two Forces balance each other.

5. That Endeavour of the Moon, or any other Body that moves in a Circle or any other curv'd Orbit, to fly farther from the Center of its Motion, is call'd a *Centrifugal Force*; and the Force acting against it to keep the moving Body in its Orbit (whether it be the Force of Gravity or Attraction,

or

or a String, driving, impelling, or drawing the Body towards the Center of its Orbit) is call'd a *Centripetal Force.* Thus when a Stone is whirl'd in a Sling, the Stone, which endeavours to (and when one of the Strings of the Sling is loos'd, actually does) fly off in a Tangent to the Curve which it describ'd before, is hinder'd by the Strings to fly off of the Sling from so doing; that Endeavour is the centrifugal Force; and the Force of the Strings holding the Stone, is the centripetal Force. By the Force which endeavours to, but cannot really, carry the Stone in the Tangent, the String is stretch'd in a Direction from the Center of the Revolution to the Circumference. Let us, for Example, suppose, that the Body F fasten'd to the String M F † is whirl'd round in the Circle F I N: whilst the Body, not held by the String, would have mov'd the Length F H in the Direction F G by the first Law, being retain'd by the String it moves in the Arc F I, being brought down from H to I; the Line H I therefore may represent the Quantity of the Centripetal Force directed towards the Point M; and the same Line I H directed from the Point M towards H, may also represent the centrifugal Force, both those Forces being equal, otherwise the revolving Body would not be kept in Orbit. For tho' in Orbits that are not circular, these Forces may increase and decrease (as shall be hereafter shewn) yet they both equally increase and decrease, always balancing one another. N. B. *The swifter a Body moves in its Orbit, the more is the String stretched, that is, the greater is the Centrifugal and Centripetal Force. If, for Example,* F G *represents the Velocity of the Body instead of* F H, *the Body will describe the Arc* F N *instead of* F I; *in which Case the Point* N *(where the Body is) being farther remov'd from the Tangent (at* G. *where the Body would go to by the projectile Force alone) the Centripetal and Centrifugal Force must be represented by the Line* G N, *which is greater than* H I.

Lect. V.

† Pl. 24. F. 7.

All these will be further prov'd and illustrated by the following Experiments.

Experiment VI. *Pl.* 24. *Fig.* 8.

6. A B C D is a round Table which may be swiftly turned upon a Pivot, as at F, (the same that is represented by the 13th *Fig.* of *Pl.* 23.) There is a little Brass Pipe screw'd in at the Center C, into the Top of which the String of the leaden Bullet B is thrust, so as to go out at an Hole in the Side of the said short Brass Pipe; thence

Pl. 24. F. 8. and Pl. 23. F. 13.

Lect. V. thence it is carried under the Table thro' the Hole *b*, and ſo faſten'd to a Pin in the Side of the Table at A. When the Bullet is laid at B, if the Table be turn'd ſwiftly round, it leaves the Bullet behind at firſt, which thereby appears to have a Motion contrary to that of the Table, till by the roughneſs of the Surface of the Table, it goes round at laſt along with the Table on the ſame Part of the Table; then if the Table be ſtopp'd on the ſudden, the Bullet goes on ſeveral Turns, till having communicated all its Motion to the rough Surface of the Table, it comes to reſt at laſt. This illuſtrates the firſt Law of Nature; for as the Part of the Table under the Bullet leaves it behind for a while, becauſe it endeavours to continue in its State of Reſt, it would for ever leave it where it was at firſt, if the Table was perfectly ſmooth: and when the Bullet is once in Motion, it would for ever go round on the Table, if (beſides the Smoothneſs of the Table) the String that holds the Ball had no Friction at the Center C. It is alſo to be obſerv'd, that the String, which is ſlack at C B, is always ſtretched as at C *b* by the Motion of the Bullet; and this ſhews the Centrifugal Force.

If we ſcrew a forked Prop toward the Edge of the Table, as at D, and put the String of the Bullet into its Slit, ſo as to let the Bullet hang down as at 1, the Force of the Bullet's Gravity may be ſo overcome by the Centrifugal Force, which the whirling of the Table produces, that the Bullet ſhall riſe to 3, the String 3 *d* becoming horizontal; as the Table turns ſlower and ſlower, the Ball comes down to 2, and ſo to 1 at laſt, Gravity becoming ſenſible as the Centrifugal Force diminiſhes.

Experiment VIII. *Pl.* 24. *Fig.* 9.

Pl. 24. F. 9. If a String be tied round the Brim of a Pot full of Water, and the Pot be whirl'd round ſwiftly about the Hand or Center K in a Circle or Curve, of which A C B is an Arc, the Water acquiring a Centrifugal Force greater than that of Gravity, will not be ſpilled when the Mouth of the Pot is downwards. If inſtead of the †P. 24. F. 10. Pot, the Glaſs W C †, (*Fig.* 10.) containing Liquors of different ſpecific Gravities be whirl'd round the Center K, (after they have been confounded together by ſhaking) they will all recover their Places and Tranſparency, even ſooner than if the Glaſs containing them had been hung up and at reſt. The Reaſon is, that as the different Subſtances in the Glaſs have the ſame Velocity given them by the Centrifugal Force, their *Momentum* will be as their ſpeci-

ick

fick Gravities, that is, their *Momentum* will be made up of the different Quantities of Matter, which they contain under equal Bulks, multiplied by the common Velocity † which the Centrifugal Force gives them, in the Line K C from the Center of the Motion towards the Circumference. Therefore the Glass Beads among the Liquors weighing more than the Drops of any of the Liquors, will have the greatest *Momentum*, and consequently go to the Part G most remote from the Center of Motion K. Then the Drops of Oil of Tartar (which is the heaviest of the Liquors contained in the Glass) having for the same Reason more *Momentum* than the Drops of the other Liquors (tho' less than the Glass Beads) will take up the Space T next to the Beads, and also fill their Interstices. The next Liquor, which is Oil of Peter, will fill up the Space P. And lastly, the Spirit of Wine, whose Drops are the lightest, will (notwithstanding its own Centrifugal Force) be brought nearer to the Center of Motion, and occupy the Space W; because the Beads and all the other Liquors having more *Momentum*, drive it from the End C, to which it has a Tendency all the while. N. B. *The Tube is hermetically sealed at both Ends.* Lect. V. † L. 2. N°

The Glass Beads, and different Liquors, settle in their proper Places when the Tube is hung up; because, as all Bodies tend downwards with the same Velocity *, the *Momentum* of Particles of equal Bulk must be as their respective Quantities of Matter in their Descent: And that the Liquors will not be so soon settled in this Case as when the Tube is whirl'd round, is because we can give as great a Velocity as we please in the Direction K C, by a Centrifugal Force; whereas that which is owing to Gravity is always the same. * L. 1. N° 8.

COROLLARY.

Hence it follows, that a Bottle of any Liquor (which after having been muddy is by length of Time become fine, and is again made foul by shaking) may sooner be brought to be fine by a Centrifugal Force, than by being set upright at Rest.

Experiment VIII. *Pl.* 24. *Fig.* 11.

Join by a String the two Balls T and M, * whose Weights are to one another as 4 to 2, (here we use a two Ounce and a * Pl. 24. F. 11.

Lect. V. four Ounce-Ball) and pass the String thro' the opposite Side-Holes of the little Pipe C; let the Length of the String, measuring from Center to Center of the Balls, be 18 Inches, and the Distances of the Centers of the Balls from the Center of the Table be reciprocally as their Masses: that is, the Center of the two Ounce Ball M must be at the Distance of 12 Inches from C, and the Center of the four Ounce Ball T at 6 Inches from C. Let the two little Squares, or rectangular Pieces S *s* and V *v* be fixed on the Table, at the Distance of about 1 Inch or 2 behind the Balls, to stop them from flying off of the Table, and the long Sides of those Pieces so fix'd alternately, that when the Table is made to turn in the Direction mark'd by the Dart, the Balls may not be left behind, but immediately put into Motion. Now let the Table be whirl'd round with any Velocity, and the Balls will remain at the Points T and M, and describe round their common Center of Gravity unequal Circles * in a reciprocal Proportion of the Masses, the *Momenta* given the Bodies by the Centrifugal Force being equal, and (because of their contrary Directions) destroying one another. But if either of the Balls be removed farther from C than in the reciprocal Proportion above-mentioned, that Ball will gradually recede from the Center of Motion and draw the other along with it, till it be stopp'd by the End of the Piece V *v* or S *s*. So the Earth and Moon turn round one another and round their common Center of Gravity, as has been already observed †.

* L. 2. N° 32.

† L. 2. N° 32.

Experiment IX. *Pl.* 24. *Fig.* 12.

Let the Table be put into an uniform whirling Motion, and that Motion continued (which may be done by a Wheel and Pulley hereafter to be describ'd) and at the same time let a Piece of Chalk be drawn pressing upon the Table in the right Line C P; and there will by those two Motions be describ'd the spiral Line C D E F P: then if the Table be mov'd again in the same Manner and the same Way (as mark'd by the Dart) and at the same time the Chalk be press'd on the Table in the same Line P C, but with a contrary Direction, *viz.* from P to C, there will be described another Spiral like the former, but directed the contrary Way, as is shewn by the prick'd Line.

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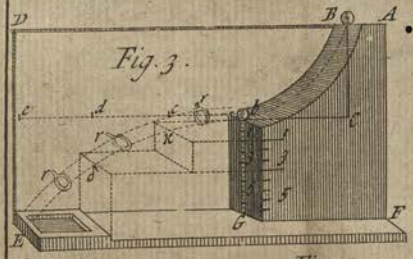
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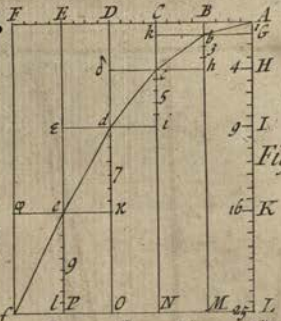
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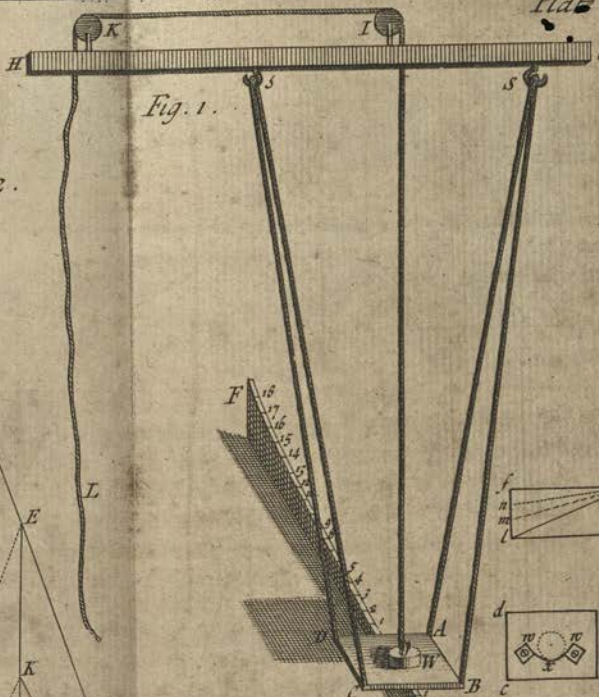
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Fig. 4.

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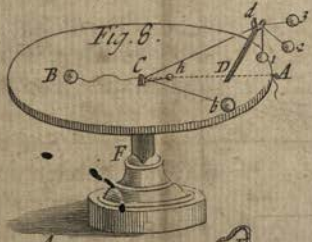


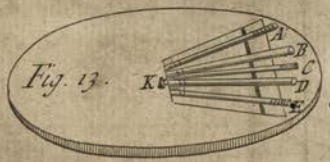
Fig. 6.

Fig. 9.

Fig. 11.

Fig. 10.

Fig. 12.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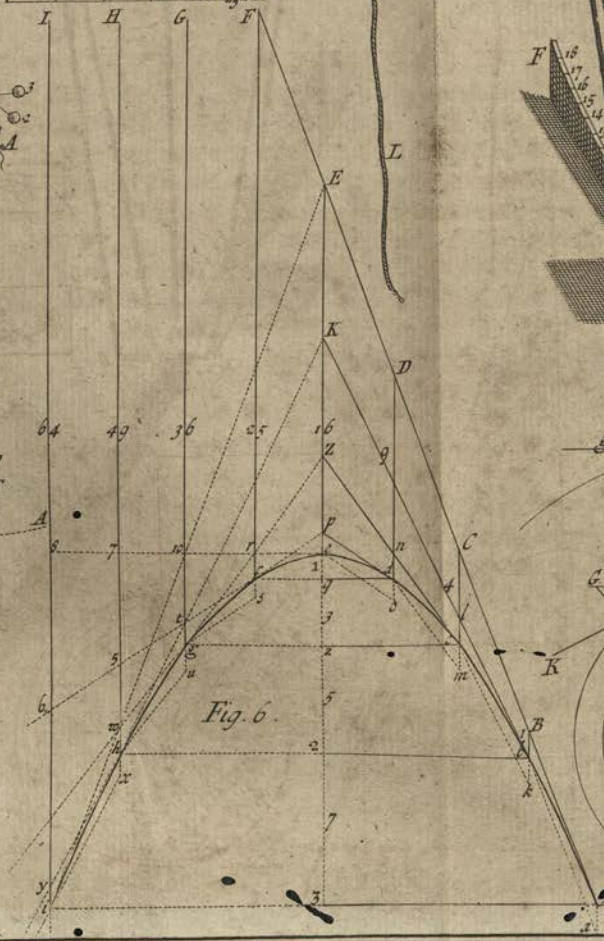
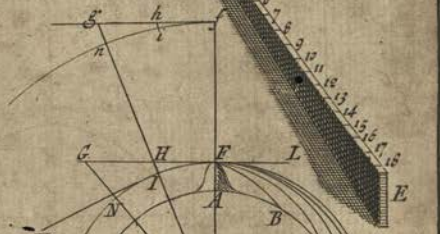


Fig. 6.



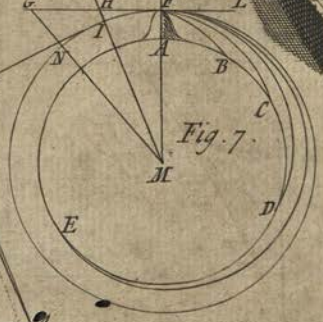


Fig. 7.

COROLLARY.

HENCE it follows, that if a Body that has a Centrifugal Force, whereby it recedes from, or a Centripetal Force, whereby it accedes to, the Center of its Motion, at the same time be carried round by a Force that gives it a circular Motion; it will fly from the Center in a spiral Line in the first Case, and run to the Center in a spiral Line in the second Case.

EXPERIMENT X. *Pl.* 24. *Fig.* 13.

ON a Piece of Board AEK*, which has a Piece under it across, to raise up its broadest End AE to an Angle of 15 or 20 Degrees above the horizontal Position, are fasten'd 3 Tubes AK, CK and EK, shut up at both Ends. In the first there is a small Cylinder of Cork, which can easily slide up and down the Tube: in the Tube CK there is a little Cylinder of Lead moveable in the same manner: and in the Tube EK, there is an Inch or two of Quicksilver shut up. This Board has a Screw under it, which going thro' one of the Holes in the Table, (such as are mark'd A and B, in *Plate* 23. *Fig.* 13.) † is fasten'd by a Nut so as to join the Board of Tubes firmly to the Table. Then when the Cork, Lead, and Quicksilver are in that Part of the Tube next to the Center of Motion K; let the Table be whirl'd round, and those Bodies will after a few Turns be carried to the Ends of the Tubes, which are fartheſt from the Center, tho' 3 or 4 Inches higher than the Ends at K. Put on the Tubes BK and DK, the first of which being fill'd with Water, has a Cork Cylinder moveable in it; and the other has in it an Inch or two of Oil, the other Part of it being full of Water. At first, (when the Table is at Rest) the Cork and the Oil will be at B and D, the higher Ends of the Tubes, and fartheſt from the Center; but when the Table is whirl'd round, the Cork and the Oil will go towards the Center to K, because the greater Centrifugal Force of the Water (being greater than either that of the Cork, or that of the Oil) must give the Cork and the Oil a Centripetal Direction, as has been explain'd in the Experiment of the different Liquors in the Glass of *Fig.* 10*,.

*Pl. 24. F. 13.

†Pl. 23. F. 13.

*Pl. 24. F. 10.

7. Monſ. *Des Cartes* endeavour'd to explain the Motion of the Planets round the Sun, by a *Vortex* or Whirlpool of Celestial Mat-

Lect. V. ter; supposing that the Sun by turning about his Axis, gave circular Motion to all the Celestial Matter about it as far as *Saturn* and beyond; and that this Whirlpool of Matter (in *French*, *Tourbillon*) being without Vacuity (for he asserted a *Plenum*) carried the Planets along with it, and so was the Cause of their Motion round the Sun. People that did not give themselves the trouble to examine Things thoroughly, believ'd the *Vortices* to be the Cause of the Motion of the Planets; because, where there are Whirlpools in Rivers and Brooks, we see that Straws, little Sticks, Saw dust, and other light Bodies, are carried round as they float; and therefore it was thought very probable, that the Planets might in the same manner be carried round in the Heavens. But, if we apply Experiments and Observations to the Doctrine of the *Vortices*, we shall find it insufficient for explaining the Motions of the Planets.

In the first Place, we'll suppose the Planets to be denser than the Matter of the *Vortex*; and then the same Thing must happen to them, as happened to the Cork, or the Lead, or the Quick-silver contained in the Glass Tubes, and whirl'd round on the Table. For the Tubes full of Air being carried round, contained Part of a *Vortex* of Air, where the Bodies carried in it were of different specifick Gravities, but all specifically heavier than the Parts of the *Vortex*; and those Bodies mov'd round, and at the same Time agitated by a Centrifugal Force, did continually recede from the Center of the *Vortex*, going off from it in a spiral Line. Planets therefore specifically heavier than the Parts of the *Vortex*, would continually recede from the Sun, moving off in a spiral Line. Since this won't do, let us suppose the Planets all rarer than the Matter of the *Vortex*; and the Consequence will be—That as the Parts of the *Vortex* are denser than the Planets, their Centrifugal Force must give the Planets a centripetal Direction, and so make them go continually towards the Sun in a spiral Line, till they fall into it; after the Manner of the Cork and the Oil in their respective Tubes of Water, where it appeared by the Experiment that the Centrifugal Force of the Water gave the Cork and Oil a contrary Tendency, whereby they continually approach'd towards the Center in a spiral Line.

There remains only now for the Support of the *Cartesian Hypothesis*, to suppose that Planets are of the same Density as the Parts

Parts of the *Vortex*, (and indeed, if there was a *Plenum*, as Monſ. *Des Cartes* ſuppoſes, all Bodies muſt be equally full, and there would be no ſuch Thing as different ſpecifick Gravities; for all Bodies of the ſame Bulk, having the ſame Quantity of Matter, would weigh equally) and in that Suppoſition it will follow, that the Planets making up Part of the *Vortex* itſelf, move along with the Parts of the *Vortex* next to them: and tho' at firſt thoſe Parts of the *Vortex* which are neareſt to the Sun will move faſteſt*; yet at laſt, the whole *Vortex* will move round like a ſolid Body, or Sphere, the Parts moſt remote from the Center making their Revolutions in the ſame Time as thoſe which are neareſt; which muſt be the Conſequence of a *Plenum*. So that all the Planets would at laſt have their periodical Times equal, which is contrary to Obſervation; for the periodical Times of the Planets Revolutions are all different. *Mercury*, the neareſt to the Sun, performs his Revolution almoſt 120 Times ſooner than *Saturn*, the moſt diſtant. In a Word, this is the Proportion of their Diſtances and Revolutions, viz. *The Squares of the Periodical Times of the Primary Planets round the Sun, and of the Satellites round Jupiter and Saturn, are as the Cubes of their Diſtances from their reſpective central Bodies* †.

Lect. V.

Pl. 24. F. 2.

* Ann. 2.

† Ann. 2.

8. SINCE then the *Carteſian Hypotheſis* is inſufficient for explaining the Cauſe of the Motion of the heavenly Bodies; we muſt ſhew what is the real Cauſe, and that not from Conjectures, but from Obſervations and Experiments. That Gravity, or an Attraction towards the Sun, keeps the Planet (or even the Comet) in its Orbit about the Sun, (and likewiſe the Satellites about their Primaries, our Moon alſo being the Satellite of the Earth) whilſt the projectile Force is continually endeavouring to throw it off along the Tangent, will evidently appear from what we have already ſaid, and from Conſequences of the ſecond Law of Nature, which we are going to explain, having firſt ſhewn ſeveral Laws relating to central Forces by the following Experiments.

But we muſt firſt deſcribe the Machine for the making the Experiments.

Plate 25. *Fig.* 1.

The Machine for central Forces conſiſts of a ſtrong wooden Frame CABDHGKEF, triangular at Top and Bottom. On the horizontal

Pl. 25. F. 1.

Lect. V. Pl. 25. F. 1.

zontal Piece G *a* at Top is a Wheel G, which (by means of the String G K H G) when turn'd round, gives circular Motion to the Pulleys and Spindles K L and H I, so as to move them either with equal Velocities, or with Velocities that are as 2 to 1, as 3 to 1, or as 3 to 2; because in the Pulley K there are two Grooves, one of 3, and one of two Inches Diameter; and in the Pulley H there are also two, one of 6, and one of 3 Inches Diameter. There are two Pieces M N, *m n*, (which we may call Planet-bearing Pieces) of about 30 Inches long, to be screw'd upon the Pulleys K, H, so as to turn round with them. These Pieces have each of them an open square Tower S, *s*, with a little Pulley at Top and Bottom to conduct a String from the Weights S and *s*, to the Brass Balls P and *p*, (which we must here call Planets) so that when P and *p* go towards N or *n*, the Weights are drawn up from their Bases, which are about an Inch above the Bottom of the Towers, and rise within the Towers till the Weight-carrying Piece strikes the Top of the Tower; each Ball having two little Wings with Holes in them, to slide easily along Iron Wires, that go from one End to the other of the Planet-carrying Piece, passing thro' the two perpendicular Brass Planes M and N, also thro' the Towers at about the Distance of $\frac{1}{4}$ of an Inch from the Surface of the Planet-carrying Piece. N. B. *Only one of the Wings and one of the Wires is here drawn upon each Piece, to avoid Confusion.* There are also Brass Collars at H and K, in which the Necks of the Spindles (which are of Steel) turn; and Iron Screws headed with Brass at L and I, with little Holes to receive the Bottoms of the Spindles.

* Pl. 25 F. 2.

THE Second Figure * represents something more than half of one of the Planet-bearing Pieces, divided into Inches both Ways from the Center. B *b* is the perpendicular Brass Plane at one End, thro' which the horizontal Wires W *w*, W *w* pass, to carry the Planet P by its perforated Wings L L; whilst the String that goes thro' the Middle of the Planet is made fast, by thrusting in the little Pin *p*, to give the Planet any certain Distance from T, the Center of its Motion, before it is moved round by turning the Wheel G, (in *Fig.* 1.) S *s* represents the Section of the Brass Tower fasten'd to the Wood by a cross Pin, whose Head is seen at *s*. T is the Base or Plate which is to support the Weight-carrying Piece, which is represented at *Fig.* 4. † and consists of a circular Plate and hollow Stem of two Ounces Weight, and on which may be slipp'd several

† Pl. 25. F. 4.

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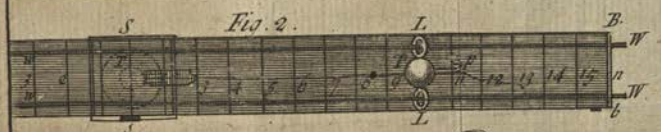
Plate 25.



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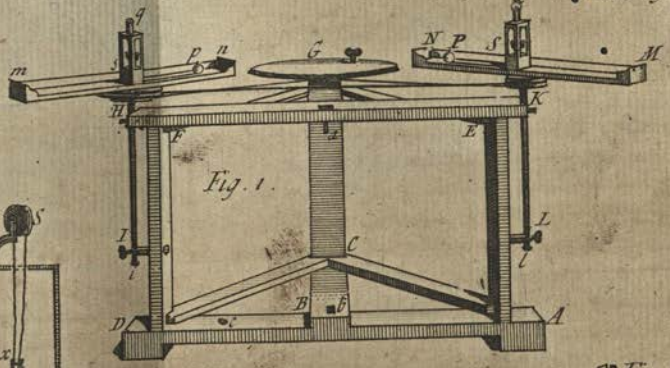

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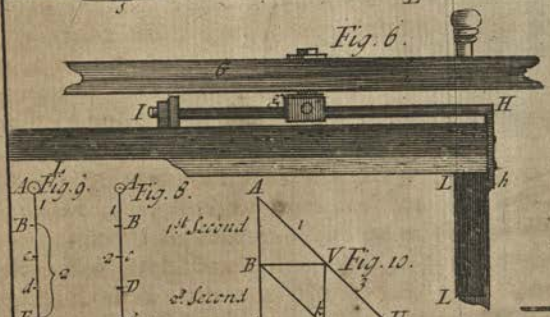

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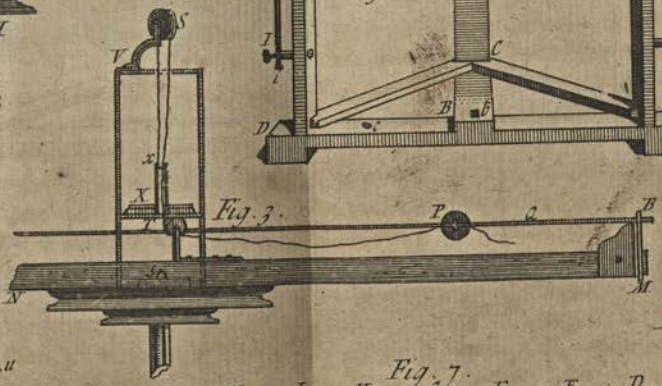

Fig. 3.

Fig. 4.

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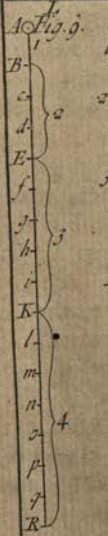

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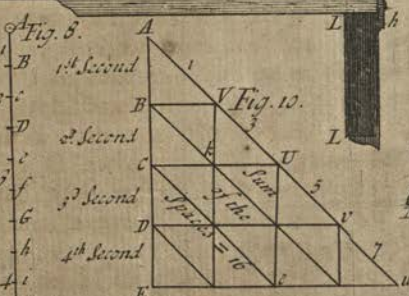

Fig. 8.

Fig. 10.

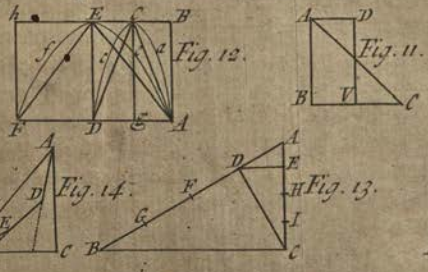

Fig. 12.

Fig. 11.

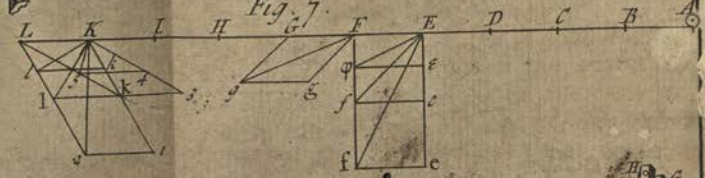

Fig. 7.

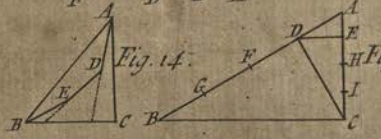

Fig. 14.

Fig. 13.

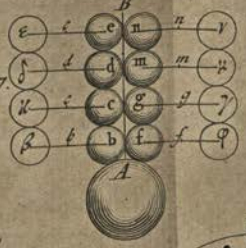

Fig. 17.

Fig. 16.

Fig. 21.

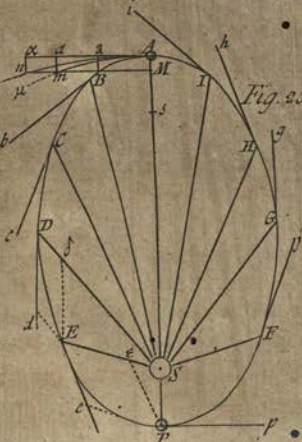

Fig. 23.

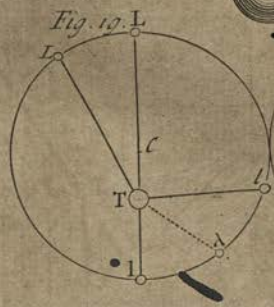

Fig. 19.

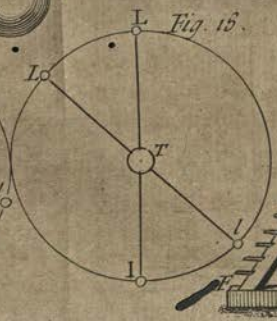

Fig. 18.

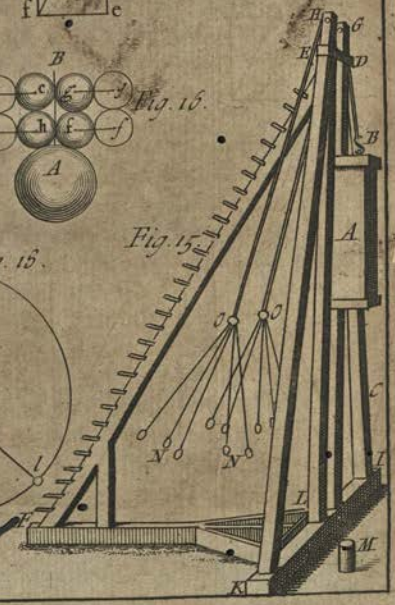

Fig. 15.

ral Leaden Weights like X, *Fig.* 5.* At T also (*Fig.* 2.) may be seen the little Pulley under which the String goes. Lect. V.

*Pl. 25. F. 5.

THE 3d Figure † is a vertical Section of one of the square Towers S *s*, with the Weight X and Weight-carrying Piece X *x* in it, and Part of the Planet-bearing Piece, Pulley and Spindle under it, mark'd M N. One little Pulley is fasten'd to the Wood at T under the Plate on which the Weights stand; another is sustain'd by an Iron Arm V S over an Hole S in the Top of the Tower. So that the String coming from the Planet P, goes first under the Pulley T, then thro' the hollow Stem of the Weight-carrying Piece, and so thro' the Hole in the Top of the Tower, then over the Pulley S, so down again to the Top of the Stem of X, where it is fasten'd. By observing this Figure, one may easily see that if the Planet P be moved ever so little in the Direction P Q, the Weight-bearing Piece X will be rais'd up towards S. †Pl. 25. F. 3.

THE whirling Table, which we have already mention'd * for making several Experiments, is best turn'd round by screwing on to the Top of either of the Spindles L K or I H, instead of the Planet-bearing Piece M N or *m n*. *Pl. 23. F. 13. and Pl. 24. F. 8, 11, 12, 13.

THE sixth Figure * represents the Section of the Wheel and Part of the horizontal Piece, the upper Part of the Frame which carries the Wheel, and the upper End of the Piece that supports it mark'd L L L, the Wheel's Axis and square-sliding Collar *g*, which is moveable on the square horizontal Iron H I, fasten'd to the Wood by a Nut and Pin at I, and two Wood-Screws H *h*. N. B. *There is a Screw in the sliding Piece* g, *to fix the Center of the Wheel, when it is brought forwards or backwards.* *Pl. 25. F. 6.

THO' it be of no Consequence of what Bigness the Machine above describ'd is made, provided its Parts are proportional; yet for the sake of those who would have such a Machine made, I give here the Measures of the principal Parts of mine in English *Inches.*

Plate 25. *Fig.* 1.

*THE Thickness of the Wood every-where about one Inch, except the Feet at A and D, where it is 2 Inches thick. Pl. 25. F. 1.

M N

Lect. V.

M N = *m n* 30 Inches.
K H 33⅙ Inches.
K L = I H = 8⅕ Inches.
A D = 34⅕9 Inches.
A C = C D = 27⅕2 Inches.
B B = 24⅕8 Inches.
Diameter of the Groove of the Wheel G = 14 Inches.
Breadth of the Planet-bearing Piece M N or *m n* = 2⅕3 Inches.
Grooves of the Pulley K, the one 2, and the other 3 Inches.
Grooves of H, the one 6, and the other 3 Inches.
The Height A K = H D = 13 Inches.
Height of the Towers S or *s* above the Board M N, *m n* 5⅕8 Inches.
Breadth of the Towers = 2⅕3 Inches.

There are 4 Brass Planets made use of, two of which weigh each 2 Ounces Troy, and the two other 4 Ounces Troy each.

The Weight-carrying Plate and Stem weighs 2 Ounces, and each leaden Weight (as represented by *Fig.* 5.) weighs 2 Ounces also.

What is to be consider'd in the Use of the Machine.

9. THE Weights in each Tower are to represent the Sun, whose Attraction is shewn by the Force with which the Weight resists the Action of the Ball P or *p* (representing a Planet) that endeavours to raise it by the String P T S *x* † (*Fig.* 3.) when it receives a Centrifugal Force by turning the Wheel G. So that by putting equal or unequal Weights in the Towers; by using equal or unequal Planets as P, or *p*; and by having their Distances equal or unequal in different Proportions; and the Periodical Times equal or unequal (as the Wheel-String goes round equal or unequal Pulleys) we may by Experiments shew those Laws of central Forces, which Sir *Isaac Newton* has mathematically demonstrated in his *Principia*.

* Pl. 25. F. 3.

IN considering the central Forces of Bodies (for Example, of the Primary Planets in respect of the Sun, and of the Moons in respect to their Primary Planets) which move round other Bodies that have an Influence upon them; we are to observe three Things. 1*st*, The Periodical Time, or Times in which the Bodies perform their Revolution. 2*dly*, The Quantity of Matter in the

the revolving Bodies. 3*dly*, The Distance of the Bodies from the Center of the Revolution.

Experiment XI.

First, make the periodical Times equal, by putting the Wheel-String into the 3 Inch Groove of each Pulley, crossing it before each Pulley to give it the more Force to move the Pulleys, but so that the Pulleys may both turn the same way that the Planet-bearing Pieces may not unscrew. Then put only the Weight-carrying Piece in each Tower: and lastly, fasten to their Strings a 2 Ounce Brass Ball, as P and *p* *, at the Distance of 12 Inches from the Center on each Planet-bearing Piece. So you will have *the Periodical Times, the Quantities of Matter,* and *Distances from the Center equal.* Give circular Motion to the Wheel G, and the Planets by their centrifugal Force will raise the Weights S and *s* at the very same Instant of Time; which shews, that *in this Case the centrifugal Forces are equal.* N. B. *If upon each Weight-carrying Piece you put on one or two, or more equal Weights, (such as are express'd by Fig. 5.) the Planets will always raise them at the same Instant, provided the Wheel be turn'd proportionally faster as there is more Weight.*

* Pl. 25. F. 1.

Experiment XII.

Secondly, Instead of *p* put on a 4 Ounce Ball, and double the Weight in the Tower *s q*; then turn the Wheel, and both Weights will rise at once. This shews, that *when the Quantities of Matter in Planets are unequal, but the Distances and Periodical Times still remain equal, the centrifugal Force is proportional to the Quantity of Matter.*

Experiment XIII.

Thirdly, Take off the 4 Ounce Ball, and make use of *p* again, but put it only at 6 Inches from the Center. Take the additional Weight from *s*, and add it to *S*; that is, let the Weight *S*, which has P at 12 Inches Distance, be = 4 Ounces; and the Weight *s*, which has *p* but at 6 Inches Distance, be = 2 Ounces: then upon turning the Wheel, they will both rise at the same Time: Hence it is plain, that *if the Periodical Times, and the Quantities of*

Lect. V. *Matter continue the ſame, but the Diſtances are different, the centrifugal Forces will be as the Diſtances.*

Pl. 25. F. 1.

EXPERIMENT XIV.

FOURTHLY, at the Diſtance of 6 Inches, where *p* was laſt, change *p* for a 4 Ounce Ball, and put equal Weights in the Two Towers; then when you turn the Wheel, both Weights will riſe at once; which ſhews, that *when the Periodical Times are equal, and the Diſtances from the Center reciprocally as the Quantities of Matter in the Planets, the Centrifugal Forces are equal.*

EXPERIMENT XV.

LASTLY, change the String on the Pulley H, putting it into the Groove of 6 Inches Diameter, ſo that the periodical Time of the Planet *p* may be double that of the Planet P, which laſt will then move twice as faſt, if its Diſtance be the ſame from the Center, which it muſt be in this Experiment. Put 8 Ounces in the Tower S Q, and only 2 in the Tower *s q*, the equal Planets P and *p* being then each at 12 Inches Diſtance from the Center. Turn the Wheel, and both Weights will riſe at once. This ſhews, *That Planets that have an equal Quantity of Matter and the ſame Diſtance from the Center, but different Periodical Times, have their centrifugal Forces reciprocally as the Square of their Periodical Times; that is, directly as the Square of their Velocities.*

COROLLARY.

HENCE follows, that if the ſame Planet changes its Velocity in the ſame Orbit, its centrifugal Force will increaſe or decreaſe according to the Square of the Velocity, which the Planet has in that Part of its Orbit.

SCHOLIUM.

WHEN we compare the laſt Experiment with the 13th Experiment, and find that the Planet *p* (going round in a Circle of 12 Inches Radius at the ſame Time that P went round in a Circle of 6 Inches Radius,) rais'd twice the Weight becauſe it had twice the Velocity; it will appear ſtrange, that in the laſt Experiment, where P (going

P (going twice thro' a Circle of 12 Inches Radius, whilst *p* goes once thro' such a Circle) has but double the Velocity of *p*, it should now raise four times more Weight. But this Proportion (which is of very great Use in explaining the Motions of the heavenly Bodies) will be very clearly deduc'd from a Consideration of the First Law, and what we have said upon it. Lect. V.

ROUND the Body supposed at M † (*Plate* 24. *Fig.* 7.) or the Center M, let a Planet revolve in the Circle FIN; it is plain, that when it describes the Arc FI, its centrifugal Force is represented by the Line HI: Then if the same Planet be supposed to describe the Circle *fin* of double the Radius, in the same Time, it is evident that it will have a double Velocity, and consequently will describe the Arc *fn* in this Circle in the same Time, that it described FI in the little Circle; so that now *gn*, which is double of HI or *hi* (because the Triangles *fg* M and FHM, as well as the Arcs *fn* and FI are similar) will represent the centrifugal Force to be double of what it was in the little Circle. But if, instead of going in the great Circle, the Planet doubles its Velocity by going twice round the Circle FIN in the same Time that it went once round before, it will describe the Arc FN, of twice the Number of Degrees, instead of FI; and as the Arc FN (tho' of the same Length is twice more curve than *fn*, the Planet at N will recede from the Tangent twice as much as if it was at *n*, consequently four times as much as when at I with half the Velocity. Therefore the centrifugal Force at N: will be to the centrifugal Force at I:: as GN: to HI; or as 4: to 1. † Pl. 24. F. 7.

N. B. *This holds good only in small Arcs, as* FI *and* FN *are suppos'd; but then it is only to be consider'd in such.*

THERE are many more Experiments relating to central Forces to be made with this Machine; but these are sufficient for our present Purpose. However, for the sake of the Curious we shall mention a few more in the Notes †. † Ann. 3.

The Second LAW *of Motion.*

10. *The Change of Motion is always proportionable to the moving Force impressed; and is made in the right Line in which that Force is impress'd.*

IF any Force generates a Motion, a double Force will generate double the Motion, a triple Force triple the Motion, *&c.* whether that Force be impress'd all together and at once, or gradually and successively. And this Motion (being always directed the same way with the generating Force) if the Body mov'd before, is added to or subtracted from the former Motion, according as they directly conspire with, or are directly contrary to each other; or obliquely join'd when they are oblique, so as to produce a new Motion compounded from the Determination of both.

Pl. 25. F. 7. 11. LET the Body A * receive an Impulse in the Direction AL, so as to go thro' any determinate Space in a determinate Time; as for Example, the Space AB or one Rod (16 ½ Feet) in a Second Minute of Time. The Body will, according to the first Law, by virtue of the Force impress'd, go on uniformly thro' the Spaces AB, BC, CD, DE, EF, FG, GH, HI, IK, KL, *&c.* so as to describe each of them (supposing them all equal) in a Second, and so on *in infinitum.* Now when the Body is in Motion, if the same Force that acted upon it at first, or one equal to it, should act upon it again in the same Direction, when it is at B, for Example; then would the Body be carried thro' a double Space, *viz.* the Space BD, in a Second, and so thro' DF, FH, HK, *&c.* in every Second, that is, with double Velocity; because, whilst it was going on at the Rate of one Rod in a Second, it receiv'd an Addition of Force capable of making it go also a Rod in a Second, and consequently that Addition of those two Forces that conspire give the Body a double Velocity. If the Body in motion, when at B, had received the second Impulse by a Force double of the first Force, it would after that Impulse go on in each Second thro' the Spaces BE, EH, HL, *&c.* that is, with a triple Velocity. If after receiving the Second Impulse with a Force equal to the first, while it is going on with double Velocity (for Example, thro' the Space BD or DF, *&c.* in a Second) it should receive a third Impulse (still in the same Direction) equal to the first, then it would go

from

from D to G, and so on from G to K, *&c.* every Second of Time; that is, with a triple Velocity, after three equal Repulses, just as it would do after two unequal Impulses in the Proportion already explain'd: as it would also do if it had receiv'd but one Impulse at first, but three times greater than we then suppos'd. Lect. V. Pl. 25. F. 7.

12. If, whilst the Body by the Force impress'd is (by the first Law) going on uniformly thro' the Spaces A B, B C, *&c.* a Force equal to the Force impress'd should act upon it in the Direction L A (that is, in a contrary Direction) the Body would lose all its Motion. But if this last Force should only act upon it after it had receiv'd several Impulses according to the first Direction, it would only destroy so much of the Motion as its own Quantity of Force would be capable of producing: for Example, if the Body, after three Impulses, was going on thro' the Spaces D G, G K, *&c.* that is, 3 Rods in a Second, and then it should receive a fourth Impulse, but in the contrary Direction, it would only lose so much Motion as to go thro' but two Rods in a Second, and continue to go on uniformly with that Velocity, just as if it had receiv'd but two Impulses. Again, if when it is going 3 Rods in a Second it should receive two Impulses at once equal to the first, but in contrary Directions, one in the former and the other in the opposite Direction; those Forces would destroy each other, and the Body would go on uniformly with the same Velocity that it had before those Impulses, *viz.* 3 Rods in a Second. N. B. *We don't consider any Elasticity that the Body might have.*

13. Now, let us once more suppose the Body to be going on uniformly at the rate of a Rod in a Second, or thro' the Spaces A B, B C, *&c.* Suppose when it is at E, a new Force equal to the first (or equal to the Force which the Body has then) should act upon it in the Direction E *e*, or at right Angles to its present Direction A L; the Body would change its Direction, and move (as has been shewn) in the Diagonal E f of the compleated Parallelogram E e f F †. If the new Force had been double, then the Body would have gone in the Diagonal E f of the double Parallelogram E e f F. But if the new Force had been only equal to half of the first Force impress'd, then the Body would only have gone in the Diagonal E φ of the Parallelogram E ε φ F.

† L. 3. N° 85. L. 5 N° 3.

COROL.

COROLLARY I.

Pl. 25. F. 7. HENCE it follows, that let the Quantity of the new Force be what it will, if it acts at right Angles, the Velocity will be greater than if the Body had continued in its first rectilineal Direction; † L. 3. N° 85. and Page 295, 296. because (as has been shewn †) the Body by the Action of the two Forces describes the Diagonal in the same Time as it would have described either Side of the Parallelogram by the single Action of one of the Forces; and in the Parallelograms E φ, E *f*, E f, the Diagonals are longer than the Sides, as they subtend the right (that is, the biggest Angles) of the rectangular Triangles E F φ E F *f*, and * By 19. 1. Eucl. E F f*. N. B. *When we make Use of these Words* the Force impressed, *or* the innate Force, *we mean the Force which the Body has when it is in Motion, without considering what first gave the Motion: That is, we mean the same Thing that Sir* Isaac Newton *calls* Vis insita.

COROLLARY II.

HENCE follows also, that if the new Force acts at acute Angles, its Effect will be more considerable the acuter the Angle is; but it will never increase the Velocity of the Body so much, as if acted at no Angle; that is, in the same Direction. (For Example, supposing the new Force equal to *the Force impress'd*, and its Direction to be in the Line F g, G F g will be the acute Angle made by the two Directions. Now it is evident, that the less the Angle G F g is, the greater will be the Angle F G *g*, and consequently the Diagonal F.*g* (expressive of the Body's Velocity) which subtends that Angle will also be greater: But whilst there is any acute Angle at F, there will be an obtuse Angle at G, and G F g will be a Triangle in which one Side F g will always be less than the two other Sides † By 20. 1. Eucl. F g and G g †, which are equal to the two F G and G H, through which the Body goes, when the two Forces act in the same Direction, in the same Time.

COROLLARY III.

IT follows likewise, that the new Force may act upon the Body in such a Direction, that the Body will not be accelerated thereby; nay, sometimes retarded. For if the new Force neither conspires with the Force impressed, by acting at acute Angles with its Directions,

rections, nor acts at right Angles with it; the Body may keep its Velocity, tho' in a new Direction, nay even go slower when the new Force acts at obtuse Angles. Let us, for Example, suppose the new Force, equal to the Force impress'd, to act when the Body in Motion is at K, in the Direction K k, so that the obtuse Angle L K k may be of 120 Degrees: then will the Diagonal K l of the Parallelogram K k l L be equal to the Side K L, because K L l and K k l are two equilateral Triangles, and therefore the Body will have the same Velocity as before. But if (every thing else remaining as we have suppos'd) the new Force should be less than the Force impress'd, as for Example, only equal to half of it, the Velocity of the Body would be diminish'd: for then K *k* would represent the new Force, and K *l* would be the Diagonal of the Parallelogram (which would now be K L *l k*;) and K *l* falling from K, one Angle of an equilateral Triangle, upon the Middle of its opposite Side, would be shorter than any of the Sides; *therefore, &c.* If the new Force was more or less than half in any Proportion, the Velocity of the Body would indeed be lessen'd, but not so much; for then the Point *l* would fall between L *l*, or between l *l*, and in any of those Cases the Diagonal would be longer than K *l*, because K *l* is perpendicular to L l, and the said Diagonal would be oblique to it, therefore longer, tho' still shorter than the Side K L.

Lect. V. Pl. 25. F. 7.

If the new Force had been greater than the Force impressed, for Example double, the Body then (supposing the Angle of Application the same) would increase its Velocity by the Action of it; for the new Force being represented by K 1, the Diagonal or Velocity would be K 2, longer than K L. But then if the Direction of this greater Force (supposing it now diminish'd in the Proportion of L 2 to L k) should be more opposite to the Direction of the Force impress'd; that is, if the Angle L K 1 should be increased, for Example 30 Degrees more, so as to become L K 2, then the Body would not change its Velocity with its new Direction, because it would go in the Diagonal K k, which is equal to K L. Nay, if the Angle of Application should be greater than K L 3, or of more than 150 Degrees, the Velocity of the Body would be diminished, the new Diagonal becoming shorter than K k.

If the new Force was only equal to the Force impress'd and its Direction K 4, that is, if the Angle of Application should be of 150 Degrees, the Velocity of the Body would be diminished by near half, the Diagonal K 5 being then but a little more than half of K I (= K L)

Lect. V. (=K L) because it subtends the Angle K L 5, which is but half of the Angle K L l. N. B. *Knowing the Angle of Application and Quantity of the new Force, we may always know what will be the Velocity of the Body after the Action of the new Force; because the Diagonal that shews it, is always known; as it is one Side of a Triangle, in which two Sides representing the two Forces are given, and the Angle between them, and consequently the 3d Side* †, *which is the said Diagonal. The Angle known does always contain the Number of Degrees that the Angle of Application wants of* 180. *Thus in the Triangle* K k L, *where* K L *and* L k (=K 3) *represent the Forces; (because of the Parallels* K 3 and L k) * *the Angle* K L k *is equal to* I K 3, *which* L K 3 *wants of* 180 *Degrees; and consequently the Diagonal* K k, *which subtends that Angle, is known.*

Pl. 25. F. 7.

† By 15. and 17. 1. Euclid.

* By 29. 1. Euclid.

14. It has been observed, that when a Body falls during one Second of Time, it goes thro' a Space equal to 16 $\frac{1}{2}$ *English* Feet, or one Rod, as we have already mentioned **: therefore the Force impressed by Gravity at the Beginning of its Fall (considering Gravity only as a Blow) is capable of making the Body go downwards at the rate of one Rod in a Second †, tho' it should act no longer upon it than during the first Second; that is, tho' the Body should for ever after cease to be heavy. For example, if the Body A* falls thro' the Space A B during the first Second of its Fall; if then it should cease to be heavy, yet it would go through the Spaces equal to A B during all the succeeding Seconds of Time, *viz.* thro' the Spaces B *c*, *c* D, D *e*, *e f*, *f* G, G *h*, *h i*, *i k*, *k* L, *&c.* But as the Body does not cease to be heavy, we must consider the Action of Gravity as an Impulse given by a new Force equal to the first, acting downwards when the Body is got to B at the Beginning of the 2d Second, and the Body during the 2d Second will go through the Space B D double the Space A B, or equal to the two Spaces B *c* and *c* D. Then if the Body should cease to be heavy, it would go uniformly thro' the double Spaces D *f*, *f h*, &c. every Second; but Gravity acting upon it also at the Beginning of the 3*d* Second, when it is at D, superadds a Force able to make it go thro' a Space equal to the first A B in a Second: consequently it will go thro' 3 Spaces or Rods (or thro' a Space D G equal to 3 Rods) the 3d Second. At the Beginning of the fourth Second, Gravity acting by a fourth Impulse superadds a Force equal to the former, whereby it will go the

† By the first Law, Page 295, 296.

*Pl. 25. F. 8.

** Bodies in reality only fall 16 *English* Feet and one Tenth of a Foot in a Second; but we call that Space here 16 and a half Feet; because one Rod (which is a Measure of 16 and a half Feet) gives us such a Number as to avoid Fractions in the Examples of the Calculations that we give.

the Length G L, that is, four Rods or Spaces the fourth Second; and so on †; and this will be a Motion uniformly accelerated. Lect. V. † N° 11.

THIS Motion of a Body thus accele rated in its Descent would be the real Motion of falling Bodies, if Gravity acted by Intervals, as we have only suppos'd, to help Conception: But as Gravity never ceases to act, we must fill up the Intervals between the Beginning and End of every Second or small Part of Time.

THUS, if we consider the Body falling thro' the Space A B * * (*Fig.* 9.) in the first Second, we must not only consider Gravity superadding a Force capable of making the Body fall one Rod farther at the Beginning of every Second; but also during the Time of every Second. For Example, at B at the Beginning of the 2d Second, the Body not only receives an additional Impulse to carry it to *d* instead of *c*, but also during the Time of the 2d Second another Impulse, which makes it go to E instead of *d*; so that the Body will fall 3 Rods during the 2d Second. Likewise at the Beginning of the 3d Second, when the Body is at E, it will again receive from Gravity one Impulse at the Beginning of that Second, and another during the Time of it; so that it will go thro' the five Spaces E *f*, *fg*, *gh*, *hi*, and *i* K, during the Time of that 3d Second. So in the Time of the 4th Second, the Body will (for the same Reason) go thro' seven Spaces or Rods from K to R, and so on, the Number of Spaces describ'd increasing by two every Second; that is, according to the Series of the odd Numbers, 1, 3, 5, 7, 9, &c. * Pl. 25. F. 9.

AS a great Part of my Auditors, tho' very curious, are unacquainted with mathematical Studies; I always found it very difficult to make them understand the Effect of Gravity in accelerating Bodies in their Fall, by *Galileo*'s Scheme of Triangles, till I had prepared them by what I have now said in the 14th Paragraph; but as that Account is not strictly true, I hope this following will satisfy every Body.

The Actions or Accelerations of any centripetal Force upon a Body, are (at the same, or equal Distances from the central Body) always proportionable to the Times; that is, equal in equal Times: And such are those of Gravity (as to Sense) near the Surface of the Earth.

Lect. V. SUPPOSE a Body free to fall *in vacuo*, receives the Impreſſions or Impulſions of Gravity during a certain Time, as for Example a whole Second; if that Time be divided into a great many ſmall Parts or Intervals (which I call Moments or Inſtants) the Velocity of the falling Body will increaſe uniformly or equally at every ſuch Inſtant; which being all ſuppos'd equal, the Spaces deſcrib'd in thoſe Inſtances by the falling Body, muſt be as the Velocities; that is, as the Terms of an Arithmetical Progreſſion, ſuch as the following.

Moments or Inſtants.	*Velocities or Spaces.*
1ſt --- --- --- --- ---	1
2d --- --- --- --- ---	2
3d --- --- --- --- ---	3
4th --- --- --- --- ---	4
5th --- --- --- --- ---	5, *&c.*

THE firſt Term of which is the Velocity of the falling Body at the End of the firſt Inſtant acquir'd by the Sum of the Impulſions of Gravity during that Inſtant. The laſt Term will be the Velocity of the falling Body at the End of the laſt Inſtant of the Fall: And in the like manner the middle Term of the Progreſſion will expreſs the Velocity acquired at the End of half the Time of the Fall, by the Sums of the Impulſions of Gravity during half the Time of that Fall. And ſince the Spaces deſcrib'd in the ſame Time, are as the Velocities, the Sum of the Terms of the Progreſſion will expreſs the whole Space deſcrib'd during the whole Time of the Fall. Now by the Doctrine of Progreſſions, it is univerſally known, that the mean Term multiplied by the Number of Terms, gives a Product equal to the Sum of the Progreſſion; that is, in the preſent Caſe, the mean Velocity (which is the Velocity that the Body has acquir'd at the End of half the Time of its Fall) multiplied by the whole Time of its Fall, will give a Space equal to the Spaces deſcrib'd by the Body during that Time, with an uniformly accelerated Motion.

That is, in other Words,

THAT, *if the Body was to move uniformly during the whole Time of the Fall, with that mean Velocity it has, when it has fallen half that Time; it would deſcribe a Space equal to that, which it deſcribes during*

during the whole Time, when it sets out from Rest, and has its Velocity accelerated all the while. Lect. V.

Now since we have already said, that the Velocities, or the Accelerations of the falling Body, are always proportional to the Times; the ultimate Velocity at the End of the Fall, must be double that of the mean Velocity, acquir'd at the End of half the Time of the Fall; and consequently, if that last, or greatest Velocity, be multiplied by the whole Time of the Fall, the Product will be the Space, which the Body would describe with that last Velocity in the same Time, and with an uniform Motion, which is double of the Space describ'd during the same Time, with the mean Velocity, or its Equal, *viz.* the Space describ'd during the same Time by an accelerated Motion.

'Tis therefore evident, that after a Body is fallen freely during a certain Time; it has acquir'd, by the equal and uniform Accelerations of Gravity, a Velocity, such as would make it describe (in a Time equal to the Time during which it fell) a Space, double of that Space it went through in the same Time, with a Motion uniformly accelerated.

From hence, it will plainly appear, how the Motion of falling Bodies is accelerated; and the Spaces they describe in any equal given Times, are as the odd Numbers 1, 3, 5, 7, *&c.* and the total Spaces as the Squares of the Times. For let a Body *in vacuo* be supposed to fall by Gravity during the Time of 4 Seconds; and if it be found, that at the End of the first Second, it has describ'd a Space equal to one Rod; was Gravity to act no longer upon it, it has already acquir'd by uniform Accelerations, such a Velocity at the End of that 1st Second, as would make it describe a Space equal to two Rods during the next Second with an uniform Motion. But Gravity continuing to act during the second Second, the Sum of its Actions or Impulsions will be (near the Surface of the Earth, where the Distance from the Center is not sensibly alter'd) equal to that of the Impulsions in the first Second, and therefore the Body will describe three Rods in the next Second. Now to find the Velocity of the falling Body at the End of the second Second; we may consider, that since from Gravity it acquir'd an uniform Velocity of two Rods *per* Second, by the Actions or Impulsions receiv'd during one Second, it will acquire a double Velocity, *viz.* of four Rods *per* Second of Time, after it has fallen, and been acted upon by Gravity, during two Seconds of Time; so that in the third Se-

Lect. V. cond (instead of the four Rods it would describe, was Gravity to cease to act) it will describe five Rods, because during that third Second, the Impulsions of Gravity are also suppos'd equal to those it receiv'd in the first. In the same manner, and for the same Reason was Gravity to cease to act, after having acted three Seconds, the Body would have a Velocity to describe six Rods in one Second, (triple of two Rods *per* Second, which is the Velocity, that Gravity gives in the Impulsions of a Second;) but Gravity acting as before, during that fourth Second, the Body will, by the Addition, or Sum of all those joint Impulsions, describe seven Rods; so that the Times and Spaces will be as follows.

Number of each particular Second.	*Rods.*	
1 = 1st	1	*or Spaces*
1 = 2d	3	*describ'd*
1 = 3d	5	*in each*
1 = 4th	7	*of them.*
4 *Seconds*	16 *Rods.*	

And it evidently appears from this Progression, that the total Spaces from the Beginning of the Fall, are as the Squares of the Times spent, during which the Bodies fall; for in one Second the Spaces describ'd are 1 Rod; in 2 Seconds, $1 + 3 = 4$ Rods (2×2); in 3 Seconds, $1 + 3 + 5 = 9$ Rods (3×3); and in 4 Seconds, $1 + 3 + 5 + 7 = 16$ Rods (4×4). Therefore it is true, that the Spaces describ'd by falling Bodies, are in a duplicate *Ratio* of the Times or Velocities; and on the contrary, that the Times of the Fall or the Velocities acquir'd at the End of those Times, are in a sub-duplicate Proportion of the Spaces, or as the square Roots of those Spaces.

This is the true Manner of Bodies falling with an accelerated Motion by the Force of Gravity, abstracting from the Resistance of Air, which we shall hereafter take into Consideration. N. B. *All Bodies fall equally swift where there is no Air, as has been proved by the Experiment of a Piece of Gold and a Feather falling in the same Time from the same Height in an exhausted Receiver* †.

† L. 1. N° 8.

15. What we have shewn (Step by Step) of the Effect of Gravity on falling Bodies, may be demonstrated another Way after *Galileo's*

Galileo's Method, thus: As Bodies (abstracting from the Air's Resistance, as we have said) increase their Velocity in falling according to the Time during which they fall, *Galileo* represents the Times and Velocities by equal right Lines, which he joins at right Angles; and then joining their other Ends by a third right Line, makes a rectangular Triangle, of which this last Line is the Hypothenuse, to represent the Space fallen thro' in a certain Time; A B, for Example, † represents the Time; B V represents the Velocity; and the Triangle A B V the Space gone thro' during that Time; as for Example, one Rod, if the Time be one Second. Now, if the Time be double, A C represents it; and then the Velocity being double also, the Line C U must represent it. Draw U A, and the large Triangle A U C will represent the Space fallen thro' in that Time; which greater Space being divided into Triangles equal to the first A B V, will appear to contain four of them. Therefore, since a Body falling one Second of Time goes thro' the Space of one Rod, the same Body falling two Seconds will go thro' the Space of four Rods. Had the Time been A D (= 3 Seconds) the Velocity would have been D *v* = A D, and the Space A *v* D = 9 Rods, or 9 of the first Space. Thus, if the Time be A E = 4 Seconds, the Velocity must be E *u* quadruple of B V, and the whole Space A *u* E containing 16 Times the first Space A V B. Lect. V. † Plate 25. F. 10.

COROLLARY I.

HENCE it follows, that if you multiply the Times by the Velocities, or if you square the Times, or square the Velocities, you will have the Number of Spaces (equal to the First) which the Body has describ'd during its whole Fall: As here, the Times being 4, and the Velocities 4, the Spaces gone thro' are 16.

COROLLARY II.

HENCE follows also, that if you mark the Times (or Number of Seconds) as the Numbers 1, 2, 3, 4, in the Scheme shew; you will have, against any of the particular Numbers, as many little Triangles as the Body has gone thro' Spaces during the Part of Time express'd by that Number. For Example, there is on the right Hand one Triangle over-against the Number 1, three over-against 2, five over against 3, and 7 over-against 4. And to know how many Rods a Body falls in any determinate Second or Part of Time, as for

Lect. V. for Example, in the 10th Second, without drawing the Scheme; we must first find how many Rods the Body falls in the whole Time, as here in 10 Seconds; then how many Rods the Body falls in 9 Seconds, and subtract the last from the first. 10 × 10 is 100, the Number of Spaces in 10 Seconds; 9 × 9 is 81, the Number of Spaces in 9 Seconds; 81 from 100 leaves 19, the Spaces or Rods fallen thro' in the 10th Second.

COROLLARY III.

IT appears likewise, as a Consequence from what has been said, that when a Body has fallen thro' a certain Space by a Motion uniformly accelerated, it has acquir'd such a Velocity as is able to carry it thro' a double Space in the same Time, if Gravity should cease to act, or have its Effect any Way destroy'd, or if the Body's Direction should be chang'd from a vertical to an horizontal one. For †Pl. 25. F. 10. Example, when the Body in the Time A B †, falling thro' a Space represented by the Triangle A V B, has acquir'd the Velocity B V; if, Gravity ceasing to act, it receives no Addition to its Velocity, tho' it continues to fall during the Time B C = A C, so that the whole Time must be represented by the whole Line A C; yet the Velocity will not be represented by the equal Line C U, but by the Line C *k* equal to B V, the Velocity the Body had when it ceas'd to accelerate its Motion: therefore to know how many Spaces the Body has gone thro' in the Time B C = A C with an uniform Motion, we must multiply B C by C *k* = B V the unchang'd Velocity of the Body, and we shall have the Rectangle B V C *k*, containing two Triangles equal each to A V B; that is, two Spaces will be run thro' by the Body moving with an equal Motion in the Time B C equal to the Time A B, in which the Body describ'd only one Space by an accelerated Motion. So likewise, if the Body having fallen thro' 4 Spaces during the Time A C, and having at the End of that Time acquir'd the Velocity C U; the Velocity be no more increas'd, and the Body continues to fall during the Time C E = A C, it will fall thro' 8 Spaces instead of 12, that it would have fallen if the Velocity had continued to increase, so as to become E *u* at the End of the Time C E; but as it is then only E *e*, equal to C U its Velocity at the Beginning of the Time C E, we must multiply C E the Time by U C or E *e* the uniform Velocity, and then we shall only have the Rectangle U C E *e* containing 8 Spaces, instead of the Trapezium C U *u* E containing 12.

THIS

Lect. V.

THIS will become visible by changing the Direction of the moving Body from a vertical to an horizontal one; for Gravity acting only perpendicularly, will neither accelerate nor retard the Body in its horizontal Motion. N. B. *The manner of doing this we shall hereafter shew.*

COROLLARY IV.

WE may also further gather from what has been said, that Gravity does not act by Intervals, (tho' we explain'd it that Way at first to facilitate Conception;) for if it did, after every Impulse the Body acted upon would go with an uniform Velocity, though it would have a greater Velocity after every Impulse. Neither after it had fallen a certain Space would it have a acquir'd a Velocity capable of carrying it thro' double that Space in the same Time, should Gravity cease to act, but only thro' a Space equal to the first.

FOR Example; after *Galileo*'s Method, the Triangle A C B (*Pl.* 25. *Fig.* 11.) represents the Space fallen thro' in any Part of Time, suppose one Second, in the Time A B with a Velocity *nascent* (or just beginning) at A the Beginning of the Second, and equal to C B at the End of the Second: Whereas if a Body should fall one Rod in a Second by an Impulse as a Blow, the Velocity at first would be equal to A D, and continue so during the whole Time, so that at the End of the Second it would only be B V equal to A D and half of B C, whereby without a fresh Impulse it would only carry the Body down one Rod more in another Second; for then the Time multiplied into the Velocity would only produce the Rectangle A D V B equal to the Triangle A C B, because it is of the same Height and has half its Base †. Pl. 25. F. 11.

† By 41. 1. Eculid.

16. IF the Direction of a falling Body be so chang'd as to make it go directly upwards, beginning that Motion with all the Velocity it had at the End of its Fall, it will go up (by a Motion uniformly retarded) exactly to an Height equal to that from which it fell; and the Spaces it goes thro' in each Part of the Time are to be seen in the Scheme over-against the Numbers representing those Parts of Time. If, for Example, the Body has fallen 4 Seconds, it will have gone thro' 16 Rods in its Fall, and have acquir'd the Velocity E *u* † capable of carrying it 32 Rods in the same Time by an uniform Motion; but as Gravity acts against it in its Rise, it will destroy all its Motion by the Time that it has risen only 16 Rods: † Pl 25. F. 10.

for

Lect. V. for it is the same Thing to make a Body from Rest move 16 Rods downwards in a certain Time, as to destroy half of the Force which was able to carry a Body upwards 32 Rods in the same Time. The same Thing will also be plain if you consider the whole Time, as divided into small Parts, and observe the uniform Diminution of
Pl. 25. F. 10. the Body's Motion. For Example; let the Body be thrown upwards, and let the Spaces which it goes thro' in every Second be consider'd. If the Velocity given to the Body projected upwards be such, that in the Time E D (which will now be the first Second of Time) it goes thro' 7 Rods, the next Second it will go but 5, because Gravity gives it an Impulse downwards at the Beginning of, and during, the 2d Second, able to carry it downwards 2 Spaces or Rods; which is (in other Words) to take away 2 of the 7 Spaces, which the Body would have gone thro' in that Time. Thus likewise, instead of going 5 Spaces the 3d Second, Gravity taking away 2, it will go but 3 Spaces; and the 4th Second (the last of its Rise) instead of going three Spaces, it will for the same Reason go but one; and then the Body will for an indivisible Moment of Time be at Rest. From the Point of Rest the Body will come down again, falling with an accelerated Motion already described, and coming down just in the same Space of Time that it went up.

SCHOLIUM, Pl. 25. Fig. 12.

Pl. 25. F. 12. If a Body, instead of going upwards in the Line A B, should rise to the same Height in the inclin'd streight Lines A C or A E, or in the Curves A *a* C or A *e* E, it would in the same Time come down in the streight Lines C D or E F, or in the Curves C *c* D or E *f* F. For we have already shewn, that when a Body is acted upon by Force to carry it in the Line A B in a certain Time, if another Force A *g* acts upon it too, the Body will go in the Diagonal A C of the Parallelogram A B C *g* in the same Time: And we have also shewn, that it would go in a Parabolic Curve A *e* C in the same Time from A to C. Now since the Force of Gravity in the Direction B A, which destroys the Motion of the rising Body, is equal to the Force of Gravity acting in the Direction C *g* to make the Body fall from its Point of Rest C, the Forces C E and C *g* are equal to A *g* and A B, and therefore the Diagonal right Line C D, or curv'd Line C *c* D, will be gone thro' in the same Time as the Lines A C and A *a* C. The same might be said of the Lines A E and A *e* E compared with E F and E *f* F. N. B. *When we throw a Body directly upwards, and it falls upon the same Place of the Earth we threw it from, then it really describes two such Lines as*

as A C *and* C D *moving from* A *to* D *by the Motion which the Earth gives it from East to West. But if we shoot it obliquely upwards, it describes the two Curves* A C, C D, *or the Parabola* A C D; *or any other Parabola, as* A E F, *according to the Angle of the Direction it sets out with.* Lect. V.

COROLLARY I.

HENCE it follows, that one may at any Time know to what Height a Projectile, as a Bomb, or Cannon-Ball, &c. (whether shot up directly or obliquely) has risen. For if you take the Time between the firing the Mortar and the falling of the Bomb, the half of it will be the Time of the Fall of the Bomb. Square the Number of Seconds in that Time, and you will have the Rods or Spaces fallen through in perpendicular Height. Thus if 20 Seconds are elaps'd from the firing of the Mortar to the Fall of the Bomb, half that will be 10, the Square of 10 is 100, which hundred Rods multiplied by 16 and a half, will give the utmost Height of the Bomb in Feet, *viz.* 1650.

N. B. *We still abstract from the Resistance of the Air, which we shall hereafter consider, and shew how to allow for it, as likewise we take* $16\frac{1}{2}$ *Feet instead of* 16,1 *for the Reason above given.*

COROLLARY II.

HENCE also follows, that knowing the Weight of the Body, and the Height from which it falls, one may know what Stroke it will give; that is, what *Momentum* it has at the End of its Fall: for the square Root of the Spaces will always give the Velocity, which being multiplied into the Mass or Weight of the Body, gives us its *Momentum* *. * L. 2. N° 3.

SOME People have imagin'd, that a falling Body has a *Momentum*, and strikes a Blow proportionable to the Height from which it falls; for Example, that a Pound falling from an Height of 4 Feet has four Times the *Momentum* that it would have in falling from one Foot. But their Error lies in not taking the Time into Consideration; for a Body spends twice the Time in falling 4 Feet that it does in falling one Foot, so that the Velocity is only double in the first Case. Indeed, if a Body could fall 4 Spaces, or in any Direction whatever go thro' 4 Spaces, while another Body of equal Mass went thro' but one Space; it would then have four Times the *Momentum*, and consequently be capable of a quadruple Effect.

Lect. V. OTHERS again allow, that the Velocity is as the Square Root of the Spaces; but alledge that the *Momentum* is not as the Product of the Mass by the Velocity of the moving Body, but as the Mass multiplied into the Square of the Velocity; which Opinion they endeavour to support by several Experiments, and various Reasonings, many of which I shall consider in the Notes *. I shall only take Notice here, that though I do not think the Experiments of hard Bodies falling upon soft Substances (to be hereafter more fully describ'd) conclusive in respect of the *Momenta* of those Bodies; yet they have very well prov'd, that yielding Substances, or soft unelastic Bodies, yield to a Blow of the same percutient Body in proportion to the Square of its Velocity; and thence may be drawn very useful Consequences applicable to the Practice of Mechanicks.

* Ann. 4.

IN explaining the Action of Gravity upon falling Bodies we have considered it as acting upon Projectiles always with the same Force, though it is certainly weaker in its Effect the higher the Body acted upon is above the Earth, as we have shewn in the 11*th* Annotation to Lect. I*. For Gravity (that is its accelerating Force) decreases, as the Squares of the Distances from the Centre of the Earth increase. But as the greatest Height to which we are able (even with Gun-powder) to project Bodies, bears no Proportion to the Distance from the Centre of the Earth, (scarce a 2000*th* Part) we could make no Allowance for that Decrease of the Force of Gravity, without making too much, since the Difference is insensible.

* L. 1. An. 11.

17. IF the Force of Gravity was greater or less than it is here, Bodies would be accelerated by it in their Fall in the same manner that we have explain'd; only the Spaces, which the falling Body goes through in the same Time, would be greater or less in Proportion. If the Force of Gravity was 4 Times greater, a Body would fall 4 Rods in the 1*st* Second of its Fall; and if the Force of Gravity was four Times less (as it would be if we were carried up to the Height of 4000 Miles, or removed farther from the Centre of the Earth than we are, the Distance of one Semidiameter of the Earth) a Body would fall but a Quarter of a Rod in the first Second of its Fall *. And if we observe, that in any Place on the Surface of the Earth a Body falls not so great a Space as one Rod in a Second; we may be sure that the Force of Gravity is less there

* L. 1. An. 11.

there than in our Country, where Bodies fall one Rod in a Second. Lect. V.
Now this Observation has been made very near the Æquator, where from Experiments made on Pendulums it appears that Bodies do not fall a Rod in a Second; and it follows from thence, that the Force of Gravity is less there than in greater Latitudes: which happens because the Surface of the Earth is higher (that is, farther remov'd from the Centre there than at the Poles) about 31 Miles; and partly, because the Increase of the centrifugal Force destroys more of the Action of Gravity near the Æquator, than towards the Poles. But I shall speak more fully of this in another Place. In the mean Time, I refer the Curious to Sir *Isaac Newton's Principia*, Book 3. Prop. 20. and the *Philosophical Transactions*, N° 386, 387, and 388.

18. When a Body runs down an inclin'd Plane, it cannot descend with its whole Gravity, because Part of it is supported by the Plane, and that in Proportion of the Length of the Plane to its Height (or of the *Radius* to the *Sine* of the Angle of Inclination) as has been shewn in the *7th Annotation* of *Lect.* 3. But then that Part of the L. 3. Ann. 7.
Weight which is not supported by the Plane, or the *relative* Gravity, being always of the same Quantity, because every Part of the Plane has the same Inclination, will cause the Body that rolls down to accelerate its Motion in the same Manner as a Body that falls freely down, but not so fast; or in the same Manner that Bodies would fall freely if the Force of Gravity was so much less.

Construction.

Upon any streight horizontal Line as C B, * raise a Perpendicular *Pl. 25. F. 13.
at C, and make A B the Hypothenuse of the Triangle A B C (which is to represent an inclin'd Plane) in Length equal to twice the Height or Perpendicular A C. Divide A B into 4 equal Parts mark'd by the Points D, F, G, and likewise A C into 4 equal Parts mark'd by the Points E, H, I. Draw E D and C D.

If A E, E H, H I, and I C be the 4 Spaces (or Rods) which a Body falls thro' in 2 Seconds, A E will be one Space which the Body goes thro' in 1 Second †. Now, if the Force of Gravity was but †N° 14, 15.
half of what it is, the Body, instead of going down from A to C in two Seconds, would only go from A to H. Let the Body be laid upon the Plane A B, and then it will be so supported by the Plane as to descend towards B, with only half its Weight; that is, it will lose

Lect. V. Pl. 25. F. 13.

half its Weight, and advance upon the Plane no faster than it would do in the Line A C if it fell freely down, and the Force of Gravity was but half of what it is. The Body therefore rolling on the inclined Plane will come down but to D, which is upon the same Level (or as near the Centre of the Earth) as E, in 2 Seconds. Whereas, if it had not been supported by the Plane, it would have fallen 4 Spaces or down to C. It has then at D the same Velocity which a Body falling freely from A has at E, but is (in this Case) twice as long in acquiring that Velocity. If the Body, when it is come down to D, continues to move along the inclin'd Plane, it will for the Reasons already alledg'd go thro' the next 3 equal Spaces D F, F G, G B, in the next 2 Seconds of Time; just as a Body falling freely in the Line A C would do if the Force of Gravity was but half; or as a Body will do now with the whole Force of Gravity in half the Time: and the Velocity of the rolling Body, when it is at B, will be as great as the Velocity of the Body at C; but will be so much longer acquiring it, as the Line A B is longer than A C. N. B. *A more geometrical Solution of this may be found in the 37th Theorem of Dr.* John Keill, *in his* Introductio ad veram Physicam.

COROLLARY I.

HENCE follows, that whatever the Inclination of the Plane is, a Body will have acquir'd the same Velocity when it comes down to the Bottom of it as if it had fallen along the Perpendicular: and if any two Points (as E and D, H and F, I and G) be taken in the same horizontal Line, one upon the inclin'd Plane, and the other upon the Perpendicular, the Body will have the same Velocity in both, tho' acquir'd in different Times. For if the Plane be less inclin'd, the relative Gravity which carries it down will be greater, and accelerate the Body so much quicker as the Plane is shorter; whereas the more inclin'd or the longer the Plane is, the more slow its Acceleration will be.

COROLLARY II.

HENCE follows also, that if a Perpendicular be drawn from any Point of the Line in which a Body falls freely to the inclin'd Plane, it will shew how far upon the inclin'd Plane another Body would roll in the same Time, if the Fall of both began from the same Point:

Point: That is, in this Case C D, which from the Bottom of the vertical Line A C comes upon the Plane at D, (to which Point we have shewn that the Body would fall along the Plane, in the same Time that it would fall freely as low as C) is perpendicular to the Plane A B. For since by Construction A B is the double of A C, and A C the double of A D, and the Angle C A D common, the Triangles C A B and C A D will be similar †, and consequently C D is perpendicular to A D. Lect. V. Pl. 25. F. 13. † By 6. 6. Euclid.

COROLLARY III.

HENCE also may be shewn, that if a Body should fall down from the same Height along several Planes differently inclin'd one after another, as for Example the Planes A D, D E, E B, † it would have acquir'd the same Velocity as if it had fallen along only one Plane as A D, or the Perpendicular A C. † Pl. 25. F. 14.

COROLLARY IV.

IT is likewise evident, that if a Body falls in an Arc of a Circle, it will have the same Velocity at Bottom, as if it had fallen in the Perpendicular; because an Arc of a Circle may be consider'd as an infinite Number of Planes differently inclin'd.

19. THE Doctrine of Pendulums is naturally deduc'd from what we have explain'd about the Fall of Bodies; but as there are some Things in it that require the understanding the 3d Law of Motion, we must defer it till that Law is explain'd. In the mean Time we will give an Account of those Mechanical Organs or Instruments which were omitted in the 2d Lecture, (when we describ'd the simple Machines commonly, but erroneously, call'd *Mechanical* Powers:) because the Principles explain'd in that Lecture were not sufficient for it, without the Knowledge of the 1st and 2d Law of Nature.

20. THE seven simple Machines or mechanical Organs which I describ'd in my Second Lecture, are only Instruments to perform the same Thing different Ways;—namely, to convey the Powers exerted or spent by some Body acting, so as to be applied for the moving of other Bodies, or to transmit or regulate the Power from one Body to another. Now the Instruments I am about to describe will also serve

Lect. V. serve to perform this, therefore they may very well be call'd Mechanical Organs; but the Difference in the acting is this: In the Machines or Organs already describ'd, a great Part of the Strength of the Men or other Animals, *&c.* (that is, the Intensity of the Power) is lost in the rubbing, sticking, stretching, wearing, and yielding of the Parts of the Machine, and the Force of the Men, *&c.* employ'd, can only be exerted by Degrees, with the Loss aforesaid. But in the Machines I am about to describe, the Power is collected and convey'd from Body to Body with little or no Loss by Accumulation continuing in the same Line.

21. If a Man weighing 140 lb presses with his whole Weight, or exerts a Force equal to his Weight, upon a Leaver or Balance-Beam equally divided by its Centre of Motion, it is impossible for him to overcome a Resistance greater than 140 lb, tho' he should act upon the Beam a whole Day with all his Force, the Man's Power or Force being destroy'd as fast as he exerts it; but if he can communicate his Force by Degrees to a Body that shall keep it all, and exert in a Moment the Sum of all the Impulsions given by the Man at several Times, that whole Force of the Body so collected, and as it were condens'd, will perform at once what the Man could never have done by an Engine in the common Way. Such a Body was the *battering Ram* of the Antients, which was a very large Piece of Timber (one of which we have describ'd in the 3*d* Annotation of the 2*d* *L. 2. An. 3. Lecture *) headed at one End with Metal, either Brass or Iron, which was contriv'd various Ways to be supported, and by the conjoin'd Strength of many Men made to be moved with the Metal-headed End forward; till having receiv'd and kept all the successive Impressions of the Force of the Men (which was wholly employed to move the Ram forward; because its Weight was suspended by Ropes or Chains from a Distance above it to allow it to swing freely) it had by little and little acquir'd a certain Degree of Velocity; and with that Degree of Velocity it met with or struck the Walls or Fortifications of Cities or Castles, and thereby remov'd or beat them down *. This Machine is by the Men

* No doubt but the first Invention of the battering Ram was copied from Nature, rather than mathematical Reasoning; that is, from what a *Ram* is observ'd to do by Instinct. If that Creature, having in vain push'd with his Head against an Obstacle by help of the Muscles that extend his Legs, his hind Feet being fix'd hard against the Ground, makes a second Attempt; and, by a sudden Stroke of his Head, removes the Obstacle which resisted too much before: in such a Case afterwards, he retires so far as to be able to accelerate his Motion

Men accelerated in an horizontal Direction in the Manner that falling Bodies are accelerated by Gravity in a vertical one. The Man therefore that should be unable to overcome a Resistance greater than 140 ℔ by the common mechanical Organs, might do it by a heavy Body swinging horizontally in such a Manner that he could give it a Motion accelerated till it reached the Obstacle; that is, he could do it by Means of a small battering Ram. And if it was requir'd to have a Force impress'd downwards, he might overcome the Resistance by the *Rammer*, or heavy Body made Use of to drive Piles into the Earth; where applying his Force successively against Gravity (that is, raising the Body to a certain Height) the Rammer would be put into such a Situation, as to receive an accelerated Motion from Gravity before it acted upon the Resistance, which at last it would do with a great Force, because the same Quantity of Matter would move with a great Velocity; so much the greater, as it had spent more Time in falling, by Reason of the Height it had been raised to. N. B. *It would be wrong to alledge here, that a Man, who could not raise a Weight, or overcome a Resistance something greater than* 140 ℔, *with the Force* 140 *by Means of a Leaver or Beam, whose* Brachia *are of equal Length, might do it by applying his Power farther from the Centre of Motion; because we are to suppose the Weight or Intensity of the Resistance to be always in a Proportion to the Power something greater than the reciprocal Proportion of Distances: that is, we are to suppose in this Comparison, a Case impossible in the common Use of mechanical Organs.* Lect. V.

22. The Mall or Hammer, which is a Body or Mass of Wood or Metal, directed or moved circularly, or in a curve Line by the Handle, so as by Degrees to receive a certain Velocity, and thereby to have a Power of moving, or making Impressions on, other Bodies, partakes of the two above-mentioned Organs; as it receives an accumulated Force from the Arm that moves it, and at the same Time from Gravity, when it is made Use of to strike downwards. Though it is mov'd circularly, yet its Stroke is made in a streight Line, namely in the Tangent to the Curve in which it moves, just at the Point of the Curve in which the Blow is made;

Motion to the utmost that his Muscles can exert their Action in running, and thereby give a prodigious strong Blow by the accumulated Force with his arm'd Head, to the Thing or Animal which he runs against; his Horns being so plac'd and fix'd on his Head, that he feels no Pain from the Shock.

because

Lect. V. *N° 5. becauſe all Bodies mov'd in a Curve endeavour to fly off in a Tangent to that Curve, as we have already ſhewn *.

* L. 4. P. 255, 256. 23. THE Fly (ſome of whoſe Uſes I have already conſider'd in the *4th* * Lecture) is an Organ whereby the Body moved is caus'd to circulate or move round about a Centre or Axis, and thereby is capable of accumulating the Powers impreſs'd upon it by Degrees, ſo long as ſhall be requir'd. This differs from the Hammer and Mall in this Particular, as the Capſtane differs from the Leaver: for whereas the Leaver can only lift or remove the Body for a very ſhort Space, the Capſtane can perform it for a given or requir'd Space. So that the Hammer or Mall can only receive ſo much Power as can be given it in the Part of an Arc or Circle, or in a ſtreight Line of a ſhort Length; but the Fly can acquire its Power by Accumulation in many Revolutions, and ſo is made receptive of a given Power and Velocity, and can exert the ſame on any other Body in a determinate Manner.

24. THIS mechanical Organ join'd to the Screw, compoſes that powerful Machine, whereby they impreſs the Stamp or Image upon Coins and Medals, which requires a prodigious as well as regular Force and Power. Now in this Engine there are accumulated Powers, by three mechanical Organs: firſt, by the Fly, whereby the Strength of the Man moving it is accumulated into the Weights at the End thereof: ſecondly, That Power accumulated is condens'd and impreſs'd on the Cylinder of the Screw by means of the *Radii* at the Fly, which are two Leavers that ſerve to condenſe the ſame in a given Proportion: thirdly, That Power ſo communicated to the Cylinder accumulated and condens'd, is again condens'd by the Slope of the Screw in a given Proportion; and ſo the whole Power (or all the ſucceſſive Powers) exerted by the Man in moving the Weights of the Fly, is accumulated and condens'd in the laſt Impulſion, which is made upon the Medal.

25. ANALOGOUS to the Fly may be accounted the *circular Pendulum*, which is a mechanical Organ, whereby Motion may be accumulated in the Body or Weight thereof: which is ſuſpended by a String from a Centre above, that keeps it at due Diſtance from the ſaid Centre.

By this Organ, the Weight to be moved by the impressed Force, is more free to receive and retain the accumulated Power, and receives none of the first Impediment of the Fly, namely from the rubbing and wearing of the Pivots or Gudgeons, and is only subject to the latter, namely, the Impediment of the Air, or of the Medium thro' which it is moved, which is also less, by reason that the String by which the Weight is suspended is less than the Arms or Branches of the Fly: and hence it is, that this Organ preserves the impressed Force much longer than the other, namely the Fly, and will continue in Motion much longer. When set a going, it will often continue to move many Hours together; and, if the Length of the spiral Space in which it moves be computed, it will amount to several Miles in Length. *But not having yet given any Account of the common or simple Pendulum, I must defer saying any more of the circular one till I come to explain the simple one.*

26. The last of these kind of Organs that I mean to describe now is the *Sling*; for the Bow in its Action depends upon the *3d Law of Motion*, and therefore cannot properly be treated of till we have explain'd that Law. The *Sling* then is an Instrument or Organ which serves, by the means of Strings, to convey gradually the Strength of an Hand moving in a small Circle to a Body detain'd by these Strings, to move in a greater Circle about the same Centre, till it has fully accumulated in it all the Power of the Hand, or the design'd Quantity of it, at which Time it discharges it with a design'd Direction and Determination. This Instrument partakes of the Fly and the circular Pendulum; for, like the Fly it can receive an Accumulation of Power for many Revolutions, and like the circular Pendulum it is free from the Friction of Pivots and Gudgeons; and none of the Force applied to it is lost, but just what is sufficient to overcome the Resistance of the Air.

It would be too long to describe the various Ways that this Organ has been contriv'd and made use of by the ancient as well as the modern Engineers. It is sufficient to observe, that they are all reducible to this (the common Sling) which is the most simple; and the Power and Effect thereof is as easy and reducible to Geometrical Computation and Calculation, as any of the others already mentioned: There being only one Way applicable to all; and it is to

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find what the Velocity of the Body is just at the Instant that the Percussion is made *, or the Machine applied to use. For it being always supposed that we know the Weight of the Body or Instrument, we have nothing to do but to multiply it by the Velocity, and it will give us the *Momentum* of our moving Body †, and thereby shew the Effect which it is able to produce in acting upon another Body, in stopping, driving, breaking or striking it, so as to shake, move, or remove the whole or any Part of it.

* Ann. 4.

† L. 2. N° 3.

27. Now all the Difficulty consists in finding what is the real Velocity of the percutient Body just at the Moment of Percussion; and it may be done in the following Manner, which we will first shew with respect to the Rammer or heavy Weight to drive Piles, it being most easily considered in that Instrument, and all Cases deduced from that.

* Pl. 25. F. 15. THE 15th *Figure* of *Plate* 25 * represents an Engine to drive Piles, consisting of the Cill K I and Frame F L, on which are fix'd the upright Pieces L H and L G, supported by the Side Braces C, C, and the hind-Brace F E (which has Pins on it to make it serve as a Ladder) and held together by the square Collar E D. The Rammer A being a very heavy Piece of hard Wood or Iron slides up and down between the Cheeks or upright Pieces L H, L G, and is drawn up by means of its Hook B with two Ropes H O, G O, having each 5 smaller Ropes with Handles at N, N, for 10 Men to pull up the Rammer to a certain Height, (the great Ropes running over two Pulleys or Rollers on the Iron Pin H G) and then let it fall again all at once upon the Head of the Pile at M, to drive it into the Ground. Now suppose the Rammer A weighs 500 lb and falls the Height of one Foot, it will fall that Height in a quarter of a Second, and consequently have a Velocity able to carry it uniformly 2 Feet in the same Time †, (that is, at the Rate of 8 Feet in a Second) at the very Instant it strikes the Pile M. Therefore multiplying the Mass by the Velocity, *viz.* 500 × 8, we shall have 4000 for the *Momentum* of the Rammer, with such a Fall. But if the Rammer be raised up to the Height of 4 Feet, it will fall that Height in half a Second, and have at the Time of Percussion a Velocity to carry it 8 Feet in half a Second, without any farther Help from Gravity; so that we must now multiply 16 Feet (the present Velocity, since it goes at the Rate of 16 Feet in a Second) by 500 the Mass of the Rammer, which will give us a

† N° 15. Cor. 3. P. 324.

double *Momentum* wherewith it will strike the Pile in this last Case; for $500 \times 16 = 8000$. If we consider any other Height from which the Rammer falls, (for one may employ a Capstane, Windlass, or Pulleys, to raise it to a very great Height, and then by an easy Contrivance loosen it at once from its Hook) the *Momentum*, with which it strikes the Pile, will always be as the square Root of the Height from which the Rammer fell; that is, as the Velocity which the descending Body has at the End of its Fall.

N. B. *I cannot say but that the Pile may enter into the Earth sometimes farther than in that Proportion; but I shall shew the Reason of it in the Notes* * Ann. 4.

28. If a Pile is to be driven obliquely, the Engine must be set so that the Cheeks may have the same Obliquity, and the Blow will still be perpendicular to the Head of the Pile; but then the Force of the Blow must not be estimated from the Length, but from the Height, of the Descent, in the Manner already shewn; because how long soever the inclined Plane is in which the Body falls, it has acquired no more Velocity than what it would do if it had only fallen perpendicularly from the Height of the Plane.

29. To find the Velocity of the battering Ram when it makes its Stroke, we must observe at what Rate its Motion is accelerated; for according to the Number and Strength of the Men that work it, it may be accelerated more or less than Gravity would accelerate it if it was to fall perpendicularly. Therefore we are to observe the Length of the Stroke from the Point most remote from the Wall (or the Thing batter'd) and the Wall, and take Notice in what Time the Stroke is made; (for when the Men, by a little Practice have got the Knack of playing the Ram, all their Strokes are made in the same Time.) If the Stroke, for Example, be of 4 Feet, and made in a quarter of a Second, the *Momentum* of the Ram is equal to what it would be if the same had mov'd uniformly 8 Feet in a quarter of a Second, or 32 Feet in one whole Second. This Force would be quadruple of what Gravity might give the falling Ram (or a Rammer equal in Weight to the Ram,) in the same Time; but only equal to what it would give it in one Second or the quadruple of the Time, and only half of what Gravity would give in two Seconds to the same Body falling from an Height of a little more than 64 Feet. If the Time of making the Stroke had been twice as long, or half a Second, then the *Momen-*

Lect. V. *tum* or Force of the Percussion would have been but half as great, *&c.* the Percussion with the same battering Ram being always inversely as the Time in which it is accelerated by the same Force of Men.

30. To find the Velocity (and consequently the *Momentum*) of the Mall or Hammer at the Instant of Percussion, we must consider it first in the most simple manner, as when being raised up it falls down again in an Arc of a Circle by its own Gravity: then we are only to consider the Height from which it fell; and we shall know, from what we have said of Bodies falling perpendicularly or on an inclin'd Plane, what Velocity it has at the End of its Fall. Thus we may know with what Force the great Hammers, raised by the Axis of a Water-wheel, fall upon the Plates or Bars of Iron or Copper that are flatten'd in Iron or Copper Battering-Mills or Forges. When the Hammer is moved by the Hand, or driven by a Spring as well as by Gravity, it will move quicker, and its Effect will be proportionably greater, therefore by observing the Time of its Fall in such a Case its Velocity may be known.

N. B. *Tho' a Body moving twice as fast in the same Circle, was able, according to what appear'd in the 15th Experiment of this Lecture, to raise four Times the Weight, we must not imagine that a Hammer mov'd twice as fast will strike with four Times the* Momentum, *whilst a Hammer of the same Weight moving twice as fast, only because of an Handle or Radius twice as long, shall have but double the* Momentum, *according to the 13th Experiment: for it is only the Centrifugal Force in the same Circle, which is as the Square of the Velocity, and not the Stroke made along the Tangent. The Effect of the Centrifugal Force will only be this, that the Hand which holds the Hammer will feel four Times the Force endeavouring to pull the Hammer out of his Hand; and would feel but twice that Force, if the Velocity was only double by having an Handle twice as long.*

It is also to be observed, that in the use of the Hammer it is better to strike with the same Mass of Matter with a double Velocity, than with double Mass and single Velocity; because yielding Substances, as hot Iron, &c. and Nails driven into Wood, give Way to the same Hammer nearly according to the Square of its Velocity, as we shall account for in the Notes. But the reverse*

* Ann. 4.

verſe muſt be done in the battering Ram, if you would have its greateſt Effect †. Lect. V.

† Ann. 4.

31. If the Fly be made uſe of only to give a Blow with one of its Weights after ſome Revolutions, the Method of finding its Velocity, and conſequently its *Momentum*, juſt at the Stroke, is the ſame as will ſerve for the Sling: for tho' the Fly has vaſtly more Friction than the Sling, we are only to obſerve what Velocity (without any Regard to the Impediments which hindered the Velocity from being ſo great as it would have been without them) it has juſt at the Stroke; which we may know by comparing ſeveral Revolutions, or Parts of Revolutions together; which will ſhew us the Degree of Acceleration.

32. Now to ſhew how to calculate the Force of theſe Organs when joined with others commonly called the *mechanical Powers*. (explain'd in the 2d Lecture) I ſhall ſhew what Force may be given by the Machine made of the Fly combined to the Screw for ſtamping the Image upon Coins, as we have above hinted.

Let us ſuppoſe the two Arms of the Fly to be 15 Inches long each, (meaſuring from the Centre of the Weights to the Axis of Motion) and the Weights to be 50 lb each, and the Diameter of the Axis preſſing upon the Dye to be one Inch. If every Stroke be made in half a Second, and the Weights deſcribe an half Circumference, which will in this Caſe be of 4 Feet, the Velocity will at the Inſtant of the Stroke be at the Rate of 8 Feet in a Second, and therefore the *Momentum* of it will be 800; but the Arms of the Fly being as Leavers, one *Brachium* of which is 15 Inches long, whilſt the other (which is the Semi-Axis) is but of half an Inch, we muſt increaſe this Force 30 Times, which will give us 24000: an immenſe Force, equal to 100 lb Weight falling 120 Feet, or near 2 Seconds in Time, or a Body of 750 lb falling 16 $\frac{1}{12}$ Feet, or one Second in Time. Some of theſe Engines for coining Crown Pieces have the Arms of the Fly 5 Times as long, and the Weights twice as heavy; and the Effect is 10 Times greater.

N. B. *We have allowed nothing here for the inclined Plane of the Screw, becauſe that Declivity only ſerved to help to accelerate the Motion of the Weights, which we conſidered in taking the Time of the Deſcription of the Semicircle by them.*

33. It

33. It would be endless to shew all the Consequences of the two Laws of Motion already explain'd, in the Practice of mechanical Operations; and to apply them to the Explication of all kinds of Motions whether of Bodies on Earth, or of the Planets and Comets in the Heavens: We shall only give a few Instances more, but first we must shew how far the Resistance of Air (which we have hitherto left out of our Computations) hinders the Effects from being such as might be expected from their Causes without it.

Tho' the Resistance of *Mediums* is to be consider'd in the Hydrostatical Part of this Work, yet we must say so much of it now as is required for understanding how to allow for the Disturbance that the Resistance of *Mediums* gives to moving Bodies, whether their Motion be owing to Gravity, or any other Cause or Causes.

34. When a Body moves in a Fluid of any Kind whatever, or resisting *Medium*, it cannot go on without separating the Parts of the *Medium* to make itself way; and so much as it bestows of its own Motion on those Parts, so much it loses of its own Motion; so that it will be retarded if its Motion was uniform before; or if it went on with an accelerated Motion, that Resistance (or the Motion given to the Parts of the *Medium*) will hinder the Acceleration from being so great as it would have been, or (according to the Quantity of it) destroy the Acceleration; that is, destroy the additional Motion as fast as it is given to the Body by the accelerating Cause: So that the Body will then move uniformly; as if the accelerating Cause had ceased to act, and the Body should move in a *Vacuum* without any Resistance at all.

35. There are two sorts of Resistance in Fluids; the first arising from the Tenacity of the Fluid, that is, from the Cohesion of its Parts; and that Resistance is always as the Velocity of the Body moving in the Fluid: for the swifter the Body moves in such a Fluid, the more Parts it has to remove from their Cohesion in the same Time, according as it goes thro' a greater Space. That sort of Resistance may be diminish'd by rendering the Medium more fluid, as Oil, Honey and Pitch, *&c.* become more fluid by heating.

N. B. *Such Fluids as have no Tenacity have none of this Resistance.*

The other Sort of Resistance arises from the Quantity of Matter to be removed, and that is always proportionable to the Density

or

or specific Gravity of the Fluid *Medium*. Thus Water resists 850 Times more than Air, because a Body moving in it thro' a certain Space, has 850 Times more Matter to remove; and if it mov'd in Mercury, the Resistance would be 11900 Times greater, because Mercury has 11900 Times more Matter than Air in the same Space. In respect to the same Body moving with different Velocities in a Fluid, this Resistance is always as the Square of the Velocity. An Instance or two explained by Number will make the Thing evident. Lect. V.

LET us suppose the Body A (*Pl.* 25. *Fig.* 16 *.) to be moving in a *Medium* at the Rate of 2 Inches in a Second, or from A to B; and that it is to remove the 4 Particles of Matter h, c, g, f, to make its Way, which Particles we will suppose to be an Inch in Diameter. Now it is not enough to consider that those Particles are to be remov'd, but we must also examine with what Velocity they are remov'd, in order to find the Quantity of Motion which they receive. Let us then suppose each of them to be remov'd an Inch in a Second, or from the Points f, h, c, g, to *f*, *h*, *c*, *g*, to make Way for the Body A to go between them. Now since it is the same to move all the 4 Particles laid upon one another from f to *f*, as to move all the four one Inch in different Lines, it is evident that the Space *ff* or one Inch is their common Velocity: One then multiplied by four, the Number of Particles, gives 4 for the *Momentum* of the Matter remov'd by the Body A, which consequently must lose as much of its Motion as it has communicated; and therefore in this Case the Resistance will be 4. Again, let the same Body be supposed to move twice as fast, that is from A to B *Fig.* 17 *.) in a Second. There must be 8 Particles (that is, b, c, d, e, f, g, m, n, or as much more Matter) removed in the same Time; but as the Body moves twice as fast, it will strike each of them as hard again, which will make them recede to the Points β, $\varkappa$, δ, ε, φ, γ, μ, ν, instead of the Points *b*, *c*, *d*, *e*, *f*, *g*, *m*, *n*, in the same Time; so that their common Velocity will be 2 Inches instead of 1. But 8 Particles multiplied by 2 will give 16, which is a *Momentum* 4 Times as great as what the Matter of the Fluid receiv'd before. Therefore the Body moving twice as fast in the same Fluid communicates four Times as much Motion to its Parts, and consequently meets with 4 Times as much Resistance. Likewise if the Body moved 3 Times as fast, it would remove 3 Times more Matter in the same Time, and also dash it 3 Times farther; therefore

* Pl. 25. F. 16.

* Pl. 25. F. 17.

it

Lect. V. it would meet with 9 Times more Resistance. And this will hold good in any Degree of Velocity of the moving Body; for the Quantity of Matter remov'd in a certain Time, and the Velocity with which that Matter is removed, will always produce a *Momentum* in the fluid *Medium*, and consequently a Resistance, proportionable to the Square of the Velocity of the Body moving in that Medium.

N. B. *This Resistance according to the Square of the Velocity is the only Resistance that the Air is found to have, by Experiments of Bodies moving in it: and therefore it has no Tenacity; a Consequence of which is, that its Parts do not touch one another.*

HENCE it is, that a Fluid will resist sometimes as much as a Solid, nay more, when the Velocity of a Body coming against it is very great, as we shall shew by some Instances that we shall give in the Notes *.

* Ann. 5.

SCHOLIUM.

36. WHAT we have said concerning the Fall of Bodies in the Air, and along inclined Planes, will not agree with Experiments, because in the Theory we abstracted from the Resistance of the Air; but when we make proper Allowances for it, the Experiments will confirm the Theory. According to the best Observations a Body falling *in vacuo* should go thro' 16 *English* Feet and an Inch and a Quarter the first Second of its Fall; but in the Air it must lose of that Space in proportion to the Motion it gives to the Air, which must be subtracted from its own Motion: so that the more Matter the Body has in proportion to its Surface wherewith it strikes the Air, the less it will lose of its own Motion. This will be best explain'd by giving an Account of an Experiment I made, by observing the Time of Leaden Balls which I let fall from the Inside of the Top of the Cupola in St. *Paul*'s Church.

EXPERIMENT XVI.

I TOOK several Leaden Balls of 2 Inches Diameter, weighing 2 lb *Troy*, which I let fall from a Board fixed 2 Feet over the Top of the inner Cupola, and observed the Time of their Fall very nicely by an Instrument which I shall hereafter describe, and found that they fell to the Bottom in $4\frac{1}{2}$ Seconds and a very little more. The Height was 272 Feet. Now according to the Theory, the Balls in that Time should have fallen 52 Feet farther, that is, 324 Feet; therefore

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therefore the Resistance of the Air continually taking off something of what Gravity superadded to the Motion of the descending Body, occasion'd the Acceleration not to be so quick, and therefore the Body was longer in falling that 272 Feet than it ought to have been: for since a Body falls a Foot the first quarter of a Second of its Fall, if we take the square Root * of 272 we shall have $16\frac{1}{2}$ quarters, that is, 4 Seconds and $\frac{1}{8}$ for the Time that a Body would fall 272 Feet *in Vacuo*. Suppose now that the Resistance of the Air took off 5 Inches of the Space which the Balls should have fallen the first Second; during the 2d Second the Resistance of the Air must be greater in proportion to the Square of the Body's Velocity; that is, as the Body then should go thro' 3 Spaces equal to what it went the first Second, the Resistance must be 9 times greater; consequently the Resistance of the Air would take off 9 times 5 Inches, that is, 45 Inches of the Spaces gone thro' by the Body in the 2d Second: so likewise in the 3d Second, the Body must lose 125 Inches; in the 4th Second 245; and in the last Half-Second above 200; which makes about 52 Feet in all, according to what was observ'd in the Experiment.

* N° 15.

N. B. *This Calculation is not exact, being given only as an Illustration rather than a Demonstration, and founded upon a Supposition of a Body falling only* 61 *Feet in the first Second of its Fall, which Number was taken to avoid Fractions.* 16

Several Consequences may be drawn from the Resistance of Air in regard to Bodies moving in it, and which are verified by Experiments. The first is, That *the Motion of a heavy Body is not always accelerated, but at a certain Height it becomes equal and uniform in the Air*; because the Resistance of the Air increasing in the same Proportion as the Spaces increase (and consequently in a duplicate *Ratio* of the Times, or of the Velocities) this Resistance may become so great, as to destroy as much of the Velocity as should be produc'd, and by that Means hinder the Velocity of the moving Body from being increased any more; *just as if the Body at that Time should cease to be heavy. The second is, That *Bodies of different specifick Gravities moving in the same Medium have not their Motions accelerated after the same Manner*, by reason of the difference of their Bulk, compared to their Weight, which meets with more or less Resistance; because those of a greater Bulk, when the Weight is the same, drive more Air before them than those of a less.

* N° 12, and 13.

THE Third is, That *the Motion of heavy Bodies is differently accelerated in different* Mediums, *and in the moſt denſe* Medium *it becomes equal ſooneſt*; becauſe the more denſe the *Medium* is, the more Difficulty it has to make its Circulations, and it reſiſts Motion the more eaſily.

THE Fourth is, That *the leaſt Bodies of the ſame homogeneous Matter fall with leſs Velocity, and come ſooneſt to an Equality*; becauſe that Body which has a greater Surface is more reſiſted than that which has a leſs, and the leſs Bodies have a greater Surface than the great ones in reſpect of their Weight or Solidity: for we are taught by Geometry, that if a Cube has its Surface, for Example, of one Foot, another Cube eight times as heavy will have its Surface but of four Feet. According to this Principle, the Duſt falls very ſlowly when it is rais'd, Birds ſuſtain themſelves in the Air by ſpreading their Wings; and a Charge of Shot will not go near ſo far as a Bullet of the ſame Weight ſhot from the ſame Gun with the *ſame Quantity of Powder*, tho' both begin to move with equal Velocities.

THE Fifth is, That *there is a determinate Height which produces in a heavy Body the greateſt Velocity that it can acquire in falling*; ſo that if it ſhould fall from an higher Place it would have no more Velocity: which is evident from the firſt Conſequence, where we have ſaid that the Motion of a heavy Body is not continually accelerated; but that at a determinate Height it becomes equal.

THE Sixth is, That *there is a determinate Height, the greateſt of all thoſe to which the Velocity which a Body has acquir'd in falling, can make the ſame Body riſe up again*; becauſe by the foregoing Conſequence, there is a determinate Height which produces the greateſt Velocity that a falling Body can acquire, and that Velocity can make it riſe up again but about to the ſame Height.

THE Seventh is, That *a Body thrown upwards by a Force greater than the greateſt that it can acquire in falling, ought to be longer in falling than riſing*; becauſe the Velocity of a Body thrown up to any Height whatever is continually diminiſh'd; whereas the Velocity of the ſame Body in its Fall increaſes but till it comes to ſuch a Height; it being certain that if it ſhould increaſe continually, the Body would be juſt as long in falling as in riſing.

THE Eighth is, That *if a Body be thrown downwards by Force that exceeds a greatest Force it can acquire in falling, it will have a retarded Motion*; because by the first Consequence the Body which falls with the greatest Velocity that its Fall would give it, meets with a Resistance in the Air, equal to its Gravity; and when it goes with a greater Force, the Resistance of the Air becomes greater still, and must destroy part of the Motion, which thus will be slacken'd and retarded.

THIS last Consequence shews why a Cannon-Ball shot downwards retards its Motion; because such a Ball is put in motion by the Force of the Powder, which gives it a greater Velocity, than that which its absolute Gravity would have given it in falling: And the seventh Consequence shews likewise the Reason of this Experiment, which Father *Mersennus* takes notice of in his *Ballistica*, or Art of throwing heavy Bodies. *Prop.* 13.

THIS Author says, that he has found by several Experiments, that an Arrow which has been three Seconds in rising, has been five in descending; and tho' he adds that an Iron Bullet of three Pound weight having been shot upwards perpendicularly by a Mortar-piece a Foot long, has spent as much time in rising as in descending, *viz.* Six Seconds; yet it does not follow, that it must always happen so, the Difference not being so considerable in a Bullet as in an Arrow, whose Motion comes soonest to an Equality, by reason of its Lightness.

A COMMON Bomb not receiving from the usual Charge of Powder a Velocity greater than the greatest it can acquire in falling, is as long in coming down as it is in going up. But a Ball of light Wood or of Cork (which in a *Vacuum* would go much higher and farther than a Bomb of the same Bigness, because it receives from the Powder so much more Velocity at first as it has less Matter) will not go so high as a Bomb, and also will be longer in coming down than in going up, on account of the Air's Resistance, which has more Effect upon those light Bodies for the Reasons above-given.

N. B. *It is not mathematically true, that a Body falling in the Air ever comes to an equal Motion; but as it always approaches nearer and nearer to it, we may take it to be such physically, and reason from it accordingly.*

Lect. V. 37. I ſhew'd, after explaining the firſt Law of Motion, how far it would ſerve to make us acquainted with the Motion of the heavenly Bodies, by ſhewing in what manner Gravity and the projectile Force keep thoſe Bodies in their Orbits; but it requir'd the underſtanding of the ſecond Law to conceive rightly how they move in Ellipſes that have the central Body in one of the *Foci*, and why their Velocities are ſucceſſively accelerated and retarded.

38. BUT before I proceed to conſider this, I muſt explain ſome aſtronomical Terms, and ſhew what is meant by ſaying, that *the Planets and Comets in reſpect to the Sun deſcribe Area's about it proportionable to the Times; as likewiſe the Satelittes in reſpect to their primary Planets.* And this is a Truth known and own'd by all modern Aſtronomers, however they differ in accounting for the Cauſes of the celeſtial Motions.

SUPPOSE a celeſtial Body to move round another in a Curve returning into itſelf as a Circle or an Oval; as for Example, the Moon about the Earth, whoſe Orbit we will conſider at firſt as circular, tho' it is really elliptical. If at any Time of a certain Day we †Pl. 25. F. 18. obſerve the Moon's Place in its Orbit to be at L †; and the Day after, at the ſame Time, the Moon is found to be at another Place as *L*, the triangular Space T L *L* (being contain'd by the Line or Ray T L drawn from the central to the revolving Body, at the firſt Obſervation, the ſame Line in the Situation T *L* at the ſecond Obſervation, and the Arc L *L* deſcrib'd by the Moon during the Interval of the Obſervations) is call'd an *Aſtronomical Area*, and the Ray T L, conſider'd as ſweeping along that Space and carrying the Moon with it, is called the *Radius vector* or ſweeping Ray. If ſome Days after, for Example 14 Days after, we obſerve the Moon at l, and the next Day at the ſame Hour and Minute obſerve it to be at *l*, the *Area* T l *l* will be equal to the former *Area* T L *L*, which was deſcrib'd by the Moon and *Radius vector* in the ſame Time: And this is what is meant by ſaying, that *revolving Bodies* in the Heavens *deſcribe about the central ones* Area's *proportionable to the Times.*

39. HERE we are to obſerve, that the Triangles or *Area*'s T L *L*, †Pl. 25. F. 18. T l *l* †; are not only equal but ſimilar, and therefore the Body L does in this Caſe deſcribe the equal Arcs L *L* and l *l* in equal Times, as well as equal *Area*'s; ſo that the Motion of L, round

T

T is equal, neither accelerated nor retarded. Such are the Motions of *Jupiter's Satellites* about his Center, their Orbits being circular (as far as Observations have been made hitherto) except so much as they disturb each other by their Gravity towards one another, and as they are disturb'd by the Sun according to the different Distances of *Jupiter* from the Sun, or by *Saturn*, whose Attraction becomes sensible at, and near, its Conjunction with *Jupiter*. But all these Inequalities may well be neglected here; because, tho' they are certain Consequences of the mutual Attraction of Bodies, they are not considerable enough to be observed with Telescopes. Lect. V.

40. Now let us suppose the Body T not to be in the Center of the Orbit, as the Earth is not in the Center of the Moon's Orbit, but to be distant from it the whole Length C T (*Fig.* 19. †) if the Moon or the revolving Body be observ'd at L and *L*, and so found to have gone through the Arc L *L* in one Day's Time; then again if it be observ'd 14 Days after at l, the next Day it will not be at λ, (to which Point it would have gone in a Day if its Velocity had not increas'd) but it will be got quite to *l*, its Velocity increasing so as to make it describe so much a greater Arc as it is nearer to the central Body T; otherwise the *Area* last describ'd would not be equal to the *Area* first describ'd: for as much as T l, the Distance from the central Body in the beginning of the Description of the last *Area*, is less than T L the Distance from the central Body in the beginning of the first, so much must the Arc describ'd in the last be greater; that what this last *Area* wants in length, may be made out in breadth. † Pl. 25. F. 19.

41. If, *instead of a Circle*, the revolving Body moves in an Ellipse, in one of whose *Foci* the central Body is plac'd, (as is the Case of the Moon about the Earth, but more strictly so of the Planets and Comets in their Motion round the Sun;) and the whole periodical Time of the Body's Revolution be divided into equal Parts, in every one of those Parts of Time the Body (by its *Radius vector*) will describe an equal *Area*, but none of those *Area's* will be similar, except those that are describ'd on each Side of the Axis of the Ellipse, in correspondent Parts of the Curve, at equal Distances from the central Body. Let A B C D E P F G H I † represent one of the Ellipses, which is describ'd by a Planet or a Comet round the Sun; P S *s* A the Axis of the Ellipse, and S, *s*, its *Foci*; S the Sun, and A a Planet at the Sun's *Aphelion* † Pl. 25. F. 20.

(that

Lect. V. (that is, at the greateſt Diſtance from it) and P the ſame Planet at the *Perihelion* (or leaſt Diſtance from the Sun) and the Time of the Revolution be divided into 10 equal Parts; the Planet ſetting out at A and going towards B, by its *Radius vector* A S, will deſcribe ſucceſſively the 10 equal *Area*'s A S B, B S C, C S D, D S E, E S P, P S F, F S G, G S H, H S I, I S A, of which only every two correſpondent *Area*'s are alike, as B S C is like I S H, D S E like G S F, *&c.*

COROLLARY I.

HENCE follows, that in an excentrick Orbit, ſuch as an Ellipſe, the revolving Body moves faſter at the *Perihelion* (the Sun being in one of the *Foci*) than at the *Aphelion*; accelerating its Motion from the *Aphelion* to the *Perihelion*, and retarding it from the *Perihelion* to the *Aphelion*.

COROLLARY II.

HENCE follows alſo, that the more excentrick (that is, the longer) the Ellipſe is, the greater is the Difference of Velocity at the *Perihelion* and *Aphelion*; ſuch is the Caſe of Comets, which moving in very excentrick Ellipſes, go thro' the lower Part of their Orbits with very great Velocity, but move extremely ſlow near their *Aphelia*.

COROLLARY III.

THIS ſhews why a Planet, tho' it be much more ſtrongly attracted in its *Perihelion* than its *Aphelion*, will not be drawn into the Sun; becauſe it acquires a greater centrifugal Force as its Velocity increaſes, and thereby balances the Sun's increas'd Attraction. So likewiſe, when the Planet goes from the *Perihelion* to the *Aphelion*, tho' the Sun's Attraction be decreas'd, becauſe of the increas'd Diſtance, the Planet will not fly out of its Orbit; for the Velocity decreaſing, the centrifugal Force decreaſes alſo. In the Ellipſe repreſented by *Fig.* 20 †, when the Planet is at P, it is ſix times nearer the Sun S than when it is at A, therefore it is 36 times more attracted *; but then its Velocity being alſo ſix times greater, the centrifugal Force increaſing as the Square of the Velocity ‡, becomes 36 times greater. So that the Attraction, or *accelerating*

† Pl. 25. F. 20.

* Lect. 1. page 9, 32.

‡ Exp. 15. page 322.

accelerating Force * (however it increases or decreases, on account of the different Distance) is always balanced by the centrifugal Force of the Planet. Lect. V. * L. 1. p. 32.

COROLLARY IV.

HENCE we see, why those Planets, which are nearest to the Sun, perform their Revolutions in shorter Time than those which are farther off; that their greater Velocity may give them a sufficient centrifugal Force to balance their centripetal Force (or Gravitation) towards the Sun; Regard also being had to their Quantity of Matter †. † Exp. 12. p. 321.

THE Satellites of *Jupiter* and *Saturn* also have their Periodical Times shorter, as they are nearer to their Primaries, as we have already hinted ‡. ‡ N° 7. p. 317.

THO' the Orbits of the Planets are nearly circular; yet as the *Foci* of an Ellipse are considerably distant from one another, even when the Curvature does not much differ from that of a Circle; the Excentricity will be sensible enough to be observ'd. Hence it is, that our Winter Half-Year (in the northern Hemisphere of our Earth) in which we go thro' the *Perihelion*, is 8 Days shorter than the Summer Half-Year, *&c.*

42. *KEPLER* was the first who discover'd, that the Planets, by a Ray drawn from them to the Sun, describ'd *Area's* proportionable to the Times; and guess'd that the Cause might be a Gravitation towards the Sun; but he did not demonstrate it. But Sir *Isaac Newton* has demonstrated it in his *Principia*, shewing, that when a centripetal Force drives one Body towards another, and that first Body has received the Impression of a projectile Force in any other Direction, it will describe round this last, *Area's* proportional to the Times; and *vice versâ*, that if a revolving Body, by a Ray drawn to a central Body, describes *Area's* proportionable to the Times, it is acted upon by a centripetal Force. Then he shews (from Observations of the Moon's Motion) that the centripetal Force is the very same as Gravity, which makes our Bodies fall by an accelerated Motion near the Surface of the Earth. We shall give his Demonstrations and a farther Account in the Notes †. † Ann. 6.

43. IN

Lect. V. 43. In the mean Time I shall endeavour, in the easiest way, to shew how Gravity makes the Planets describe their excentrick Orbits with a Motion uniformly accelerated and retarded: for tho' I shall give no strict Geometrical Account of every thing relating to this Motion, and the Nature of the Curve, yet every Assertion will be a Consequence of the two Laws of Motion, already explain'd, and their Corollaries.

Plate 25. Fig. 20.

Pl. 25. F. 20. The Ellipse represented by this *Figure*, is more excentrick than any of those that are described by the Planets, but not so excentrick as those that are described by the Comets. I took it at a Mean; because, as both the Comets and Planets are retain'd in their Orbits, and continue their Motions from the same Causes, one Explication might serve both.

Let S represent the Sun, A the Planet or Comet, which Gravity (or the mutual Attraction of the Sun and revolving Body) drives towards the Sun in the Direction A S, and let A M represent the Quantity of that Force; that is, the Space, which that Force alone acting, would cause the Planet to go thro' in a given Time. Let the Planet A be supposed to have receiv'd a projectile Force in the Direction A α at right Angles to A S. If A *a* expresses the Space, which the projectile Force alone would make the Planet describe in the said given Time, and the Quantity of that Force be such, as acting jointly with Gravity, will make the Planet (setting out in the Diagonal A *m* of the compleated Parallelogram A *a m* M) describe the Circle A *m* μ *&c.* whose Centre is S the Centre of the Sun; then a greater *Vis empressa*, or projectile Force, such as A α (the Force of Gravity A M remaining the same) will make the Planet set out in the Diagonal A *n* of the compleated Parallelogram A α *n* M, and describe an Ellipse A *n* ν, *&c.* whose nearest *Focus* is S: then will the Point A become the *Perihelion* instead of the *Aphelion*, and the Motion will be retarded from A till it comes to the *Aphelion* on the other Side of S; and then from that *Aphelion* accelerated till it comes back to A, from whence it began its Motion. But if the projectile Force be less than what we have supposed capable of making the Planet move in a Circle, and it be expressed by A a instead of A α, then the Planet (begin-

(beginning its Motion in the Diagonal A B of the compleated Parallelogram A a B M) will describe the Ellipse A B C D E P F G H I A, the Sun S being in the farther *Focus*, and the neareſt Focus will be at *s*; ſo that now A will be the *Aphelion*, and P the *Perihelion*. Lect. V. *Pl. 25 F. 20.

* Now to ſhew how the Planet is accelerated in going from the *Aphelion* to the *Perihelion*; let us obſerve that at firſt ſetting out, the Direction of the Projectile Force A *a* is at right Angles with the Direction of Gravity (or the centripetal Force) A S. When the Planet by the joint Action of the two Forces is come to B; the projectile Force has its new Direction along the Tangent B *b*, which makes an acute Angle with the new Direction of Gravity which now is B S; therefore the Planet will deſcribe a longer Diagonal in the ſame Time *, *viz.* B C; that is, increaſe its Velocity ſo that the *Area* B C S may by a Breadth proportionably greater be equal to the longer *Area* A B S. When the Planet is come to C, the Direction of the projectile Force along the Tangent C *c* ſtill making an acute Angle with C S the Direction of Gravity, the two Forces conſpiring ſtill accelerate the Planet and carry it from C to D in the ſame Time that it went at firſt from A to B. The ſame Forces conſpiring ſtill in their Directions D *d* and D S, when the Planet is at D, will in a Space of Time equal to the former carry it to E: and laſtly, the ſame Forces with their conſpiring Directions will ſtill accelerate the Planet in its Motion from E to its *Perihelion* P, where its Velocity is the greateſt of all. At P the *Perihelion*, the Direction of the projectile Force is along the Tangent P *p*, and makes a right Angle with P S the Direction of Gravity; and the Planet from the Action of thoſe two Forces will go to F. When the Planet is at F, the Direction of the projectile Force along the Tangent F *f* makes an obtuſe Angle with F S the Direction of Gravity, and therefore the Motion of the Planet muſt be retarded, * becauſe the Diagonal F G will be ſhortened, as the Angle *f* F G opens, and the Forces begin to act againſt one another. The Angle above-mentioned will ſtill increaſe at G, ſo that the Force of Gravity in the Direction G S will check the projectile Force which now acts in the Line G g, and ſtill retard the Planet more. The Angle continuing to be obtuſe at H and I, the Planet is ſtill retarded till it comes to A the *Aphelion*, where its Motion is the ſloweſt of all. *It may perhaps be objected here, that ſince the Angles* a A S, *b* B S, *c* C S, *&c. only decreaſe half way from* A *to* P; * *and the Angles* S P *p*, S F *f*, * N°. 13. * N°. 13. * Ann. 7.

Lect. V.

Pl. 23. F. 20.

* Ann. 7.

S F f b, *&c. only increase half way from* P *to* A *; *the Planet descending towards the* Perihelion *should not increase its Velocity after it is come half way, because then the Angles, as* S E *e*, *&c. increase again: neither should it retard its Motion after it has moved half way from* P *to* A, *because the Angles made by the Directions of the two Forces no longer increase, but gradually decrease. But then we must consider, that there is another Cause of Acceleration and Retardation which does not depend upon the Quantity of the Angle abovementioned, but continues to increase after the Angles cease to diminish in the Descent of the Planet towards the Sun; and that Cause likewise continues to decrease in the Ascent of the Planet from the* Perihelion *to the* Aphelion, *even after the Angles of the Directions of the Forces cease to increase: and that Cause is the diminished or increased Distance of the Central Body (or the Sun* S) *the Power of Attraction changing continually in a reciprocal Proportion of the Square of that Distance. If, for Example, we would compare the Velocity of the Planet at* D *and at* E, *we must compleat the Parallelogram* D ♌ E d, *of which the two Sides* D ♌ *and* D *d represent the Force of Gravity and the projectile Force when the Planet is at* D; *then we must also compleat the Parallelogram* E ♍ P *e*, *in which* E ♍ *is greater than* D ♌ *in the same Proportion as Gravity is greater at* E *than* D; *likewise* E *e must be greater than* D *d*, *in Proportion as the projectile Force by its continual Acceleration is there also become greater; and tho' the Angle* ♍ E *e is greater than* ♌ D *d*, *we shall have a greater Diagonal* E P *on account of the greater Length of the Sides in the Parallelogram* ♍ E *e* P.

4. To shew how agreeable the Gravitation of Planets and Comets (for what we have said of Planets is as applicable to Comets) towards the Sun, is to the Gravitation of heavy Bodies towards the Earth; let us take a View of the 21st *Figure* † representing a long Ellipse, or the Orbit of a Comet about the Sun S, when π represents the *Perihelion*. The Semi-Ellipse $\pi\ \alpha$ is the Line in which the Motion of the Comet is uniformly retarded, and the Semi-Ellipse $\alpha\ \pi$ the Line in which the Motion of the Comet is uniformly accelerated. Now, if instead of including the Sun S in the Orbit, we should from one Part of the Surface of the Sun P, project a Body upwards, so that it should rise as far as A, its Motion would be uniformly retarded till it came to A, where it would have its least Velocity; then it would turn again, and fall in the

Pl. 23. F. 21.

Line

Line A *p* accelerating its Motion till it came to *p*, taking up just as much Time in its Ascent as Descent. And this is what we have shewn concerning the Action of Gravity upon Projectiles, abstracting from the Resistance of the Air. Lect. V.

45. But now let us take in the Resistance of the Air, and compare it with any resisting Medium, to see what would happen to the Planets, if they moved in such a Medium.

We have shewn ‖, that when Bodies move in the Air, they lose of their Motion by the Resistance of the Air, in proportion to the Square of their Velocity; and that that Resistance hinders falling Bodies from accelerating their Motion, as they would do, if they fell in a *Vacuum*; because that Resistance continually taking away some Part of the Velocity which Gravity superadds to the falling Body, continually brings the Motion of the Body nearer and nearer to a Motion of Equality. Now, if the Planets moved in a resisting *Medium*, such as the celestial Matter, which the *Cartesians* suppose; the Resistance of that Matter would hinder a Planet from acquiring that Velocity in its Descent to the Perihelion, which is necessary to make its centrifugal Force balance the Force of Gravity: for this last Force would always increase in proportion to the Square of the decreased Distance, let the *Medium* encompassing the Sun be of what Nature it would; but the Centrifugal Force would want its proper Increase, if the Planet wanted its required Velocity. The Consequence therefore would be, that the Planet would change its Track and come nearer to the Sun, and revolve in a longer Ellipse. The next Revolution, the Planet coming towards the *Perihelion*, and wanting its due centrifugal Force, would be brought nearer to the Sun by the Attraction (not diminished, but increased, because now the *Perihelion* would be nearer;) then again would the Ellipse be changed into a longer, and the next *Perihelion* would be still nearer, till the Planet for want of its due Velocity in a Direction along the Tangent, approaching nearer and nearer every Revolution, would at last fall into the Sun. ‖ N°. 35. Pag. 350, 351

Now, since no such Thing happens, it is evident, there is no such resisting *Medium*, or *celestial Matter of a Vortex*, as the *Cartesians* suppose to be the Cause of the Motion of the Planets round the Sun. So far from that, that such a Fluid would destroy the Motion of the Planets, as we have shewn.

But what destroys the *Cartesian* Hypothesis at once, (as well as the Opinion of those antient Philosophers, who supposed solid Orbs of Crystal to belong to every Planet, and carry it round) is the Observation of Comets, which are neither Meteors, as some of the Antients suppose them, nor Planets straying from one *Vortex* to another, as the *Cartesians* assert; but Planets moving in very excentrick Orbits, which we shall consider more fully in another Place*. Only here we are to observe, that they move freely to and from all Parts of the Heavens; and therefore there can be no crystal spherical Shells, which would stop them; nor Whirlpools of Matter, which would change their Direction by degrees, and at last make them move nearly in the same Plane as the Planets, whose Orbits have all their Planes contained in the Breadth of a Zone of a few Degrees. But what is most contrary to the *Cartesian* Hypothesis, is the Motion of a retrograde Comet, such as that of the Year 1682, which moving from East to West, was carried directly against the supposed Stream of celestial Matter; and, instead of having its Motion first diminish'd, and then quite stopp'd, and afterwards being carried in a contrary Direction, (which must follow, when a Body moving from East to West falls into a Vortex, whose Matter moves from West to East) it accelerated its Motion in its Descent towards the Sun.

* Ann. 8.

46. Tho' we may very well call the *Medium*, in which Planets move, a *Vacuum*; yet, since Light is propagated thro' all the celestial Spaces, and some fine *Effluvia* may be separated from the Comets and Planets, there will (strictly speaking) be some Resistance to the Motions of the Planet, tho' not so much by many thousand times as our Air would make; and that Resistance after a great Number of Years, must so alter the Motion of the Planets, as to require the Author of Nature's mending Hand ‖. If any Alteration has been found in their Orbits, tho' ever so small, since Astronomers began to make accurate Observations (as a great many assert that there has) that will be sufficient to shew, that the World is not *eternal*, if there were no other Arguments against its Eternity.

‖ This is Sir *Isaac Newton*'s Opinion. See the Queries at the End of his *Opticks*.

The Sun has been observed to have a considerable Atmosphere, as its Surface, on account of the prodigious Heat, must always be throwing out *Effluvia*, those *Effluvia* (except such as are small enough to become Particles of Light, and be darted off with immense

menſe Velocity) floating about round the Sun's Body, muſt make a *Medium*, at leaſt as denſe as our Air. Now, if a Comet comes near enough to go into the Sun's Atmoſphere, it will on the account of the Reſiſtance it meets with, come nearer and nearer to the Sun every Revolution, and at laſt fall into it. Such may in Time be the Fate of the Comet obſerved in 1680, which came ſo near the Sun as to be, at its *Perihelion*, no farther diſtant from the Sun's Surface than the 6th Part of the Sun's Diameter. For what we know, many a Comet may have fallen into the Sun without our Knowledge and Obſervation; and perhaps thoſe Bodies may ſerve as freſh Fewel to repleniſh the Waſte of the Sun in ſupplying the Syſtem with Light. For tho' it has been objected, that a Comet would be but a ſmall Supply, yet if it be as big as the Earth, it will be in Diameter the 100th Part of that of the Sun; that is, in Solidity the Ten hundred thouſandth Part; and that may be as much as the Sun in many Years may loſe in Light: nay, tho' the Comet ſhould be no bigger than the Moon (as moſt Comets are ſuppoſed to be of that Bigneſs) yet it might ſtill be a ſufficient Supply for the Waſte of Light. Lect. V.

47. THO' theſe are but Conjectures, yet it may not be unacceptable to the Reader, to ſhew by a Scheme, how a Comet, when once it comes into the Sun's Atmoſphere, will at laſt fall into it.

Plate 26. *Fig.* 1.

LET A B P π be the Orbit of a Comet, S and F its *Foci*, S the Sun, and D β C the Sun's Atmoſphere. When from the *Aphelion* A the Comet is come towards the *Perihelion* as far as B, the Reſiſtance of the Sun's Atmoſphere hindering ſome of the Acceleration, which the Comet ought to have, the Sun's Attraction will give its Orbit more Curvature at the *Perihelion*, bring it nearer to it ſelf, and make it come to *b* inſtead of π in its going off; ſo that it will then have leſs Curvature, the Sun's Attraction at its going off acting more directly againſt the Direction of the projectile Force. This will make the Ellipſe longer, carry the *Aphelion* to *A*, and make the *Focus* at *f* be farther off from the Sun than when it was at F. The next Revolution, when the Comet comes down to B, it will ſtill come nearer to the Sun in its *Perihelion*, and quit it at *B* in a new Direction, ſo as to go off in an Ellipſe ſtill longer, whoſe farther *Focus* is at φ, and *Aphelion* at *a*; and ſo on, till Pl. 26. F. 1.

till at laſt it comes down to the Sun in the Line *a* S. But if a Comet or Planet moves in the Orbit *a p*, whoſe *Foci* are at the Sun S and at F, and whoſe *Perihelion p* is quite out of the Sun's Atmoſphere, the Motion of the revolving Body will not be ſenſibly diſturbed in many thouſand Years.

OTHER Things relating to Aſtronomy cannot be well underſtood, till we have explained

The Third LAW *of Motion.*

48. *To every Action there is always oppoſed an equal Reaction; or the mutual Actions of two Bodies upon each other, are always equal, and directed to contrary Parts.*

WHATEVER draws or preſſes another, is as much drawn or preſſed by that other. If a Man preſſes a Stone with his Finger, the Finger is alſo preſſed by the Stone. If an Horſe draws a Stone tied to a Rope, the Horſe (if I may ſo ſay) will be equally drawn back towards the Stone: For the ſtretched Rope, by the ſame Endeavour to relax and unbend it ſelf, will draw the Horſe as much towards the Stone, as it does the Stone towards the Horſe, and will obſtruct the Progreſs of the one as much as it advances that of the other. Suppoſe, for Example, that the Horſe is able to overcome an Obſtacle equal to 1000 lb Weight, preſſing againſt it with his Breaſt; when the Horſe draws a Stone of 100 lb Weight, he will then be able to ſurmount an Obſtacle but of 900 lb, the Stone taking away from the Force of the Horſe as much as ſerves to bring it ſelf forward. We muſt therefore take care rightly to underſtand the Term *as much*, and diſtinguiſh it from *as far*. If a Body ſtrikes upon another, and by its Force changes the Motion of the other, that Body alſo (becauſe of the Equality of the mutual Preſſure) will undergo an equal Change, in its own Motion towards the contrary Part. The Changes made by theſe Actions are equal, not in the Velocities, (unleſs in ſuch Caſes as the two Bodies have the ſame Quantity of Matter) but in the Motions or *Momenta* of the Bodies; that is to ſay, if the Bodies are not hindered by any other Impediments. For, becauſe the Motions are equally changed, the Changes of Velocities made towards contrary Parts, are reciprocally proportional to the Bodies. *This Law takes place alſo in Attractions.*

Fig. 4.

Fig. 3.

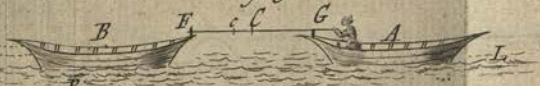


Fig. 1.

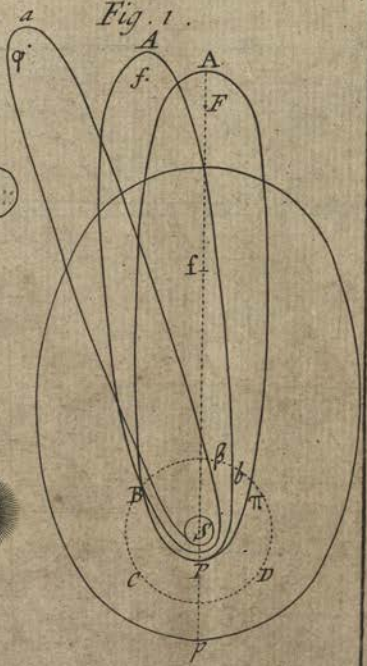


Fig. 5

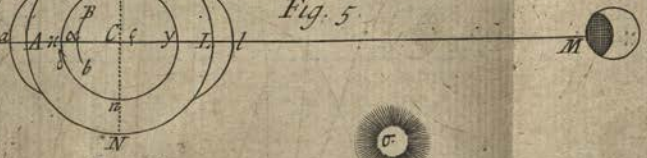


Fig. 6

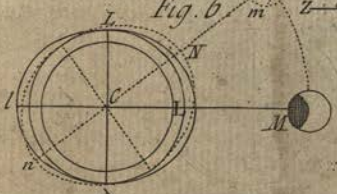


Fig. 9.

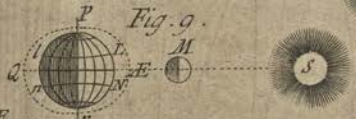


Fig. 7.

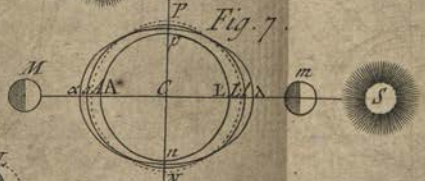


Fig. 8.

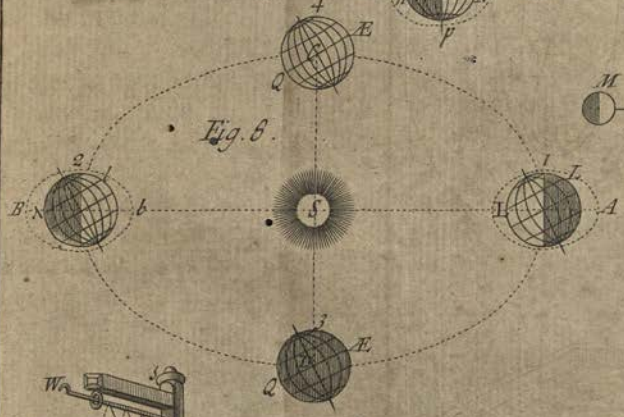


Fig. 2.

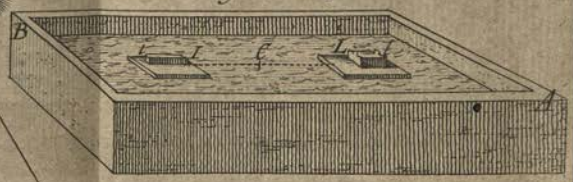


Fig. 13.

Fig. 10.

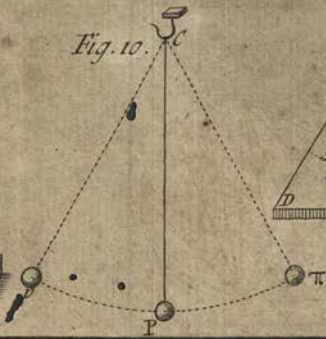


Fig. 12.

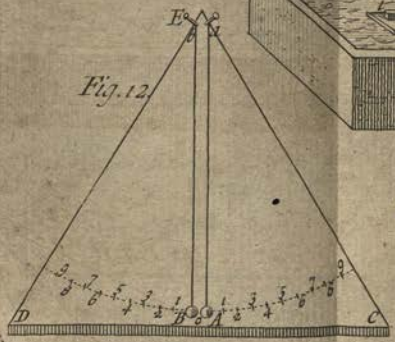


Fig. 11.

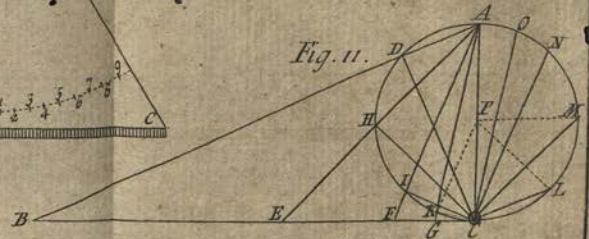


Plate 26. *Fig.* 2. Lect. V.

49. If in a large Vessel of Water, A B, a Loadstone L *l* be set afloat on a Piece of Cork, and a Piece of Iron or Steel I *i* of the same Weight be likewise set afloat on another Piece of Cork, they will come towards one another and meet at C the middle of their Distance; which shews, that the Attraction is mutual between the Loadstone and the Iron.

It is well known, that any long Piece of Steel, which has had each End drawn over the Poles of a Loadstone, will with one End attract the Pole that gave it the Virtue, but repel the other Pole; and so likewise with the other End. If then I *i* be the touched Piece of Steel, whose End I, has received its Virtue from the Pole L of the Stone, and *i* from the Pole *l*; the Steel and Stone will come together in the manner above-mentioned, when I is placed towards L: but if either *i* of the Steel, be placed towards L of the Stone, or *l* of the Stone towards I of the Steel, and they be brought as near to C as the Corks on which they float will allow, then as soon as they are left to themselves, the Stone and Steel will repel one another; which shews, that Action and Reaction are equal and contrary as well in the Repulsions as Attractions §. § Ann. 9.

50. A G and B F * are two Boats of equal bigness and weight, floating on the Water and at rest, at the Distance G F; a Man in one of them at G, pulling a Rope fasten'd at F, by pulling will bring both the Boats together, and they will meet at C their common Center of Gravity, which happens also here (because they are equal) to be the Middle of their Distance §. When the Boats are together at C, if the Man pushes the Boat F B from him, that Boat and his own Boat will recede from each other to equal Distances from C. But if the Boat F B had been as big again (for Example, had been F B H) and the common Center of Gravity of the two Boats had been at *c*, the Distance of the Boats being G F; then by the Pull of the Man, the Boats would have met at *c*, the biggest Boat going thro' but half the Space which the least would go thro': likewise if they were pushed asunder from the Point *c*, the Velocity of their Recess would be reciprocally proportionable to their Mass; that is, A G would recede as far again as F B H, the common Center of Gravity in both Cases being at rest §. It is plain here, that * Pl. 26. F. 3. § L. 2. N°. 31. § L. 2. N°. 31.

Lect. V. that the *Momenta* of both Boats will always be equal, tho' their Velocities are only equal when the Boats are so; and these equal *Momenta*, whereby the Boats are carried towards contrary Parts, shew that Action and Re-action, in all Cases where Bodies act upon one another, are equal and contrary. If the Boat A G had been close to a large Ship, and the Man had pushed his Boat off; he would have given as much Motion to the Ship as to the Boat: I say, the Ship would have mov'd *as much*, but not *as far*; because the Velocity of the Ship being as much less as the Ship is bigger than the Boat, would have been insensible to Sight; and therefore the Vulgar in such a Case imagine, that the Ship does not move at all; and much less, that when a Man pushes against the Shore to shove off his Boat, he moves the whole Earth as must as his Boat is moved; tho' it is certainly true. In firing a Cannon, the Explosion of the Powder pushes the Cannon backward *as much* as it pushes the Ball forward, only the Quantity of the Matter being vastly more in the Cannon than in the Ball, the Recoil is but of a few Feet, whilst the Ball goes perhaps 10000 Feet; in considering the Recoil, we must add the Friction of the Carriage of the Cannon against the Earth, which will still diminish the Velocity of the Cannon's Motion. If the Cannon be fastened to a Ship, we feel only a Shock as we stand in the Ship when the Cannon is fired, because all the Matter of the Ship being added to the recoiling Cannon, the Velocity diminishes in proportion to the Matter added to the Cannon, which makes it insensible to the Sight, and only to be felt by a Shock.

51. Action and Re-action are very plainly seen in rowing, swimming, and flying; as for Example, when the Man K in the *Pl. 26. F. 4. Boat I K (*Fig.* 4. *.) pulls his Oar, he drives the Water towards H, and the Water drives the Boat as much towards D. In swimming, which is nothing but rowing with the Hands and Feet, we are as much pushed forward by the Water as we push the Water back. The same thing explains the flying of Birds, who are pushed forward by the Re-action of the Air against their expanded Wings, when they strike the Air with them. As for Example, if a Bird strikes the Air downwards with his Wings, with a Force equal to what would raise 10 Pound, the Re-action of the Air will push him up with the same Force; but if the Bird weighs one Pound, the Effect of the Re-action of the Air will make the Bird rise with the Force of only 9 Pound; that is, the Bird will rise just as one Pound would

would do tied to a String running over a Pulley, by the Force of the Descent of 10 Pound at the other End of the String. If the Bird should strike the Air only with a Force equal to his own Weight, he would for some be suspended in the Air without Motion, as we often see Kites, Hawks, and other Birds of Prey. Lect. V.

52. The above-mentioned Laws of Motion once understood, the *Phænomena* of the Tides will be easily accounted for; but to make the Matter still easier, let us take the following Consideration or *Lemma* along with us.

If, when three Bodies are moving after one another the same Way, with the same Velocity, there be an additional Force impressed upon each of them, but greater in the First, less in the Second, and yet less in the Third; their Distances from each other will continually increase, tho' they all continue to move the same Way, and all of them faster than they did before.

Let us suppose the three Boats, AG, FB, *and* IK † *(Fig.* 3, *and* 4.*) to be carried along a Stream from* L *towards* D, *floating down with equal Velocity, and that there is but one Man to row the Boat* AG, *two Men to row the Boat* FB, *and four Men to row the Boat* IK. *Now whilst none of the Men row, the Boats, as they are carried down by the Stream, continue to be at equal Distances from one another; so that if a Man, sitting in the middlemost Boat at* B, *can with a long Rod reach the Head of the hindmost Boat at* G, *and the Stern of the foremost Boat at* I, *he shall continue to be able to do it while the Men in the Boats do not row: but if we suppose, that all the Men at once begin to row, the Motion of all the Boats towards* D *will be accelerated, but differently, for the four Men in the foremost Boat will accelerate it faster towards* D *than the two Men in the middlemost Boat, and these two last will make their Boat go on faster than the single Man in the last Boat* AG; *so that the Person, who holds a Rod in the middle Boat* FB, *will no longer be able to reach either the Boat that goes before him, or the Boat that follows him; but will be apt to fancy (when he does not attend to his own Motion) that the Boat before him hastens away from him, and that the Boat behind him goes backward.* This Consideration will help us to explain the Cause of the Tides. † Pl 26. F 3, 4

Lect. V.

53. If the Earth was perfectly smooth, without Mountains or Vallies, the Sea would make a watery Shell about it, which Shell would be concentrick to the Earth, if no Body was near it, to alter the Figure of this Fluid by its attraction. Let $\alpha p \gamma n$* represent the Figure of the Earth in such a Supposition, C its Center, and A P L N the Surface of the Sea, which is concentrick to the Earth, because equally gravitating towards the Center of the Earth in every Part. Now, let us consider, what Effect the Moon at M (C M being a Distance of 60 Semi-diameters of the Earth) must have. Since Action and Re-action are equal, as much as the Moon gravitates towards the Sea at L, so much does the Sea gravitate towards the Moon; but as the Sea does also gravitate towards C the Center of the Earth, with much more Force (as it has 40 times more Matter, and is 60 times nearer) the Moon at the Distance M L attracting it in a contrary Direction, can only take off from the Gravitation towards the Earth so much as amounts to its accelerating Force at that Distance M L. This will make the Water at L swell up to *l*; and at the same time it will be High-Water at A, the Water also swelling up to *a* on the opposite Side of the Earth, whilst the Water falls at P and N to supply the Rise at *l* and *a*. If we consider the Water at L, the Earth at C (reducing all its Weights into its Center of Gravity, as it is not a Fluid to change its Shape) and at the *Antipodes* of L the Water at A, we shall come to the Case of the three Bodies, or three Boats in the preceding *Lemma:* for all these three gravitate towards the Moon at M; but differently according to their Distance, in the following Proportions. The Water at L is distant from the Moon M, 59 Semi-diameters of the Earth; but the Center of the Earth C is distant from the Moon 60 Semi-diameters; therefore as much as 3600, the Square of the Moon's Distance from the Center of the Earth, is a greater Number than 3481 the Square of 59, or the Moon's Distance from the Sea at L; so much is the Attraction of the Moon (that is, the accelerating Force towards the Moon) greater on the Sea at L, than the Earth at C, which makes it come forwarder towards the Moon to *l*; or in other Words, this makes High-water at *l* under the Moon. There is likewise at the same time High-water at the *Antipodes*, or opposite part of the Earth at *a*; because the Water there being less attracted than the Center of the Earth (in the reciproccal Proportion of the Square of the Distances, that is as much as 3600, the Square of the Distance of the Center of the Earth, is a less Number than 3721 the Square of A M=61 the

* Pl. 26. F. 5.

Diſtance of the Sea at the *Antipodes* from the Moon) muſt raiſe at *a* by being left behind, or not advancing towards the Moon ſo faſt as the Center of the Earth. For as the three Bodies L, C, and A, all tend towards the Moon, but L with more Force than C, and C with more Force than A; the Diſtance C L, as well as the Diſtance C A, muſt increaſe from thoſe Inequalities of Force acting the ſame way. Lect. V.

54. I HAVE often heard it objected, that it did not ſeem probable, that the Moon ſhould raiſe the Water at one Part of the Earth, as L, by attracting the Water more than the Earth; and at the ſame time raiſe it at the oppoſite Side of the Earth, as at A, by attracting it leſs than the Earth; but the whole Objection will vaniſh, by explaining the meaning of the Word *to raiſe* the Water, which here is equivocal. In reſpect of the Earth, what is farther removed from the Center of the Earth is ſaid to be *raiſed*, and in that Senſe the Water at *a* is raiſed as well as at *l*; but in reſpect of the Moon at M, if the Water at L is ſaid to be *raiſed*, becauſe it comes to *l* nearer to the Moon, the Water at A going to *a*, farther from the Moon, ſhould rather be ſaid to be *depreſſed*, or left behind, as it is leſs attracted than the Earth. If we conſider the Earth drawn towards M, ſo that the Part $\beta \alpha \delta$ of its Surface, is brought to B *a b*, whilſt the Water remains at A, or comes on towards M more ſlowly than the Surface at B *a b*, there will happen the ſame thing to an Inhabitant at *a*, as if the Water (without any regard to the Moon) had riſen at A from A to *a*. To make this ſtill plainer; let us ſuppoſe an Inhabitant of the Earth at A to ſtand in the Sea near the Shore, ſo as to have the Water up to his Middle, his Feet being towards *a*, and his Head towards Z the *Zenith*; if the Moon be in the *Nadir* at N (that is, under the Earth in reſpect to the Man at A) the Earth on which the Man ſtands will be drawn downwards whilſt the Surface of the Water does not deſcend ſo faſt; the Conſequence of which will be, that the Man will be up to the Neck in Water, whereas he was only up to the Middle before; and if he knows nothing of the Moon's Action in drawing down the Bottom on which he ſtands, he will ſay that it is the Water that riſes.——And this is really what is called the coming in of the Tide when we have the Moon at our *Antipodes*

Lect. V.

55. For the sake of such Readers as may not be so well satisfied with what has been said of the Tides hitherto; I'll give an hydrostatical Solution of it, anticipating only one plain hydrostatical Proposition, namely, that *all Liquors of the same specifick, Weight**, which being contained in different Vessels have a Communication, stand at the same Height; but if in any of the Vessels there be a Liquor specifically lighter than the rest, it will rise so much higher as it is specifically lighter, to restore the Æquilibrium.* Now we must observe, that the Points LN AP being equally distant from the Center of the Earth C, are equally high, and the Waters at those Places, which we suppose to communicate with one another do gravitate towards the Center of the Earth in the Lines L C, N C, A C and P C in the equal Columns L γ, N n, A α, and P p: whilst no other Body but the Earth attracts the Waters. But when the Moon is at M, the Columns of Water at L γ become less heavy towards C than they were, because the Moon attracts them in a contrary Direction (*viz.* from L to M) which will have the same Effect as if the Water there was specifically lighter, as it would be if it lost its Saltness. But the Columns at P and N do not become less heavy towards C; because the Moon drawing at right Angles to P C and N C, does not diminish the Tendency of the Waters at P and N towards C; and therefore they retain their full Gravity, which makes them overweigh the Columns at L, which have lost some of their Gravity towards C: the Waters therefore will fall at P and N, and rise at L, till the Waters from L coming up to *l*, have as much added to them in Quantity as they have lost in specifick Weight; and thereby the *Æquilibrium* will be restored. Likewise the Columns of Water at A, which have less Tendency towards C, than the Waters at P and N, do (as it were) become specifically lighter; and therefore, in order to maintain the *Æquilibrium*, they must receive an additional Quantity of Water from P and N, which raises them up to *a*.

56. I am aware of an Objection here, which seems at first to be of great Force; namely, That since the Waters at L by the Moon's Attraction, have a Tendency towards the Moon in the Direction L M, the Waters at A must also receive a Tendency towards the Moon in the Direction A M by the Moon's Action, tho' that Action is less at A than at L; and consequently that instead of their Gravity or

** One Body is said to be specifically heavier than another, when it contains more Matter under the same Bulk, or as much Matter under a less Bulk.

or Tendency towards C being diminished, it ought to be increased, Lect. V.
be the addition ever so small; and therefore they ought to press more towards C than the Waters at P and N, and so be lower at A, instead of rising up to *a*.

In answer to this we are to consider that as the Earth as C is more attracted towards the Moon than the Water at A, it is the same thing as removing the Center of Gravity of the Earth from C to *c*, and by that means the Water at A will be so much less attracted towards C on account of the greater Distance A *c** instead of AC, that even with the additional Force given by the Moon in the same Direction, it will have less Tendency towards the Center of the Earth than the Water at P and N, and consequently it must be raised up to *a* by the Flow of Water from P and N.

* L. 1. No. 17.

57. There is yet another Objection which seems to destroy the whole Foundation of this reasoning upon the Tides, and to make the Case quite different from the Example of the 3 Boats which we have given to illustrate it. And it is this, *viz.* If the Earth C and the Waters at L and A (as in the Case of the 3 Boats) were all carried towards the Moon, the Earth and Moon must in time come together; but we cannot observe the least Approach towards it, the Distances of the Earth and Moon varying only according to the Nature of the Curve in which the Moon moves, and the joint Actions of the Sun and Earth according to the different Positions of those 3 Bodies, and the different Distance of the Earth from the Sun as it goes round in its elliptical Orbit.

In answer to this, we must observe that it is an undeniable Consequence of the mutual Attraction of all Bodies, and the 3d Law, that the Earth must gravitate towards the Moon as well as the Moon towards the Earth; but we are not to expect to see the Earth go nearer to the Moon, because another Cause destroys that Effect. For the Moon and Earth (as we have before observed *) describing similar Ellipses round their common Center of Gravity, by the Velocity of their Motion acquire such a centrifugal Force as balances (or takes off the Effect of) their centripetal Force towards the common Center of Gravity. Hence only a Tendency is left, and not a visible Effect in the Earth itself. But in respect of the Waters, that Tendency must produce a visible Effect, tho' they do not leave the Earth; because there is a Communication from

* L. 2. No. 32.

Lect. V.

from the Water at P and N to the Water at L and A. And as we have already demonstrated that the Waters at L and A have a less Tendency towards C the Center of the Earth than the Waters at P and N; the Effect of that Tendency will be, that the Waters must rise at A and L, and fall at P and N, to maintain the *Æquilibrium*. But if there was no Communication between P, N, and A, L, there would be no visible Effect; that is, the Water would not rise at L and A, as it really does not in Lakes, and such Seas as have no Communication with the Ocean, unless they have a vast Extent.

58. The next Thing we have to shew relating to the Tides, is the Rise and Fall of the Water successively in any one Place; for Example, if it be high Water at *Portsmouth* at Noon, it will be low Water a little after Six in the Evening; then about Midnight, or six Hours after, high Water again; at Six in the Morning low Water; and, about Noon the next Day, high Water again, *&c.* From what has been said it appears, that the Waters about the Earth do by the Moon's Attraction form themselves into an oblong Spheroid, whose longer Axis produced goes thro' the Center of the Moon; as the Spheroid *l* N *a* P*, whose Axis is *a l*, and which produced goes thro' the Center of the Moon at M. Now let us suppose L to be any Sea-Port on the Ocean, as suppose at *Lucaio* one of the *Bahama Islands* in the Latitude of 27 North, and that C is the Axis as well as Center of the Earth; and that *Lucaio* is at Noon at L, where the Water is under the Moon M. As the Earth turns from West to East, in six Hours *Lucaio* will be carried to P, where the Water is low, (for the Spheroid of Water will be immoveable in respect of the Moon, which we suppose at rest at M, whilst all the Countries of the Earth go round in the Circle L P A N or other Circles parallel to it); in six Hours more *Lucaio* will come to A, where it will have high Water; six Hours after that, it will be at N, where the Water is low; and lastly, it will return to L the next Day at Noon, and so have high Water.

*Pl. 26. F. 5.

The real Fact is, that if it be high Water in any Place this Day at Noon, it will not be high Water next Day till about 53 Minutes, that is, near an Hour after Noon to-morrow. The Cause of which is, that the Moon does not remain at rest in the Heavens as we have just now supposed, but moves in its Orbit 13 Degrees and 10 Minutes, as for Example, thro' the Arc M *m* in 24 Hours

(*Fig.* 6.)

(* *Fig.* 6.) so that when any Place of the Earth, as *Lucaio*, has gone from L to *L*, then to *l*, and to λ, and lastly to the same Place at the same hour at L; the Moon being no longer at M, but at *m*, the high Water has changed with the Moon's Place, and the long Axis of the Spheroid is no longer L*l*, but N*n*; and therefore *Lucaio* must move quite to N where the high Water is towards the Moon, the Spheroid of Water being now represented by the prick'd Oval, whose Axis produced is the Line *n* N *m*; and the Time of that Motion will be about 53 Minutes, at the rate of 15 Degrees in an Hour, which every Country of the Earth moves from West to East. Lect. V. *Pl. 26. F. 6.

59. Besides this alternate Rise and Fall of the Water twice in 24 Hours, it is observed that at the Time of full and new Moon, the Tides (then called Spring-Tides) are greater than at the Time of the Quadratures, that is, when to us the Moon appears halv'd; and as the Water raises higher at the Tide of Flood, so it sinks lower at the Tide of Ebb, when the Moon is Full and when it is New. Those Tides that happen at the Quadratures, are called *Neap-Tides*: and both are accounted for by the joint Action of the Sun and Moon.

If the Action of the Sun on the Waters, and the Re-action of the Waters be considered, as we have done in respect of the Moon, we shall find (for the same Reasons) that the Waters will rise under the Sun and at the *Antipodes*; but as the Attraction of the Sun, on account of its immense Distance, is above five times less than that of the Moon **, all their Effects, *cæteris paribus*, will be above five times less. Now, since both the Moon and Sun act upon the Waters, when their Actions co-incide (as they do at *Full* and *New Moon*) the Water rises one fifth higher, and falls one fifth lower; and when their Actions are contrary to each other (as they are at the Quadratures) the Tides rise and fall one fifth less. Sir *Isaac Newton* has shewn * that the Moon is capable of raising the Water of the Ocean 10 Foot, and the Sun only 2 Foot; therefore when both their Actions join, the Water will rise 12 Foot; and * Princip. L. 3. Prop. 38.

** The visible Surface of Bodies decreases as the Squares of their Distances increase, and the Power of the Attraction of Bodies (that is their *accelerating Force*) decreases in the same Ratio; * therefore if the Sun and Moon were of the same Density, their Attractions on the Earth and Seas would be equal when their apparent Diameters are equal. *But the Sun having about 5 times less Matter than a Moon of the same Bulk would have (that is, being about 5 times less dense than the Moon) has 5 times less *absolute Force*, and therefore must have 5 times less Power to move the Sea. * L. 1 Art. 11.

Lect. V. and where they act against each other, the Water will rise only 8 Foot. But this will be better understood by a Figure.

Pl. 26. F. 7. *Plate* 26. *Fig.* 7.

A C L is the Earth, whose Center is at C; M or *m* the Moon, whose Action reduces the Water to the oblong Spheroid *l n a p*, whose Axis produced goes thro' the Moon. S is the Sun, which, if there had been no Moon, would have raised the Water up to *A*, and on the opposite Side to *L*, so as to reduce it into the Spheroid *A P L N* less oblong than that of the Moon. Now when the Sun and Moon are in the same Line, (as when the Moon is full at M, or new at *m*) the Axis of the two Spheroids co-incide, so that the Water, which would have been raised by the Moon but to *a* and *l*, by the Sun's additional Force will be raised up to *α* and *λ*, 12 Foot instead of 10 Foot, and consequently depressed so much lower at *p* and *n*. But if the Moon being at M or *m*, the Sun was at *s* or *σ*, (that is, at the Quadratures, when the Moon appears halved) the Sun's Action would be capable of raising the Water up to P and N, if it acted alone; but as the Moon at the same time raises the Water at *a* and *l*, all the Effect of the Sun's Force will only be to hinder the Water from rising so high under the Moon and on the opposite Side of the Earth, as the Moon would occasion it to do without that Check: so that the Water, instead of raising up to *a* and *l*, only rises up to *A* and *L*, 8 Foot instead of 10, and the Water is only depressed to P and N, instead of falling as low as *p* and *n*. Thus are the *Neap-Tides* produced by the contrary, as the Spring-Tides are by the joint Actions of the Sun and Moon.

As the Sea-shores are very irregular, the Sea of various Depths, and Rivers run swifter or slower into the Sea with and against Tide, and as the Water runs differently into Bays, Gulphs, and Streights, we must not expect the same Regularity of Tides as we have mentioned, any where but in the free Ocean; but if Circumstances and Facts are rightly examined, we may solve all the *Phænomena* relating to the Tides from the Principles above explained. That learned and indefatigable Astronomer Dr. *Halley*, has given an Instance of it in the first Volume of the *Miscellanea Curiosa*, and *Philosophical Transactions*, N°. 162. where he has enlarged upon Sir *Isaac Newton*'s Theory, and applied it to several Cases, which without it would be altogether unaccountable; shewing by a Solu-

tion of every difficult *Phænomena*, that the Theory will reach all Cases known and faithfully related. I refer those who are curious to know more of this Matter to the 37th Proposition of the 3d Book of Sir *Isaac Newton*'s *Principia*, and Dr. *Halley*'s Dissertation above-mentioned. There they will see why the Water rises to prodigious Heights, as 40, 50, nay above 60 Foot in several Places, as at *Chepstow* above *Bristol*, at St. *Malo*, at *Auranches*, in *Normandy*, *Cambaja* and *Pegu* in the *East Indies*. Why there are no sensible Tides in some Places, as the *Mediterranean*, the *Caspian Sea*, the *Black Sea*, and the Lakes. Why the Tides are so extraordinary at *Tonquin* in *China*, where there is but one Flood and Ebb in 24 Hours, and twice in each Month no Tide at all. Why the Moon must be a little past the South to make high Water in some Places; and why the highest Spring Tides are not precisely on the *New* and *Full Moons*, nor the *Neaps* on the Quarters, but generally the third Tides after them, on account of the Resistance that the Water meets with, which hinders it from following the Moon so fast as it would do if it covered the Surface of a smooth Earth. I shall explain but one Thing more before I leave this Subject, *viz.* the *Phænomena* of the *Æquinoctial* Tides, which are higher near the Vernal and Autumnal *Æquinox* than at any other Time; and least of all at the *Solstices*. Lect. V.

Plate 26. *Fig.* 8.

60. Let 1 C 2 D represent the Orbit of the Earth, and S the Sun in the Middle; 1 and 2 the Earth at the Time of the Summer and Winter *Solstice*; 3 and 4 the Earth at the *Æquinox*. First, let us consider what must happen to any particular Place, as *Lucaio* above mentioned, when it is Summer *Solstice* for that Country, as at 1. If the Moon be in the Line A S, that is, whether it be *New* or *Full*, in the *Syzygy*, as Astronomers call it, A L will be the long Axis of the watery Spheroid, and the Waters will be highest at L and A. Now *Lucaio* being at Noon at L will have high Water, and that in a Place where the Spheroid is highest; but at Midnight *Lucaio* will come to L, where indeed it will have high Water, but not near so high as if it had been at A, the *Antipodes* of L; the Parallel of *Lucaio*, that is the Plane of it (as well as those of all the other Parallels or Circles in which the several Countries in the World revolve about the Earth's Axis) making an Angle with the Axis of the watery Spheroid, the Whirl of the Surface of the Earth is oblique to the Motion of the Pl. 26. F. 8.

Lect. V. Water caused by the Sun and Moon's Action, and consequently the Rise of the Water is abated by the Shore's carrying it another Way. At the Winter *Solstice* when the Earth is at 2, *Lucaio* being at *l* at Noon has indeed high Water; but the Water is not near so high as at *b* the *Antæci* of *Lucaio*: but at Midnight *Lucaio* is carried to λ, where it has a great Tide, the Water being there in the highest Place of the Spheroid.

COROLLARY.

HENCE follows, that at the Summer *Solstice* the Northern Countries have the Flood in the Day-time higher than at Night; but the Reverse at the Winter *Solstice*: and the same happens to the Southern Regions in the opposite Months.

WHEN the Earth is at 3 or 4, where the *Æquinoxes* are, the Water is highest at C and D, and the Sun being then in the Plane of the *Æquator* E Q; so that every Country, that has had high Water in the Day-time, as in the Hemisphere E 4 Q C, will have as high Water in the Night 12 Hours and a half after; as in the Hemisphere E 3 Q D. The Centrifugal Force of every Place helping also to raise the Water, every Parallel of Latitude having its Plane parallel to the Axis of the Spheroid. But this will appear still plainer by the 9th Figure *, where P *p* is the Axis of the Earth, Æ Q the *Æquator*, Æ N *p n* Q P the watery Spheroid raised by the Sun S and *New Moon* M, L *Lucaio* having high Water at Noon; it is evident that when *Lucaio* comes to *l* at Midnight, it will have the Water nearly as high as it had it at Noon. The same may be said of any other Place, as the *Antæci* of *Lucaio* at N, which, when they come at Midnight to *n*, will have nearly as high Water as they had at Noon at N. So likewise will those that inhabit at Æ find the Water in 12 ½ Hours as high at Q.

* Pl. 26. F. 9.

N. B. *As the Sun is nearest the Earth in the Winter, we have the highest Tides in that* Solstice *which makes the Winter of our Northern Regions; and hence also follows that the greatest Spring-Tides are not precisely at the Time of the Equinoxes, but a little before the Vernal, and a little after the Autumnal Æquinox.*

Concerning

Concerning PENDULUMS.

61. A *Pendulum* is a heavy Body of any kind, which being fastened to a String or Wire hangs down from a Point where it is suspended; as the Pendulum P * hangs from the Center C by the String C P. Now if such a Body be removed from its lowest Point P to any other Point as p; as soon as it is loosed, it will not only come down again to P, from whence it came, but will go on till it rises up to π, to an Height equal to that from which it fell, except so much as it is hindered by the Resistance of Air and Friction at C; otherwise it would move forwards and backwards for ever. That Descent and Rise of the Pendulum, as for Example, from p to π, is called an *Oscillation* or *Vibration*. * Pl. 26. F. 10.

The Uses of Pendulums are many; but they are most commonly applied for measuring Time. The whole Doctrine of Pendulums, that is, all that relates to their *Phænomena*, naturally follows from what we have said concerning the Fall of Bodies.

62. I have already shewn, that if there be an inclined Plane, as A B (* *Fig.* 11.) whose Height is A C and Base C B; a Body falling along the inclined Plane from A will be got no farther than D, whilst another Body setting out from the same Point A is fallen down to C the whole Height of the Plane; and that the Point D is found by drawing a Perpendicular from C (the Bottom of the vertical Line which measures the Height of the Plane) to the Plane †. If therefore there be several other inclined Planes from the Point A to the horizontal Line C B, as A E, A F, A G, &c. the Perpendiculars C H, C I, C R, will shew the Points on those Planes to which a Body falling from A will come in the same Time; now since all the Angles in a Semi-Circle are right Angles, upon the Line A C with the Distance A P (the half of A C) may be described round the Point P a Semi-Circle which will pass through the Angular Points D, H, I, K, &c. Let the whole Circle be completed. The intercepted Parts of the Lines (or Planes) A B, A E, A F, A G, namely A D, A H, A I, A K, are now Chords of the Circle. Draw in the other Semi-Circle, the Lines O C, N C, M C, L C, respectively parallel to (therefore equal and equally inclined with) the Chords above-mentioned. Now, from what has been said, it is evident that a Body will fall through all those * Pl. 26. F. 11. † No. 18. P. 329, 330, 331.

Lect. V.

laſt Chords in the ſame Time as well as through the others. Therefore if a Circle be ſet upright, and there be ever ſo many Chords drawn to the lower Part of the Diameter, a Body will fall in the Diameter (the whole length of it) and in any of the Chords in the ſame Time, thoſe Chords that are moſt inclined being ſhorteſt, and ſo *vice verſâ*. Now, ſince ſmall Arcs of Circles differ but little from their Chords, Bodies falling in ſmall Arcs of Circles, inſtead of falling in their Chords, will deſcribe thoſe Arcs in times that are ſenſibly the ſame**. For Example; let a pendulous Body C by a String C P hang at P the Center of the Circle; then let it be raiſed from C to M, or any Point between C and M, (or the ſame on the other Side of C) as ſoon as you let it go, it will fall to C in an Arc of the Circle inſtead of the Chord, and ſenſibly in the ſame Time, whether it fall from L, K, or I; becauſe the Chord L C or I C differs from its Arc L C or I C very little more than the Chord C K differs from its Arc C K. But if the Pendulum was to fall in the Arc N M C or M L C, it would be ſomething longer in getting to the Bottom C, than if it had only fallen from K, becauſe the Arc N M C or M L C does exceed in Length the Chord N C or M C much more than the Arc C K does exceed its Chord C K. But if the Pendulum falling from M, be only hindered in its Motion by the Air, it will riſe nearly up to H, which is as high as M, and then there will ſcarce be any Difference between the Time of the Pendulum's Riſe and Fall.

COROLLARY I.

HENCE follows, that if a Pendulum be made to ſwing in great Arcs of a Circle, and being left to itſelf does continually diminiſh its Vibrations by the Reſiſtance of the Air, the Times of the Vibrations will be a little longer at firſt than at laſt; but if the Body makes but ſmall Arcs at firſt, all its Vibrations without any ſenſible Error may

** I don't mean that a Body will fall in the Arcs in the ſame Time that it falls in the Chords; but that in Arcs not very unequal, a Body will fall ſenſibly in the ſame Time. For a Body will fall faſter in an Arc than its Chord, becauſe it ſets out with a greater Declivity; the Time of the Fall in the Arc being to the Time of the Fall in the Chord nearly in the Proportion of 785 to 1000. The Deſcent along the Chord being perform'd in the ſame Time as the Deſcent along the Diameter; it muſt follow that the whole Vibration, or the Fall and Riſe in two Chords, taking up twice the Time, muſt be perform'd in the ſame Time that a Body would fall 4 Diameters, or 8 times the Length of the Pendulum: But Mathematicians have demonſtrated, that the Time in which a Pendulum vibrates in an Arc of a Circle, is to the Time in which it would fall the 4 Diameters above-mention'd, as the Periphery of a Circle to thoſe 4 Diameters; or the Time of the Fall in an Arc: is to the Time of the Fall in a Chord: : as a Quarter of a Circle: to its Diameter; that is, nearly, as 785: to 1000.

may be reckoned *isochronal*, (that is, performed in the same Time) till it come to press. Lect. V.

COROLLARY II.

HENCE follows also, that whatever be the Metal or Kind of the Body which is made a Pendulum, provided the String be of the same Length, all the Vibrations if small, will be performed in the same Time (since all Bodies tend to fall with the same Velocity †) by counting the Vibrations one may measure Time very exactly. For though such Bodies, as are specifically lighter than others, will sooner come to rest by the Resistance of the Air, and so they will have fewer Vibrations, yet every one of those Vibrations will be performed in the same Time. † L. 1. No. 8.

N. B. *This, and what has been said before, will be made more evident by Experiments.*

EXPERIMENT XVII. *Plate* 26. *Fig.* 12. Pl. 26. F. 12.

63. E C D is a triangular Board of Wood set upright, with an Arc of a Circle drawn round the Center *b* from *o* to 9, and another drawn round the Center *a* to 9 on the other Side, and the Divisions on each side are equal. From *a* and *b* let the two Balls or Pendulums hang so as just to touch one another when they are at lowest, which they will do because the Points *a*, *b*, are just distant from one another the Distance of the Balls Centers. If the Ball A be raised to 7, 8, or 9, and the Ball B to 1, 2, or 3; and you let them go both at the same Time, they will meet exactly at *o*, though they have described unequal Arcs; and so they will, let the Arcs be as different as they can be made by this Instrument, at least as far as Sight can determine, when the biggest Arc is not of above 20 Degrees, as it cannot be in this Instrument.

EXPERIMENT XVIII. *Plate* 26. *Fig.* 13.

64. TAKE 3 Balls P, *P*, and π, one of Lead, one of Ivory, and one of Cork, and, having fastened each of them to Strings of equal Lengths, let them hang from the horizontal Wire W on the Top of the Stand S*s*; then raising them all up to equal Heights, they will, when you let them go at the same Time, perform their Vibra-

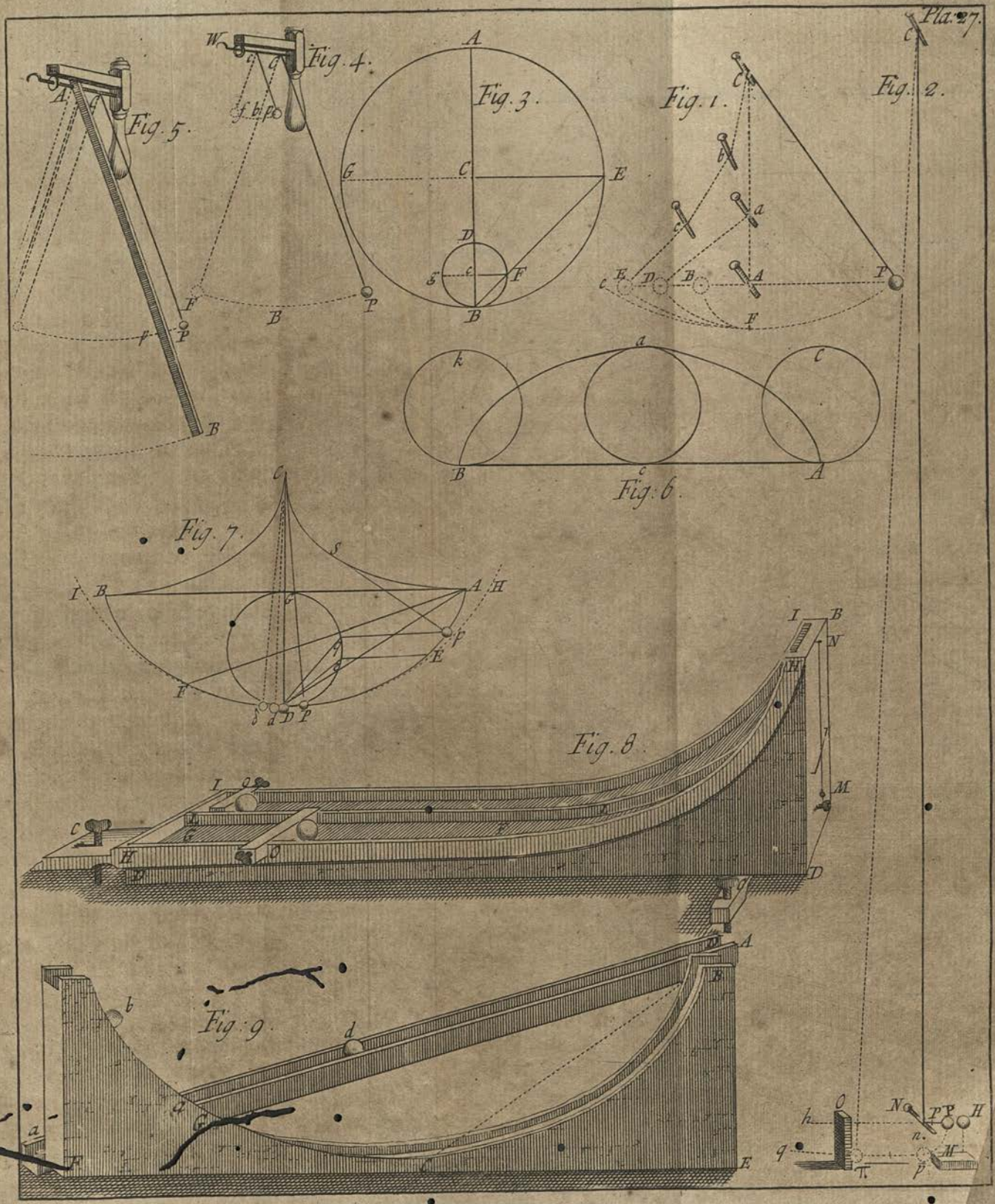
Lect. V. tions in the same Time; for tho' the Cork will come sooner to rest than the Ivory, and the Ivory than the Lead; and while the Lead Pl. 27. F. 1 is still vibrating in the Arc P p, the Ivory vibrates only in the lesser Arc *P p*, and the Cork in an Arc still less, *viz*, $\pi\,\pi$, one may observe that they always come to the lowest Part of the Vibration at the same Time.

Experiment XIX. *Pl. 27. Fig. 1.*

65. About the Pin C for a Center let the Pendulum P swing, having first lifted it from F to P, and it will describe the Arc P F *e* rising up to *e* sensibly as high as P in the horizontal Line P *e*. Then at the Point A, the middle of the said horizontal Line, stick in the Pin A, and the String of the Pendulum will be check'd and stopped by that Pin; but the remaining Part of the String below from A to F will be at liberty, so as to let the Pendulum rise up to B, a Point in the horizontal Line above mentioned in an Arc of a Circle whose *Radius* is A F. If the Pin be fixed at *a*, the Pendulum will rise in the Curve F D; and if two Pins be fixed at *b* and *c*, the Pendulum will rise in the Curve F E, made up of three different Arcs; of which the first has its Center at C, the next at *b*, and the last at *c*. This shews (as has been already explained) That *whatever Curve or Curves a Pendulum rises in after its Descent, it will go up to the same Height from whence it fell, except what must be allowed for the Resistance of the Air*; the Velocity of the Pendulum at the lowest Point F being the same as it would have acquired in falling directly from A: and, that that Velocity is capable of carrying the Body (in such a Direction as Gravity cannot act against it) twice the Space A F, by an uniform Motion, in the same Time that it falls thro' the Space A F by an accelerated Motion, as has † N8. 15. p. 334. been already proved †; but it will be made visible by the following Experiment.

Experiment XX. *Plate 27. Fig. 2.*

66. To a String about 40 Foot long from the Center C hang the Pendulum P, which when at liberty will hang down to *p* and swing in the Arc *p q*. From the lowest Point *p* on the Arc *p q*, measure two Foot or the Arc *p* π, which will be a Line sensibly horizontal, the versed Sine of such an Arc being but about half an Inch. At the Distance of half a Diameter of the Pendulum beyond π fix a


Pla: 27.
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.

vertical Obstacle or wooden Plane O π; and another such on the other Side of p at the Distance of about one Foot and an half, but in an horizontal Position, so that the Surface of it may be even with the Bottom of the Pendulum, namely, in the Line $p\pi$. Let the Pin N be fixed perpendicular to the String (that is, in an horizontal Position) at the Height of one Foot above p. A Mark or strong Point, as n, must also be made in the Wall or Plane behind the String of the Pendulum a little above p. Then having raised the Pendulum up to P in the Line H h, that is, just one Foot high, let it go again, and observe the very Instant of Time that a Mark in the String of P comes to the Point n, that you may just then let fall the Body H from the horizontal Line H h to the Obstacle M; and you will find that the Ball H strikes M at the very Instant that P strikes the Obstacle O π at π, P moving horizontally the Space $p\pi$, which is two Foot, while H falls the Height HM, which is but one Foot. This may be done so exactly, that the Sound of the two Strokes will be but as one. Lect. V.

67. It has been observed, that a Pendulum, whose Length is 39 Inches and 2 Tenths (English Measure*) from the Center of the Ball to the Point of Suspension, will perform one Vibration in one Second of Time; that is, 3600 in an Hour. It has been tried with a Pendulum of 50 lb Weight, made of a lenticular Figure, the better to cut the Air, and a fine Steel Wire was used instead of a String; and with such an Instrument the Vibrations have continued a whole Day. Hence it is that Pendulums will serve to measure Time equally and exactly, Regard being had to the Length of the Pendulum, by the Weight of any kind, or bigness. * Ann. 9.

68. The shorter Pendulums are, the shorter will be the Time of their Vibrations; tho' the Times of Vibration will not be as the Lengths of the Pendulums, but as the square Roots of their Lengths; as for example, a Pendulum of 13 Foot and near an Inch long (that is, 156,8 Inches) which is 4 times as long as a Pendulum that swings Seconds, will perform its Vibrations in 2 Seconds; and if we would have a Pendulums to go twice as fast as the Second-Pendulum, that is, to swing twice in a Second, we must make it 4 times shorter; that is, 9 Inches and 8 Tenths. The reason of it is evident from the following.

Demonstration. *Pl.* 27. *Fig.* 3. Pl. 27. F. 3.

A E B G, and D F B are two Circles whose Diameters A B and D B are to one another as 4 to 1. Now it has been demonstrated*, *L. 5. No. 15.

that

Lect. V. that if a Body fall thro' a certain Space, as A B, in any determinate Time; it will fall thro' only ¼ of that Space, as D B, in half that Time. But it has alſo been proved, that Bodies will fall in the
*L. 5. N° 59. Chord, or a Circle, in the ſame Time that they fall in the Diameter *; therefore a Body at E will fall thro' the Chord E B in twice the Time that a Body at F will fall thro' the Chord F B. Now as the Chord E B differs from the Arc E B (or is ſhorter) in the ſame Proportion as the Chord F B differs from its Arc F B (both being ſuppoſed Arcs of the ſame Number of Degrees) the Difference will be proportionable in the Deſcent of the two Pendulums C E and *c* F; conſequently E will be twice as long coming down to B, as F coming down to B: but the whole Vibration which carries up E to G, and F to *g* will, in both be double the Time of their reſpective Falls. Therefore a Pendulum, to vibrate twice as faſt as another, muſt be 4 times ſhorter. *Q. E. D.*

P. 27. F. 4. EXPERIMENT XXI. *Pl.* 27. *Fig.* 4.

ON the horizontal Wire W of the Stand repreſented in the 13th *Figure* of *Plate* 26, whoſe Top is here expreſſed in *Fig.* 4, hang a Pendulum C P whoſe Length is 39,2 Inches, and upon the ſame Wire a little forward hang the Pendulum *c p* of a quarter of that Length. Let both Pendulums fall at the ſame Time from any Height to which they have been raiſed, as P and *p*, and you will find that *p* will be got to *f* (having performed its whole Vibration) when P is only come to B; when P is come to F, then *p* is come back from *f* to *p*; when P is come back to B, *p* is got to *f*; and at laſt when P is come back to P, *p* will alſo be come to *p* from whence it firſt ſet out: ſo that the two Pendulums will begin to fall together every other Stroke, the ſhort one vibrating in half Seconds, whilſt the long one vibrates in whole Seconds.

P. 27. F. 5. EXPERIMENT XXII. *Plate* 27. *Fig.* 5.

TAKE an Iron Bar, either ſquare or round, provided it be of the ſame Thickneſs in all Parts, whoſe Length muſt be 58,8 Inches, meaſuring from a little Hole to receive the Wire of the Stand to its End; and having thruſt the Wire thro' its hole at A, and ſuſpended a Second Pendulum at C, let both go down from P and B at the ſame Time, and their Vibrations will be *iſochronal*, that is, performed in the ſame Time. This ſhews that any ſtraight even Rod, ſquare or round, of any Metal whatſoever, will perform its

3 Vibra-

Vibrations in the same Time that a Pendulum of $\frac{2}{3}$ of the Length of it vibrates; as if the whole Matter of the Bar had been collected into the Point *p*, which is therefore called the Center of *Oscillation*. Lect. V.

N. B. *If any Person holding a Bar, as* A B, *at its End* A, *should strike a Blow with it, the greatest Stroke possible with the same Strength would be made when the Point* p *strikes against the Obstacle; for this Reason, that Point which is here the* Center of Oscillation, *may also be called the* Center of Percussion (*a*.)

69. A Pendulum is very successfully applied to Clock-Work for measuring equal Time, as may be seen by looking into any Clock. But to shew in what manner it regulates the Motion, one Example will be sufficient, which I give here in describing a little Clock or *Chronometer*, which Mr. *George Graham* contrived and made for me some Years ago, so exact as to measure a small Part of Time very nicely even to the 16th Part of a Second.

To one End of an horizontal Axis is fixed a contrate Wheel of 120 Teeth, and at the other End there is a little Barrel, which on its outside receives the String of a little Weight to turn the Wheel round; the Wheel with its Teeth moves an horizontal Pinion of 15 Leaves (or Teeth) fixed to the Bottom of a vertical Axis on whose Top is the Balance Wheel which is a contrate Wheel of 15 Teeth cut askew: that is, one Side of every Tooth is perpendicular to the Plane of the Wheel, and the other cut obliquely with a Curve. Just over this last Wheel there is a little Axis or Rod of Steel placed horizontally, which has two little Pallets at right Angles to each other, so placed as to be alternately struck by the upright Part of the Skew-Teeth on the opposite Sides of the Balance Wheel, in such Manner that not one of the Teeth of that Wheel can pass by without a Stroke be made against each Pallet to give a Quarter of a Turn to the Axis above-mentioned. At the other End of this Axis is fixed a Wire, at whose End is a Brass Bob, whose Center of Gravity is distant from the horizontal Axis 2,45 Inches, that is, near two Inches and a half, or the 16th Part of the Length of a Pendulum vibrating Seconds. Therefore, as the Weight carries the Wheel-work round, the Skew-Teeth of the Balance Wheel must strike the Pallets fixed to the horizontal Axis, about which the little Pendulum moves; and as that

(a) In some sort of Bodies the Center of *Oscillation* and the Center of *Percussion* is not the same; but the Discussion of that Matter being foreign to my present Purpose, I shall say no more of it here.

Lect. V. Pendulum, on account of its Length vibrates 4 times in a Second *, and there muſt be 2 Strokes againſt the Pallets for every Tooth that paſſes by in the Balance Wheel, ſo there muſt be two Swings of the Pendulum, or half a Second in Time, for every Tooth of the great Wheel that is brought forward by the Pinion, (whoſe Number of Teeth is equal to that of the Balance Wheel;) and as the great Wheel has 120 Teeth, there will be performed 240 Vibrations of the Pendulum whilſt the great Wheel goes round; therefore an Hand or Index fixed to the great Wheel will go round in One Minute, pointing to 60 large Diviſions for Seconds on the Dial-Plate, which Diviſions are again ſubdivided into 4, to ſhew the Quarters of the Seconds. But then beſides this, there is a Quadrant of a Circle of a *Radius* equal to the Length of the Pendulum, which is divided into 4 Parts by 5 little Braſs Pins, fixed to an horizontal Axis; which Quadrant ſerves not only as a Detent to ſet the Machine a going in a 16th Part of a Second, but alſo to ſtop it in the ſame Time, the Pins ſtopping the Pendulum in any fourth Part of its Vibrations; for in fixing the Pins, Regard was had to the Time of every fourth Part of the Vibration, the 2 Spaces between the Pins next to the loweſt Part of the Quarter of the Circle which the Pendulum vibrates in $\frac{1}{4}$ of a Second, being ſo much greater as the Pendulum moves ſwifter in that Part of its Vibration.

* No. 67,68.

THIS Chronometer is of great Uſe for meaſuring ſmall Parts of Time in Aſtronomical Obſervations, the Time of the Fall of Bodies, the Velocity of running Waters, and fit for many other Purpoſes where a ſmall Space of Time is to be meaſured nicely, as from 3 or 4 Seconds to a Minute or two: but it is not adapted to meaſuring long Spaces of Time very exactly; becauſe, though it ſeems to vibrate exactly a Quarter of a Circle every Vibration, yet it does not really do ſo, and the Difference of the Lengths of the Vibrations in ſuch large Arcs makes a Difference in Time; and however ſmall that Difference is, yet a greater many of them create a ſenſible Error. In this Machine, there is ſometimes an Error of $\frac{1}{4}$ of a Second in 14 Seconds. This is a Fault which all Clocks are liable to, that have ſhort Pendulums which ſwing large Arcs of a Circle.

70. BUT there is a Curve, which is neither a Circle nor an Ellipſe, in which all the Vibrations of the ſame Pendulum, whether long or ſhort, are performed in the ſame Time, and when Clock-Work

is

is regulated by a Pendulum moving in that Curve, a long Space of Time may be exactly measured. This Curve is a *Cycloid*, of which here follows the Description. Lect. V.

71. If a Circle in a vertical Position rolls along the horizontal Line A B * (as the Wheel of a Carriage does upon the Road) till it has made one Revolution, that Point of the Circle, as A, which touched the Plane, will rise up from the Line and be at *a* when the Circle has rolled half way, having described the Curve A *a*, from whence (as the Circle goes on) it goes downwards in the same kind of Curve, till it comes to touch the Line A B at B; the Point C, which was uppermost at the Beginning of the Motion, having been down at *c* when A was at *a*, and now being returned to *k* when the Circle is in the same Position as before its Revolution began. The whole Curve A *a* B so described is called a *Cycloid*, and the Circle A C the *generating Circle*, and the line A B the *Base of the Cycloid*. †. * Pl. 27. F. 6. † Ann. 10.

It is evident from the Formation of the *Cycloid*, that the Base of the *Cycloid* is equal to the Circumference of the generating Circle. Several other Properties of the *Cycloid* have been demonstrated by Mathematicians †, whose Demonstrations it would be tedious to repeat here; therefore we shall take them for granted, and mention them as we have Occasion for them in our Conclusions about Pendulums. † Dr. Wallis, Craig. Ditton, Bernouilli, &c.

72. Let us now invert the *Cycloid*, so as to have A B the Base at Top and the *Cycloid* underneath *. It has been demonstrated, that if from any Point in the *Cycloid*, as E or *p*, there be drawn a Line parallel to the Base, and from the Point where that Line cuts the generating Circle when it has performed half its Revolution (that is, when it is at G) there be drawn a Chord such as *e* D, or *q* D, the intercepted Arc of the *Cycloid*, as E D or *p* D, will be double the Chord *e* D or *q* D; and so one half of the *Cycloid*, as A D, will be double the Diameter G D of the generating Circle. Now since it has been shewn that a Body will fall in the same Time in the Diameter G D, and in the Chords *q* D, *e* D, a Body must fall in the Arcs A D, *p* D and E D in equal Times; because each of them is the double of the aboveſaid Lines G D, *q* D and *e* D; but more especially because the Tangents are every where parallel to the correspondent Chords. So that when a Pendulum swings in a *Cycloid*, all the Vibrations, however unequal, are isochronal. * Pl. 27. F. 7.

Lect. V. Pl. 27. F. 7. Now in order to make a Pendulum ſwing in a *Cycloid*, there have been contrived cycloidal Cheeks on each Side of the Center of Suſpenſion, that the String applying itſelf to them ſhall ſhorten the Diſtance of the Pendulum from the Point of Suſpenſion, ſo as to make it deſcribe a Cycloidal Arc inſtead of a Circular one. The cycloidal Cheeks are thus made: Let the half *Cycloidal* A D be transferred to C B, ſo that the Point D may be on B, and the half *Cycloid* B D be transferred to C A, ſo that the Point D may be on A. Now if there be a Couple of Cheeks of Wood or Metal cut convex ſo as exactly to fit thoſe Semi-cycloids, and a Pendulum as P of the Length C D, double the Diameter G D, has its Point of Suſpenſion juſt between the ſaid Cheeks at their Point of Contact C, ſuch a Pendulum will no longer vibrate in the Arc H D I (whoſe Center is C and Radius the Length of the Pendulum) but in the *Cycloid* A *p* D F B, by the ſhortening of the String as it applies itſelf to the cycloidal Cheeks as at C S *p*.

THERE is another Way to make a Pendulum vibrate in Arcs ſo nearly cycloidal, as to have no Error in their Meaſure of Time; and that is, by having the Pendulum very long, and making it only vibrate very ſmall Arcs. For Example, let C D be a Second Pendulum, that is, 39,2 Inches long, and let it only vibrate from P to *p* about 4 or 5 Degrees. It is plain that the Circle and Cycloid co-inſide at D and ſome Degrees on either Side; therefore the Pendulum that ſwings in the circular Arc H D I, deſcribing only that Part of it P *p*, may be conſidered as ſwinging in a cycloidal Arc; and if it ſhould ſometimes go from D only to *d*, it would deſcribe that little Arc in the ſame time as the greater Arc D *p*. N. B. It is better to uſe the long Pendulum than the cycloidal Cheeks; becauſe if the String or Wire, by which the Pendulum hangs, ſtrikes only on the cycloidal Cheeks, the Reaction of thoſe Cheeks, which are in ſome meaſure elaſtick, will add to the Force of Gravity, by which alone the Pendulum muſt ſwing, to have its Vibration iſochronal in a *Cycloid*; ſo that when any ſuch Shock happens, the Vibration will be too quick. But the long Pendulum without a very great Shock (ſuch as would ſend it up to *p*) will not ſo ſenſibly vibrate out of the *Cycloid* as to cauſe any Error in the Meaſure of Time. So that it is now uſed very ſucceſsfully in Clocks for keeping Time at Land; the Shocks at Sea being too great to have a long Pendulum Clock go ſo well in a Ship.

2. 73. BE-

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73. BEFORE we quit this Subject, it may not be improper to mention a remarkable Property of the *Cycloid*, which is this, *viz.* that it is *the Line of the swiftest Descent*; that is, if a Body is to move from A to D*, there is no Line of any kind which can be drawn from the upper to the lower of those Points, along which a Body will descend so fast as in the Semi-cycloid A D; neither an Arc of a Circle, nor even the streight Line A D, though it be so much shorter: for the Body at first setting out from A descends in so steep a Direction (that is, so much acted upon by Gravity) that it acquires a great Velocity, so as to carry it the more swifter in the lower Part of the Curve which is less steep; and a circular Arc which would be more steep at Bottom, is less so at Top. If an inclined Line as A F, be drawn from A beyond the lowest Point of the *Cycloid*, and a Body goes down that Line, whilst another runs from the same Point along the *Cycloid*, the greater Celerity of the Body in the *Cycloid* will still appear more evidently. This Property, and that of having the Descent of a Body from any Part of the (inverted) *Cycloid* to the Bottom, exactly in the same Time, will be illustrated by the following Experiments.

*Pl. 27. F. 7.

EXPERIMENT XXI. *Plate 27. Fig. 8.*

Pl. 27. F. 8.

74. The Machine B H M D D, represented by the Figure, is made of Wood about 10 Inches high, two Foot long, and two Inches thick. From I to F it is cut into a Channel as wide at Bottom as at Top, in the Form of a Semi-cycloid inverted, the lowest Point being at F, from whence the Channel is continued horizontally for the Length of a Foot from F to G, the Walls of the Channel being H H and I I; and this Channel is very smooth and divided into two Channels by a thin upright Brass Partition L L, from the Top between H and I to the farther End at G. Upon the Partition are marked Divisions, which from F to G are equal, but unequal from F to I, in such Manner as to denote equal Heights; H the Beginning of the Channel is 9 Inches above the Level of F, O O are two wooden Stops for the Channel, and to be fixed in any Place by means of a Skrew at the End of Each of them. The whole Instrument may be set upright and horizontal by means of three Skrews such as C, C, and the Plummet N M.

Two Brass Balls, of half Inch Diameter each, are to move in the two Channels.

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Lect. V. Fix the two Stops exactly at F, and the Brass Balls, though you let them go from different Parts of their respective cycloidal Channels, will strike them at the same Time; which will also be very sensible to any one who holds a Finger in each Channel at F, whilst another Person lets the Balls run down from unequal Heights.

Now let the Stops be shifted, and one of them be fixed 4 Inches beyond F towards G, and the other 6 Inches beyond F; and let the two Balls fall at once, the one from the Height of 4 Inches, and the other from the Height of 9. That which falls from 4 Inches in Height, will go 4 Inches beyond F; and that which falls from 9 Inches in Height will go to 6 Inches beyond F, each striking the Obstacle in its own Channel exactly in the same Time as the other. *Four* and *Nine* are the Spaces fallen through, whose Roots 2 and 3 express the respective Velocities of the Balls, which are shewn by one of them running 4 Inches in the horizontal Part of its Channel, the other runs 6 Inches in the other horizontal Channel.

Pl. 27. F. 9. EXPERIMENT XXII. *Plate* 27. *Fig.* 9.

A E C.*b* is another Machine with a Cycloidal Channel, wherein the whole *Cycloid* is hollowed in from B to *b*, its lowest Part being at C. There is a streight Channel A *a* moveable round the Point A as a Center to apply on the Back-side of the *Cycloid* as the Machine stands upright, so that its Channel may be in the Plane of any Chord drawn from the Point A to any Part of the *Cycloid*. Let this Trough or Channel be first set in the Line A C; then let one of the Balls fall from B in the cyloidal Channel, whilst the other begins at the same Time to fall from the same Level in the streight Channel, and you will observe, that the Ball running down in the *Cycloid* will be a good way beyond C, when the Ball running in the streight Channel is just got to it; as may still be made more sensible by fixing one of the Stops above-mentioned (in the Explanation of the last Figure) over-against C in the streight Channel, and the other between C and *b* in the Cycloidal Channel. If the streight Channel be fixed beyond the Point C as at A *a*, the Ball in the *Cycloid* will come from B to *b*, and be down again at C, when the other is come from A in a streight Line to G.

75. WHEN a Pendulum of a proper Length to Swing Seconds is applied to a Clock; there is yet another Error which it is subject to,

to, arising from the Nature of the Materials. For all Metals are subject to be dilated by the Heat and contracted by Cold; so that if the Rod of the Pendulum (that is, the Wire whereby it hangs instead of a String) be adjusted to its right Length in the Winter-Time, it will become too long in Summer by the Heat, and therefore vibrate too slow; as on the contrary, if it had been adjusted in Summer, it would be shortened in Winter by the Cold, and by its quicker Vibrations make the Clock go too fast. The Difference of Length in a long Pendulum Rod will sometimes amount to about the 40th Part of an Inch, which in many Vibrations will make a very considerable Alteration in Time: But there is a Contrivance of a Skrew to raise or sink the Weight of a Pendulum; so that one may keep it always to its due Length, regulating it with a Thermometer. Lect. V.

The Way to measure the least Alteration of Dimensions in Metals so as to make it sensible, I shall shew in the Notes *. * Ann. 11.

76. Besides all this, Pendulums are liable to alter the Times of their Vibrations on account of the Figure of the Earth, which is an oblate or flatted Spheroid like a Bowl, the Earth (as we have already mentioned) being about 31 Miles higher at the Æquator than the Poles. For when we travel Southward, we shall find our long Pendulum Clocks (besides what we allowed for the Heat) to go too slow, as Dr. *Halley* and several other Astronomers have observed, who were obliged to shorten the Pendulums of their Clocks before they could make them keep true Time. Mr. *Richer*, in order to examine this *Phænomenon* fully, made Observations on the Vibrations of a simple Pendulum (that is, a Pendulum not applied to a Clock) for ten Months together, and found that a Pendulum to vibrate Seconds under the Æquator must be about one 10th of an Inch shorter than in our northern Latitude; without which the Clock, which it regulates, will go too slow between two and three Minutes in a Day.

From these Observations on Pendulums Sir *Isaac Newton* shews what the true Figure of the Earth is; though contrary to the Opinion of some Gentlemen of the Royal Academy of Sciences at *Paris*, who assert that the Earth is an oblong Spheroid (like an Egg) higher at the Poles than the Æquator about $\frac{1}{578}$ of the whole Diameter. *I shall, in the Notes, shew how these latter came to be mistaken* *. * Ann. 12.

77. When

77. When we conſider what Velocity the diurnal Rotation of the Earth muſt give to Bodies on its Surface, eſpecially under the Æquator or near it, (where Bodies are carried round at the Rate of about 1040 Miles in an Hour) we may eaſily ſee the Neceſſity of its being higher under the Æquator; becauſe unleſs there was a Deſcent towards the Poles, the Centrifugal Force would drive all the Waters towards the Æquatorial Regions, which would thereby be drowned, whilſt not only the Polar Regions, but even the Temperate would be left dry. And if the Earth was ever in a fluid State, or its Subſtance was ſoft and yielding, the Author of Nature giving it the diurnal Rotation that it has, it muſt from a ſpherical put on a ſpheroidical Figure, ſwelling under the Æquator till it came to have the Figure that it now has. A ſwifter Revolution round the Axis would ſtill make the Difference greater between the Equatorial and Polar Diameters. And this is obſerved in *Jupiter*, which, by ſome nice Obſervations of that late ingenious Aſtronomer the *R.* Mr. *Pound*, appears to have its Æquatorial Diameter $\frac{1}{12}$ longer than its Polar Diameter, agreeable to its ſwifter diurnal Revolution, which is performed in about 10 Hours.

Experiment XXIII.

Upon an Axis of Iron, that could be made to turn ſwiftly (by means of the Wheel of the whirling Tables, whoſe String went round a Pully fixed to the ſaid Axis) I ſlipped on two Iron Hoops, whoſe Planes interſected at right Angles, repreſenting two Colures, which, being of a Spring Temper, ſtood at firſt in ſuch manner as to be $\frac{1}{96}$ part longer in that Diameter that co-incides with the Axis, than in their Æquatorial Diameter; this Proportion being the ſame that Mr. *Caſſini* ſuppoſes to be between the Axis and Æquatorial Diameter of the Earth. Two circular Plates, to which the ſaid Hoops were riveted, had ſquare Holes, through which the Axis paſſed, ſo that the two Poles of the oblong Spheroid, which the Hoops deſcribed in their Revolutions, might approach together in ſuch manner, as to let them put on the Form of a true Sphere, when by the whirling, the Æquatorial Diameter of the Machine ſwelled and over-powered the Elaſticity of the Hoops. A greater Degree of Swiftneſs turned the Sphere into an oblong Spheroid of Sir *Iſaac Newton*'s Figure: A Velocity ſtill greater makes the Diſproportion of the Diameters ſuch as that of *Jupiter*; and ſtill the Æquatorial Diameter increaſes with the centrifugal Force.

Another

ANOTHER Hoop with a Catch, representing the Æquator, shews (in the Experiment) the Increase of the Æquatorial Circumference; and an Index applied to the Frame shews the Increase of the Diameter. Lect. V.

As soon as the Revolution of the Machine ceases, the Colures, Meridians or Hoops, return to their elliptical Figure, whose vertical longest Diameter is the Axis of Revolution.

THOSE that would know more concerning Pendulums, may consult *Huygens*'s Posthumous Works, and Dr. *John Keil*'s *Introductio ad veram Physicam*, at the latter End; where there are several Propositions concerning the circular Pendulum, which I have not room to say any more of in this Place, having already made this Lecture too long; but yet I cannot put an end to it, before I have acquitted myself of my Promise, to give an account of the *Bow* or *Spring*, so far as it is a Mechanical Organ.

78. THIS Engine is so universally known, and the Practice of it so generally understood, and withal so natural and easy, that all Mankind, in all Ages of the World, and in every Quarter of the yet habitable Earth, even among the most rude, ignorant, and barbarous, as well as among the most civil, polite, and learned, have been found expert and skilful in the Use thereof; and even in all that vast Tract of Land, extending so far towards the North, and towards the South Pole, wherein scarce any of those mechanical Engines which depend upon the Balance, or Beam, was known, namely the Leaver, the Capstane, the Pulley, the Skrew or the Wedge, nay, where the Use of the Roller and Wheel was altogether unthought of; yet even through all this vast Tract of Land, and in all the adjacent Islands, the People were found very dexterous and skilful in the Use of the *Bow*, and knew how to convey the Arrow by means thereof so as to hit and pierce the designed Mark; so that it seems to be an Engine most obvious and easy to be contrived by Mankind: And, yet after all, I do not find that any one of the Ancients or Moderns has reduced the Nature, Properties, and Power of Springs to a certain Theory, and Calculation; tho' divers have written much concerning it, and have made many Attempts and Experiments about it; till Dr. *Hook* of late reduced it to a Certainty, by shewing,

Lect. V. *That the Power of a Spring*, of what kind ſoever, *does increaſe in the ſame Proportion with its Tenſion*, whether it be by Compreſſion or Condenſation, Diſtenſion or Rarefaction; ſo that if the Strength of one Pound moves it one Degree out of its natural State, then two Pounds will move it two Degrees, and three Pounds move it three Degrees, and ſo on. Now this mechanical Organ, contrary to all others I have already ſpecified, receives all the Power or Motion which is impreſſed on it, and retains it till it be left at liberty, or no longer any farther extended, or forced with greater Power; but then, inſtead of proceeding and going forwards in that way in which it had been moved, it recoils, and goes quite back again in a contrary Direction; and by *Re-Action* exerts all the Power and Motion it has received upon the Bodies it meets with in its way as it goes back, and carries them with great Swiftneſs; as having received all the Power that had by Tenſion been impreſſed on the *Bow* by the Strength that bent it. By this means all the Strength of the Arm exerted to bend the *Bow* one way, is by the *Bow* ſuddenly communicated the contrary way to the Arrow, which makes it move with that Swiftneſs and Speed after it is diſcharged, continuing the Motion by the firſt Law. Now, here poſſibly may be inquired, how it comes to paſs, that the *Bow*, which in its bending, or drawing, is only moved by a ſlow and heavy Motion (nay, it may be, is kept ſome Time ſtill, and without any Motion at all) ſhould yet retain ſo great a Quantity of ſo ſwift a Motion or a Power of producing it, in another Body, tho' as to Senſe it ſeems to have no Motion at all of its own; whereas all the other Organs, which produce or effect ſuch a powerful or rapid Motion, have that Motion ſenſibly acting in them, when they produce their Effect, proceeding directly forward in the ſtreight Line in which they moved before the Stroke; ſo there is nothing but the Continuation of the whole acquired Motion.

To which I anſwer, that tho' the phyſical Reaſon of the Power of Springs, may not poſſibly appear ſo evident to Senſe as that of a Weight lifted up and then let fall; yet it is much of the ſame Nature, as it may be made plainly evident: for as in the lifting up of a Weight to a certain Height, and thence letting it fall, the Power of Gravity makes that Weight deſcend back again to the Place whence it was lifted, with greater Force as it is lifted higher; and ſo in its Deſcent enables it to exert as much Force or Power

as was spent in the lifting it up to so great a Height: So a Spring, which has a Power like Gravity, or an Activity equivalent to it, and within itself, but with this Difference, that it acts any way as it is directed (whereas Gravity only acts downwards) so as in its Return to exert all that Force and Power, which was spent in bending it to that Tension; and as the Motion of Gravity or Descent, whatever the Velocity of Motion was that raised the Weight, carries it back to the Place whence it was raised, by one uniform Progress of Acceleration, which is peculiar to the Power of Gravity, as we have shewn; so the Recoil of the Spring, with whatever Velocity it has been bent, unbends itself by such Degrees of Power and Velocity, as are proper and peculiar to the Nature of Springs, which, as I mentioned before, are so much the more powerful, by how much the more they are bent: so that to make the Comparison exactly to answer, we must not conceive a single Weight to be lifted up, as in the Example of Gravity, because the Power of Resistance of Gravity, in such a Weight, is the same at the last as at the first, as to Sense; for the same Power that lifted such a Weight one Foot high, at beginning, will lift it one Foot higher after it has lifted it to that Height, or a third Foot after a second: for Instance, if the Force of a Pound Weight will lift it one Foot high, that Force of one Pound continued, will lift it another Foot high, and (continued) a third Foot high. But in a Spring it is otherwise; for if the Force of one Pound be required to bend the Spring one Inch, then there must be the Force of two Pounds to bend it two Inches, and three Pounds to bend it three Inches. So that the Power of a Spring may be more properly represented by the lifting up of a Chain, or a String of Beads, or Weights that lie on the Ground; for in lifting up the first Link, or Bead, there is required one Pound, for instance, but lifting it higher there rise two Links or Beads, and so the Force of two Pounds is required; and if higher, three Links or Beads rise, and so on. Or more exactly yet, 'tis represented by lifting a Cylinder of Timber endwise out of the Water, which is of near the same specifick Gravity with the Water, or forcing Water to rise in a Pipe by a forcing Plug; for in these Cases, as the Cylinder is raised higher above the Surface of the Water, out of which it is raised, so does the Gravity of it increase, and the Power that lifts it must increase in the same Proportion, which is exactly the same as that of the Spring; and so either of these Examples becomes a most proper Instance to illustrate the Power of the Spring, and the

Lect. V.

Lect. V. Manner of its acting: and tho' the Cauſe of the Neceſſity of the Increaſe of the Power to move the one, be more ſenſible and intelligible than the like Neceſſity of Increaſe to bend the other; yet if we further proceed to ſhew the true Reaſon and phyſical Explication of the Power of Gravity, we ſhall find it to be as difficult, and as far removed from our Senſe, as the phyſical Cauſe and Reaſon of the Power and Increaſe thereof in the Spring; it is enough therefore for Geometry to know, that the *Phænomena* are ſo, and thence to examine and demonſtrate the Conſequences that will follow. Now the Power of Gravity in accelerating the Velocity or Motion of falling Bodies, as it was firſt found out, and diſcovered by *Galilæo*, ſo was the perfecting of the Theory thereof, as to the Motion of Deſcent of heavy Bodies in inclined Planes, and the Curvature of a Parabola, very far promoted by him, and many curious Theories concerning it deduced, as we have ſhewn in this Lecture; and the ſame Theory has been farther improved, and carried on by *Torricellius*, and others alſo ſince him. But all theſe proceed upon a Suppoſition, that the Power of Gravity in all the Points, or the Perpendiculars of Deſcent, are mathematically equal and uniform; and indeed, as to all Motions of terreſtrial heavy Bodies, they are found ſo as to Senſe; but as to mathematical Exactneſs and phyſical Theory, they are really otherwiſe: that is, the Perpendiculars of Deſcent are not parallel one to another, but converging and growing nearer together, as they approach nearer to the Center of the Earth; and the Power of Gravity in every Point of thoſe Lines is not equal and uniform, but ſtronger as it is nearer the Earth, and weaker as it is farther off, * according to a certain Proportion, which makes the Curve Line of a projected Body not Part of a Parabolical Line, as *Galilæo*, *Torricellius*, and the reſt have ſuppoſed; but perhaps, Part of an Ellipſe, one of whoſe *Foci* is in the Center of the Earth. Notwithſtanding which, the Theories and Problems deduced geometrically from them upon ſuch *Phænomena* as they had then diſcovered, and upon the Hypotheſis they had thereupon taken, are highly valuable; and the Deductions truly made and demonſtrated, which is the utmoſt Performance that can be expected from Geometry. But now theſe Things have been carried much farther by Sir *Iſaac Newton*; the Power of Gravity is determined and brought to a Certainty, by what Degrees it decreaſes the farther it acts from the central Body; by which means the true Reaſon of the Motion of the Celeſtial Bodies, and the Lines or Orbits in which they

* L. 1. No. 17.

paſs

paſs, and the Velocities with which they move in their Orbits, is perfectly adjuſted, as we have fully explained. Lect. V.

BUT in this, the Example taken from Gravity is (as I ſaid before) defective, for ſolving the Effect of a recoiling Spring, upon a Body moved by it; for in the Spring, the farther it is bent the more powerful it is; but in Gravity it is the contrary.

THE Effect therefore of the Power of the Spring, in this Particular, muſt be ſought out from the Theory of the Nature of Springing, which is, as I but now mentioned, ſtronger and ſtronger, as it is the more bent, that is, in the ſame Proportion with the bending; and conſequently (ſince Action and Re-action are equal) the Velocity of the Body moved by it, will always be in the ſame Proportion with the Degree of bending, whether it be longer or ſhorter, that is, the farther the Spring is bent, the quicker is the Motion of its Recoil; and the leſs it is bent, the ſlower. If it be bent one Degree, the Velocity is as One; if two Degrees, the Velocity is as Two; if Three, as Three; and ſo for any other Degree: And conſequently, the Velocity being always proportioned to the Space it is moved by it, the Time of the Spring's moving, whether a longer, or a ſhorter Space, will always be the ſame, or *iſochrone*, which is the ſame with the Motion of a Pendulum, whoſe Weight is moved in a Cycloid, as we have ſhewn.

THIS would lead me to explain ſeveral Things concerning *Elaſticity*, and the Laws of elaſtick Bodies in their Congreſs; as alſo the Effect of that Property in the Air for the Conveyance of Sounds; but theſe Things I ſhall treat of in the ſecond Volume, this being already ſwelled to too great a Bulk.

ANNO

ANNOTATIONS *upon the* 5*th* LECTURE.

1. [3. Page 295.—*the Body will go thro' the whole Diagonal by the Action of the said two Forces.*]

IF Three, Four, Five, or any Number of Forces act upon a Body at the same Time, the same Rule will serve to shew the Line in which the Body will move, which will be the Result of all the Forces compounded. As for Example, if four Forces, whose Quantities and Directions are represented by the Lengths and Positions of the Lines A B, A C, A D, Pl. 28. F. 1. A E, * act at once upon a Body at A; first let us draw B a and a C respectively equal and parallel to the Lines A B, and A C, and having so compleated the Parallelogram A B a C, let us draw the Diagonal A a, in which the Body would move if only the two Forces A B and A C acted upon it. Now, having shewn, that the Body, by only one Force, whose Quantity and Direction are represented by the Length and Position of A a, would move just as it does by the Action of the two Forces A B, and A C; we may consider them as only one Force A a, which comparing with the Force A D, we shall have the Diagonal A *a* of the compleated Parallelogram A a *a* D, being the Line in which the Body acted upon by the three Forces A B, A C, and A D will move, just in the same manner as if only two Forces, as A a, and A D had acted upon it. Three Forces therefore being thus reduc'd to one, represented by A *a*, we may combine A *a*, with A E the fourth Force, and so have the Diagonal A *α*, of the compleated Parallelogram A *a α* E, the Line in which a Body will move by the joint Action of four Forces.

N. B. *Which soever of the four Forces you begin with, the Line* A *α will always be the Diagonal of the last Parallelogram.*

In the same manner may be found the Result of the Actions of any Number of Forces.

[——*the Space in which the Body moves is carried*, &c.]

When we do not attend to this, we often make very wrong Judgments concerning Motion and Percussion, mistaking an oblique Stroke often for a perpendicular one, and a perpendicular for an oblique one, and drawing false Consequences from *Phænomena* on account of those Mistakes. Pl. 28. F. 2. This will more evidently appear by a Scheme. (*See* Plate 28. Figure 2.) G H B A and A B D C, are two equal Squares or Parallelograms standing upon

Plate 26.

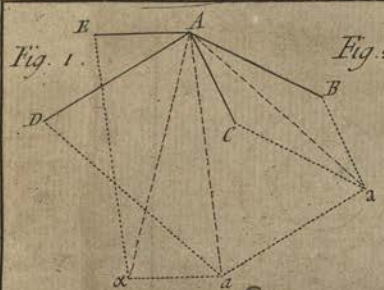
Fig. 1.

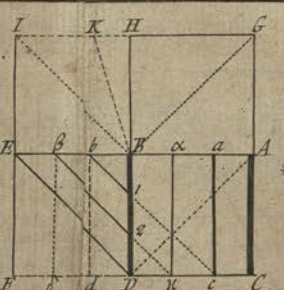
Fig. 2.

Fig. 3.

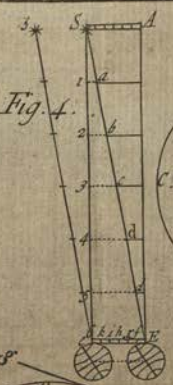
Fig. 4.

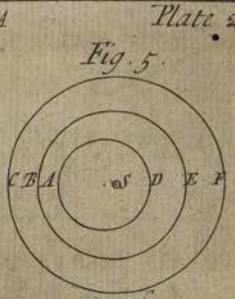
Fig. 5.

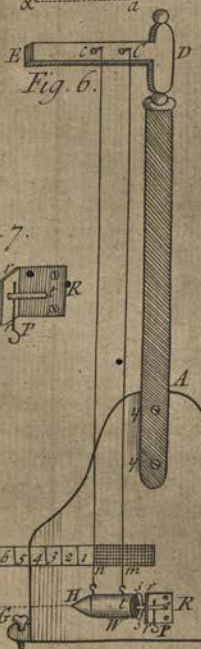
Fig. 6.

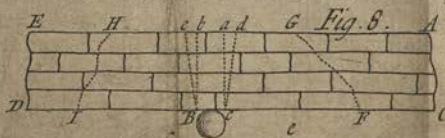
Fig. 8.

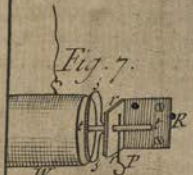
Fig. 7.

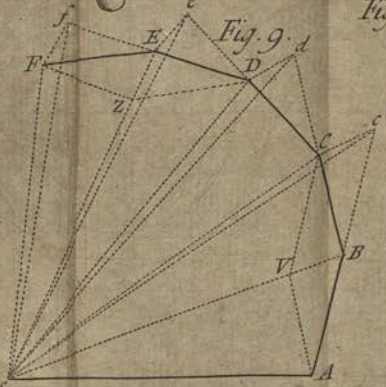
Fig. 9.

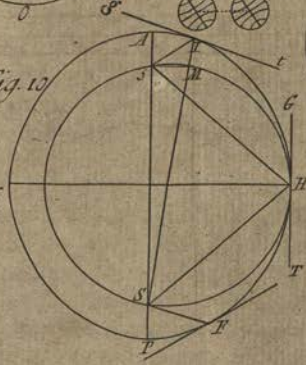
Fig. 10.

Fig. 11.

Fig. 12.

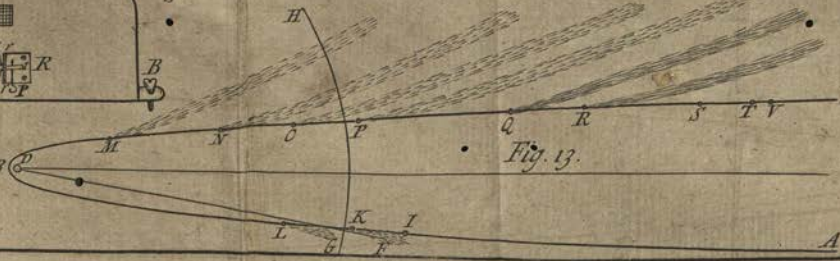
Fig. 13.

Fig. 14.

upon another, and ſeparated from each other only by the Plane or Line A E; and there is a Body moveable upon the Line G A, from G to A. Annotat. Lect. V.

First, let us ſuppoſe the Body to fall ‡ from G to A, in this Situation of the Parallelograms; and the Parallelogram A B D C will receive the Blow of the falling Body at A, the Stroke being made in a Direction perpendicular to A E. Then let the Parallelogram G H B A be ſet before the other, ſo that the Line G A may be directly before A C, and the Line H B before B D (or, to conſider it more mathematically, let thoſe Lines reſpectively co-incide) then the Body falling from G to A, will alſo deſcribe the Line A C, perpendicular to A E. Thirdly, the Parallelograms being ſtill before each other, let the Body fall again along the Line G A, now before A C; but at the very Beginning of the Fall, let the Parallelogram G H A B (which has the Body) be moved horizontally in the Direction A E, ſo as to come into the Situation B E F D, when the falling Body has mov'd its Length G A; whereby it will be got to D inſtead of C, the Parallelogram A B D C being all this while immoveable. By this means the Body, carried in its Motion by its own Parallelogram, will have deſcribed on the Parallelogram A B D C, the Diagonal A D, whilſt it has deſcrib'd on its own Parallelogram only the Line G A, now repreſented by B D. So that a Perſon, ſuppoſed to ſtand in the Parallelogram G H B A when it is before A B D C, and not to be ſenſible of the Motion of the Parallelogram in which he is carried, will ſuppoſe the Motion of the Body to have been *abſolutely* in the Line B D, perpendicular to A E, when it was only *relatively* ſo, but *abſolutely* in the Line A D, oblique to A E, as we have ſhewn in the Lecture †. Fourthly, let us ſuppoſe the Parallelogram G H A B, to ſtand upon the other A B D C, and to be moved horizontally on the Line A E, whilſt the Body is falling from G to A, ſo as to be in the Poſition H I E B, when the Body is come down to A, which is now transferred to B. A Perſon, who does not attend to this lateral Motion of the Parallelogram in which he is placed, will imagine the Stroke at B to be made in the Direction H B, perpendicular at A E; and the more ſo, becauſe the Force of the Stroke will be no greater than if the Body had mov'd in the Perpendicular H B, on account of its Obliquity. For tho' by the Motion of the Parallelogram, which made the Body go in the Diagonal G B, its Velocity (conſequently its *Momentum*) is increaſed in the Ratio of G A to G B; yet the Force of the Stroke, on account of the Obliquity, muſt again be diminiſh'd in the Ratio of G B to G A, or of the *Radius* to the *right Sine* of the Angle of Inclination, as we have ſhewn in the Caſe of oblique Traction*.

Fifthly, Let us ſuppoſe the Parallelogram G H B A upon the other Parallelogram A B D C, as we ſuppoſe at firſt; but the Body to move down the Line H B. In that Caſe the Body will ſtrike the Parallelogram A B C D at B, in the Direction H B, perpendicular to A E; and if the upper

* Pl. 28. F. 2.

† N° 3.

* L. 3. Annot. 5, &7.

‡ Tho' we make uſe of the Word *(to fall)* we muſt ſuppoſe the Body to come down by an equal, not accelerated Motion to render the Explanation the eaſier; tho' it is equally true, when we conſider the Motion as accelerated.

Annotat. Lect. V.

upper Parallelogram be set before the lower one, the Line B D and H B will be described at the same Time by a Motion of the Body perpendicular to A E.

Pl. 28. F. 2.

Sixthly, Let the Parallelogram G H B A be fix'd before the Parallelogram A B D C, which we will suppose to move behind it whilst the Body goes down from H to B, which Points are now in the Position of B and D; so that the Body may be come down its whole Fall to D, when the Parallelogram A B D C is come to the Position B E F D; the Point D being come to F, and the Point C to D, where it meets the Body, instead of the Point D, to which the Body came before. In this Case, a Man at B, carried along in the Parallelogram A B D C, if he was not sensible of his own Motion, would imagine the Body to have really described the Diagonal E D oblique to A E, which was only *relatively* describ'd; for the Body *absolutely* describ'd no other Line, but B D perpendicular to A E. For when the Parallelogram A B D C, is come into the Position *a b d c*, the Body will have descended from B to 1, which is a Point common to the Diagonal *b c*, and the perpendicular Line of Descent B D, and consequently seem to have describ'd *b* 1, when it has only described B 1: then, when the moving Parallelogram is come into the Position $\alpha\beta\delta\varkappa$, the Body will be at the Point 2, common to the Diagonol $\beta\varkappa$, and the Line of Descent B D, seeming to have described β 2, when it has only described B 2: and when the Parallelogram A B D C is come to the Position B E F D, a Person carried in it, and not sensible of his own Motion, will imagine the Body to have describ'd the whole Diagonal E D oblique to A E, when it has really describ'd the Line B D perpendicular to A E.

Lastly, let us suppose the Parallelogram G H B A immoveable upon A B D C, which last is to move horizontally the Length B E, whilst the Body falls from H to B. In this Case, the Stroke will be received by the Parallelogram A B D C, at the Point A, instead of B, the Point A being now come to the Place of B, just as the Body comes to it. The Stroke will also seem oblique as if it had come in the Diagonal I B, and the more so, because its Force will be less than if the Stroke had been made at the Point B, of the moving Parallelogram, in the Ratio of I B, to H B; that is, of the *Radius* to the *right Sine* of the Angle in which the Parallelogram A B D C by its lateral Motion receives the Stroke.

N. B. The Difference betwixt this Case and the 4th is this: That tho' the Obliquity of I B is the same as that of G B, yet the Force of the Stroke is greater in that Case than in this. For in that, the lateral Motion of the Parallelogram, that carries the Body, is a second Force added, the Combination of which Forces do really carry the Body in the Diagonal and increase its *Momentum*, whose Effect is only diminished on account of the Obliquity of the Angle G B A: but in this last Case, the Body describing no more than the Perpendicular H B, has no Increase of *Momentum*, the lateral Motion of the Parallelogram A B D C adding no Force to the Body, but on the contrary lessening the Effect of its Force by receiving

ceiving the Stroke of it obliquely, and that in the Proportion of I B to H B. Annotat. Lect. V.

If the Parallelogram ABDC had mov'd slower: as for Example, if it had only been in the Position *a b d c*, when the Body had run down its whole Length HB, it would have receiv'd the Stroke on its Point *a* now at B, and the Obliquity would have appear'd less, the Direction of the Motion seeming now to be in the Line K B, which also would have taken away less from the Force of the Stroke. But if ABDC had mov'd swifter than we first suppos'd; then the Obliquity of Motion would have seem'd greater in Proportion to the Obliquity of the Stroke, whose Force would have been more diminish'd. Plate 28. Fig. 2.

Many are the Uses of the Truth which we have been explaining, not only in Mechanicks, but all mix'd Mathematicks. I'll give one Instance of it in Opticks, by shewing how the annual Motion of the Earth about the Sun is a certain Consequence of some Observations on the fix'd Stars, made by that curious and accurate Astronomer the Rev. Mr. *James Bradley*, *Savilian* Professor of Astronomy in the University of *Oxford*. Those that understand Astronomy and spherical Trigonometry, may read his own Account of his Observations, and the Consequences he draws from them, in the *Philosophical Transactions*, N° 406. But as the Principles hitherto explain'd in this first Volume of my Course, do not qualify the Reader for understanding Mr. *Bradley*'s Account; I shall just explain so much of it as will shew the Application of what we have said of oblique Motion, and at the same time put that important Discovery of the Motion of the Earth out of doubt.

It is proper, in order to be the better understood, that I should make a Digression here, concerning the Opinions of Philosophers about the Motion or Stability of the Earth, and the Methods which have been used for the settling that Point.

Tho' the first Notion, that the earliest Observers of the Heavenly Motions, after discovering the Roundness of the Earth, must (in all probability) have, is, that the Sky with the Sun, Moon, and Stars, turn round the Earth, as a fix'd Center: yet some Philosophers very early taught, that the Sun was in the Center of the System, and that the Earth and all the Planets revolv'd about the Sun; as *Pythagoras*, *Philolaus* and others. But *Ptolemy* and his Followers maintain'd the different Opinion, which agrees with the common Notion of the Vulgar, who imagine the Earth a vast Body in comparison with the Sun and Planets, and immovable, as their Senses seem to inform them. But as Astronomers came to observe the Motions of the Planets, whose apparent Irregularity could not be accounted for upon the Supposition of the Stability of the Earth, and the Sun was discover'd to be vastly bigger than the Earth; likewise that the Diurnal Motion of the Earth about its Axis, would solve all the *Phænomena* of Day and Night, and all the other Motions ascrib'd to the *primum Mobile*, as the annual Motion solves all the *Phænomena* of the Seasons, and clears up all the Difficulties relating to the Stations and Retrogradations of the Planets; most People

Lect. V. Annotat.

receive the Opinion of the Earth's Motion reviv'd by *Copernicus*, as the most reasonable Supposition; but still without a Demonstration, this could be but a Supposition; tho' the Use of Telescopes, invented in our latter Ages, very much increased the Probability of the *Copernican Hypothesis*, by shewing by the Phases of *Mercury* and *Venus*, that they must indubitably move about the Sun, and not the Earth. Philosophers therefore endeavour'd to make such Observations as might give a Demonstration one way or other; and this was chiefly aim'd at, by endeavouring to find an annual *Parallax* in the fix'd Stars; the Manner of which will be easily understood by the 3d *Figure* of *Plate* 28.

Plate 28. Fig. 3.

E O *e* represents the Orbit of the Earth, whose longest Diameter is E *e*, S a fix'd Star, which an Observer at E sees by the visual Ray S E, which is at right Angles with the Diameter E *e*, and with the Plane of the Ecliptick E O *e*. Let us suppose this Observation to be made in *June*; then half a Year after, when the Earth and Observer are gone from E to *e*, at the Distance E *e* from the Point E, where the visual Ray S E is perpendicular, if the same Star be again observ'd, it will no more be seen by a perpendicular Ray as *s e*, but by the oblique Ray S *e*, the Angle from the Perpendicular being S *e s*, which is equal to E S *e*, the Angle under which the Diameter of the Earth's Orbit would be seen if the Observer was at the Star S; and this is called the parallactick Angle. It is evident, that if the Star was as far again from the Earth, as at Σ instead of S, the Star would be seen by the Ray Σ *e*, and the parallactick Angle would be as little again; and therefore, that the farther the Star is in Proportion to the Diameter E *e*, the less will the parallactick Angle be: So that if the Star be at an immense Distance from the Earth, the Diameter E *e* seen from the Star (that is, in respect to the Distance of the Star) is but a point; therefore the parallactick Angle vanishes. Now the Fact is really so; for when the Earth is come to *e*, the Star being at an immense Distance, is seen by the Ray *s e*; which, tho' it makes an Angle with the Ray S E, yet as to Sense it seems parallel to it. This has made the *Ptolemaicks* triumph in their Opinion, and say—— that the Orbit E O *e* is only imaginary, and that the Earth never stirs from the Place E; because the Star appears always in the same Place, the right Angle which S E makes with the Plane of the Ecliptick (describ'd by the Sun, and not the Earth) never changing. The *Copernicans* have always answer'd, that the Want of Instruments sufficiently exact has been the Reason, that the Angle of the Parallax could not be observ'd, and accordingly have endeavour'd to fix the most accurate Instruments in the best manner for that Purpose. Mr. *Flamstead* fix'd an Arc of a Circle to a Wall, in order to observe a Parallax in the Pole Star, which he thought to be of 40 or 45 Seconds; but that Observation could not be depended upon, because it is not probable, that the Wall, to which he fix'd his Instrument, should remain exactly in the same Position in all its Parts, from one Solstice to the other. Dr. *Hook* pretended to have found a sensible Parallax in the fix'd Stars, observing it in the bright Star in the Head of *Draco*, by a better Method, not depending upon a fix'd Instrument; but with a Telescope

Telescope of 36 Foot, which could easily be rectified for that Purpose; but his Observations, which had been look'd upon as very accurate, were erroneous, as has appear'd by Mr. *Bradley*'s, which we are going to mention. Annotat. Lect. V.

Sir *Isaac Newton* was the first that demonstrated the Motion of the Earth; but he did it from the Laws of Gravity, and not from astronomical Observations. He was very cautious of giving Credit to such Observations as others deduc'd the Parallax of the fix'd Stars from; and when Mr. *John Rowly* had ground an Object-Glass to fix in one of the Towers of St *Paul*'s Church, at the Top of the Newel of the Stair-Case, which would serve as a very long Telescope to observe the Stars near the Zenith, in order to determine their Parallax, he forbad the Glass to be put up, lest Conclusions should be drawn in so material a Point from precarious Observations, looking upon them to be such; because the Building might in settling alter the Situation of this Telescopick Tower. Another way is mention'd by Dr. *Gregory* in the 9th Section of the 3d Book of his Astronomy, which is this: Suppose the two Stars S and ♑* to be in a Line, so that an Observer in the same Line at E, mistakes them for one Star S; it is certain, that if the Observer be remov'd to *e*, or any sensible Distance from E, the Star will appear double, because now the two must subtend the Angle S *e* ♑; and such an Observation does not depend upon the fixing of an Instrument to an immovable Point, or much Apparatus; a Telescope arm'd with a Micrometer, being sufficient for that Purpose. There was Hope of making an Advantage of such an Observation, because Mr. *Cassini* had observ'd several Stars appearing double or triple at one time, and single at another; as the first of *Aries*, that in the Head of the first of the *Gemini*, that in *Orion*'s Sword, and some of the *Pleiades*: but upon further Examination they did not appear single and double at six Month's Distance, and in a manner agreeable to what must be expected from the Parallax; but the *Phænomenon* of the Stars appearing single or double, was owing to some other Cause: And the Distance of the fix'd Stars is so very great, that even the Removal of the Earth from its Place 160 Millions of Miles, is not able to make us discover Stars that are behind one another. * Plate 28. Fig 3.

As a great many learned Men did not understand Sir *Isaac Newton*'s *Principia*, and therefore did not acquiesce in his Proof of the Earth's Motion deduc'd from the Laws of Gravity; and those that did understand his Book, were yet willing to have a collateral Proof from astronomical Observations; several Persons have since attempted to observe a Parallax in the fix'd Stars: and in the Year 1725, the late honourable *Samuel Molyneux*, Esq; with an Instrument, made and contriv'd by that curious Member of the *Royal Society*, Mr. *George Graham*, (so much more exact than Dr. *Hook*'s, that without it we should still remain in great Uncertainty as to the Parallax of the fix'd Stars) began to observe the bright Star in the Head of *Draco* (mark'd γ by *Bayer*) as it pass'd near the Zenith. Mr. *Bradley* also observ'd it along with him; and from many Observations made with great Care, it appear'd, that the Star was more northerly 39 Seconds of a Degree in *September* than in *March*, just the contrary way to what it ought to be from the

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annual Parallax of the fix'd Stars: That is, the Observers, who in *September* saw the Star at S in the Line E S, did in the *March* following, instead of seeing it at S in the Line *e* S, see it at *σ* in the Line *e σ*. This strange Appearance puzzled the Observers, and Mr. *Molyneux* died before the Cause of it was discover'd. Mr. *Bradley*, with another Instrument of the same kind, made also by Mr. *Graham*, observ'd the same Appearances, not only in that but several other Stars; and being by many Trials fully satisfied, that the *Phænomenon* was not owing to any Error in the Instrument, he consider'd what might be the Cause; and found that it was really owing to the Motion of the Earth, and the progressive Motion of Light. So that now the Motion of the Earth is settled by astronomical Observation. This Discovery, of so great Consequence to Astronomy, which has been attempted in vain for many Ages, and at last made in the Year 1728, is yet mention'd nowhere, that I know of, but in the *Philosophical Transactions*, in a Letter to Dr. *Halley*, from the Discoverer, whose Modesty is equal to his great Skill in Astronomy. Had this been found out by some Foreigners, we should have had whole Volumes written about it in a pompous Manner, before this Time. It is surprising, that in the time of five Years, since the Publication of it, none of the learned Gentlemen abroad have taken any notice of it, either to object against, or acquiesce in it.

This shews us, that the Parallax of the fix'd Stars is much smaller, than has been hitherto suppos'd by those who have pretended to deduce it from their Observations; for instead of many Seconds, it does not amount to one. Hence follows, that the above-mention'd Star in *Draco*'s Head, is above 400,000 times farther from us than the Sun; that Light moves as far from the Sun to the Earth in 8 Minutes and 13 Seconds, and that the Motion of the Earth in its annual Orbit, is slower than that of Light 10,210 times; tho' the Earth moves at the rate of about 56,000 Miles in an Hour.

Tho' Mr. *Bradley*'s Letter is very clear to Astronomers, yet for the sake of those, who have but a superficial Knowledge of the Celestial Science, I will (according to my promise) endeavour to shew in the plainest Manner, how the progressive Motion of Light, compar'd with the Motion of the Earth, makes a Star (which, after having been observ'd in the Zenith, is left behind by the Motion of the Earth) to appear, as if it had mov'd forward, contrary to what we might expect.

Plate 28. *Fig.* 4.

Plate 28. Fig. 4.

Let us suppose a Star to be at S, from which Light is propagated every way in right Lines; but we will here consider only two of those Rays or Lines, *viz.* S E and S *e*. E *e* is a Line, in which the Observer, together with the Earth, is carried in the annual Orbit; which Motion (to make our Explanation the easier) we will suppose rectilinear; and also consider the Motion of Light to be only 6 times swifter than the Motion of the Earth;

so that a Particle of Light shall go through the Line A E or S *e*, in the same time that the Earth goes through the Line E *e*. It will then follow, that whilst a Particle of Light goes through any Part of the Line S *e*, as S 1, 1 2, 2 3, *&c.* the Earth will go through a correspondent Part of the Line E *e*, as E *f*, *f g*, *g h*, *&c.* Now if the Earth was at Rest at E or *e*, an Observer from it would see the Star in the Line E S, or S *e*, by means of the Particle of Light which should come from S to E, or from S to *e*; but he would never see the Star in the Line *e s*, as if it was at *s*. But if we suppose the Earth setting out from E to move in the Line E *e*, at the same time that two Particles of Light set out from the Star S, the one to move in the Line S E, and the other in the Line S *e*;* it will follow, that the Earth will have left the Point E, before the Particle of Light coming in the Line S E can be arriv'd at the Observer's Eye, which will be at *e*, when the above-mention'd Particle of Light is at E; but then the other Particle which came in the Line S *e*, will come to the Observer's Eye, just as the Earth comes to *e*, and strike it in the Direction *s e*, as if the said Particle had come from *s*; because, tho' it really comes in the Line S *e*, perpendicular to E *e*, the Motion of the Observer in the Line E *e*, makes *him* receive a perpendicular Stroke, as if it really was oblique; as we have shewn in the last Case of *Fig* 2. (*Page* 400.) For if we suppose the Parallelogram A S *e* E with the Earth at the Angle E, to move laterally in the Direction E *e*, whilst the Particle of Light coming from the Star moves in the Line S *e*, with such a Velocity as to go through S 1, whilst the Earth goes through E *f*; 1 2, whilst the Earth goes through *f g*; 2 3, whilst the Earth goes through *g h*; 3 4, whilst the Earth goes through *h i*; 4 5, whilst the Earth goes through *i k*; and lastly, 5 6, whilst the Earth goes through *k e*, or what is the same, to have 6 times the Velocity of the Earth; then the Diagonal S E being carried laterally, when the Particle of Light is come to 1, and the Earth (carrying the Parallelogram A S *e* E) to *f*, the Point *a* in the Diagonal, will co-incide with the Point 1, in the Ray S *e*, or Line of Motion of the Particle of Light, which by that means will seem to have mov'd in the Line S *a*; when the Earth is at *g*, *b* in the Diagonal will co-incide with 2 in the Perpendicular, and the Particle seem to have describ'd *a b*; when the Earth is at *h*, *c* will co-incide with 3; when the Earth is at *i*, d will co-incide with (or be brought to) 4; when the Earth is come to *k*, *d* will co-incide with 5; and lastly, when the Earth is at *e*, E co-incides with 6, and the Particle of Light seems to have described the Ray or Diagonal S E, which must be now transfer'd to its Parallel *s e*, because the Earth is now remov'd to *e*. So that the Star which becomes visible to the Observer by the Line or Ray S *e*, does (on account of his Motion in the Line E *e*) appear to him to come in the Line *s e*; whereas if the Observer's Place had been immoveable, he must have seen the Star at S, and not at *s*. If then the Ob-

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Plate 28. Fig. 4.

Plate 28. Fig. 2.

* Because of the immense Distance of the Star, we must consider the Line S E, as equal to S *e*.

server,

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ſerver, as the Earth carries him along, looks through a Teleſcope, whoſe Axis makes the Angle S E *e* with the Line of the Obſerver's Motion E *e*, when he comes to *e*, he will ſee the Star at *s*, which he could not do, if his Teleſcope was directed in the Line E A or *e* S perpendicular to E *e*.

Plate 28. Fig. 4.

N. B. *Since the Line* S *e is really* 10210 *times longer than the Line* E *e, becauſe Light moves ſo many times faſter than the Earth, how ſmall muſt the Angle* A E S (=S *e s*) *be? And how nice the Inſtrument to obſerve ſuch an Angle? Yet we find by repeated Trials, that the Inſtrument does perform it; which ſhews Mr.* Graham's *great Accuracy, which will be remember'd as long as Mr.* Bradley's *great Sagacity in drawing the Conſequence.*

2. [7. Page 317. *Thoſe Parts of the Vortex, which are neareſt to the Sun, will move faſteſt*, &c.———*The Squares of the Periodical Times*, &c. *are as the Cubes of their Diſtances*, &c.] Sir *Iſaac Newton* has in the 9th Section of the ſecond Book of his *Principia*, conſider'd all poſſible Caſes of *Vortices*, of which I have only taken that, which is mentioned in the 11th Corollary of Prop. 52. as the moſt obvious. I refer the Curious to that Section, which would be too long to inſert here at full length; but the latter part of it (*viz.* the 53d Propoſition, which is applied to the *Carteſian* Hypotheſis to ſhew its Inconſiſtency, making it appear, that it is impoſſible for the Planets to move in *Vortices*) I thought proper to tranſcribe here; as well as part of his general *Scholium*, at the End of his Book, which is to the ſame Purpoſe.

PROPOSITION 53*d*, *of Sir* Iſaac Newton's Principia, BOOK 2*d*.

Bodies carried about in a Vortex, *and returning in the ſame Orb, are of the ſame Denſity with the* Vortex, *and are moved according to the ſame Law with the Parts of the* Vortex, *as to Velocity and Direction of Motion.*

For if any ſmall Part of the *Vortex*, whoſe Particles, or phyſical Points, preſerve a given Situation among each other, be ſuppos'd to be congeal'd: this Particle will move according to the ſame Law as before, ſince no Change is made either in its Denſity, *vis inſita*, or Figure. And again, if a congealed or ſolid Part of the *Vortex* be of the ſame Denſity with the reſt of the *Vortex*, and be reſolv'd into a Fluid, this will move according to the ſame Law as before: except in ſo far as its Particles now become fluid may be moved among themſelves. Neglect therefore the Motion of the Particles among themſelves, as not at all concerning the progreſſive Motion of the whole, and the Motion of the whole will be the ſame as before. But this Motion will be the ſame with the Motion of other Parts of the *Vortex* at equal

equal Distances from the Center; because the Solid, now resolv'd into a Fluid, is become perfectly like to the other Parts of the *Vortex*. Therefore a Solid, if it be of the same Density with the Matter of the *Vortex*, will move with the same Motion as the Parts thereof, being relatively at Rest in the Matter that surrounds it. If it be more dense, it will endeavour more than before to recede from the Center; and therefore overcoming that Force of the *Vortex*, by which being, as it were, kept *in Æquilibrio*, it was retain'd in its Orbit, it will recede from the Center, and in its Revolution describe a Spiral, returning no longer in the same Orbit. And by the same Argument, if it be more rare, it will approach to the Center. Therefore it can never continually go round into the same Orbit unless it be of the same Density with the Fluid. But we have shewn in that Case, that it would revolve, according to the same Law, with those Parts of the Fluid, that are at the same or equal Distances from the Center of the *Vortex*. Annotat. Lect. V.

Cor. 1. Therefore, a Solid revolving in a *Vortex*, and continually going round in the same Orbit, is relatively quiescent in the Fluid, that carries it.

Cor. 2. And, if the *Vortex* be of an uniform Density, the same Body may revolve at any Distance from the Center of the *Vortex*.

SCHOLIUM.

Hence it is manifest, that the Planets are not carried round in corporeal *Vortices*. For, according to the *Copernican* Hypothesis, the Planets going round the Sun, revolve in Ellipses, having the Sun in their common *Focus*; and by *Radii* drawn to the Sun, describe Areas proportional to the Times. But now the Parts of a *Vortex* can never revolve with such a Motion. Let AD, BE, CF, (* *Pl.* 28. *Fig.* 5.) represent three Orbits described about the Sun S, of which, let the utmost *Circle* CF be concentric to the Sun; and let the *Aphelia* of the two innermost be A, B; and their *Perihelia* be D, E; therefore a Body revolving in the Orb CF, describing by a *Radius* drawing to the Sun, Areas proportional to the Times, will move with an uniform Motion. And, according to the Laws of Astronomy, the Body revolving in the Orb BE, will move slower in its *Aphelion* B, and swifter in its *Perihelion* E; whereas, according to the Laws of Mechanicks, the Matter of the *Vortex* ought to move more swiftly in the narrow Space between A and C, than in the wide Space between D and F, that is, more swiftly in the *Aphelion*, than in the *Perihelion*. Now these two Conclusions contradict each other. So at the beginning of the Sign of *Virgo*, where the *Aphelion* of *Mars* is at present, the Distance between the Orbits of *Mars* and *Venus*, is to the Distance between the same Orbits at the beginning of the Sign of *Pisces*, as about 3 to 2; and therefore the Matter of the *Vortex* between * Plate 28. Fig. 5.

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those Orbits ought to be swifter at the beginning of *Pisces*, than at the beginning of *Virgo*, in the *Ratio* of 3 to 2. For the narrower the Space is, through which the same Quantity of Matter passes in the same Time of one Revolution, the greater will be the Velocity with which it passes through it. Therefore, if the Earth being relatively at Rest in this celestial Matter, should be carried round by it, and revolve together with it about the Sun, the Velocity of the Earth, at the Beginning of *Pisces*, would be to its Velocity at the Beginning of *Virgo* in a *sesquialteral Ratio*. Therefore the Sun's apparent diurnal Motion at the Beginning of *Virgo*, ought to be above 70 Minutes; and at the Beginning of *Pisces* less than 48 Minutes. Whereas, on the contrary, that apparent Motion of the Sun is really greater at the Beginning of *Pisces*, than at the Beginning of *Virgo*, as Experience testifies: and therefore the Earth is swifter at the Beginning of *Virgo* than at the Beginning of *Pisces*. So that the Hypothesis of *Vortices*, is utterly irreconcileable with Astronomical *Phænomena*, and rather serves to perplex than explain the heavenly Motions.

GENERAL SCHOLIUM.

The Hypothesis of *Vortices* is press'd with many Difficulties, that every Planet by a *Radius* drawn to the Sun, may describe Areas proportional to the times of Description, the periodick Times of the several Parts of the *Vortices* should observe the duplicate Proportion of their Distances from the Sun. But that the periodick Times of the Planets may obtain the sesquiplicate Proportion of their Distances from the Sun, the Periodick Times of the Parts of the *Vortex* ought to be in the sesquiplicate Proportion of their Distances. That the smaller *Vortices* may maintain their lesser Revolutions about *Saturn*, *Jupiter*, and other Planets, and swim quietly and undisturb'd in the greater *Vortex* of the Sun, the periodick Times of the Parts of the Sun's *Vortex* should be equal. But the Rotation of the Sun and Planets about their Axes, which ought to correspond with the Motions of their *Vortices*, recede far from all these Proportions. The Motions of the Comets are exceeding regular, and govern'd by the same Laws with the Motions of the Planets, and can by no means be accounted for by the Hypothesis of *Vortices*. For Comets are carried with very excentric Motions through all Parts of the Heavens indifferently, with a Freedom that is incompatible with the Notion of a *Vortex*.

3. [9. Page 323. *There are many more Experiments relating to central Forces*, &c.——*For the sake of the Curious we shall mention a few more in the Notes.*] To shew experimentally how a Planet accelerates its Motion as it approaches to the Sun, and retards, as it recedes from the Sun; fix the whirling Table (in the Condition represented by the 11th *Figure* of *Plate* 24. *) to its Frame, that it may be easily turn'd round in its horizontal Situation. Then take off the *Square*, or rectangular Piece S *s*, and fasten

* Plate 24. Fig. 11.

faſten it to the Table at the Place M *m*, by means of a Screw in the under Edge of it, which goes through the Table, and is fix'd by a Nut under the Table. Having ſo done, inſtead of the two Balls T, M, take one of them as M, and having plac'd it within an Inch of the ſhort Part of the *Square* at M, paſs its String through one of the Side-holes of the Center-piece C, and ſo up through the middle Hole: then having tied a little Wire *w* croſs the String, ſo that when the centrifugal Force acts upon M (while the Table turns round) the Wire may hinder the Ball ſtriking againſt the End of M *m*; let the Table be whirl'd about in the Direction of the Dart, whilſt the Hand is holding the other End of the String looſely at *x*, and the Ball will preſs on the *Square* V *v* at M. Pull the String a little, to make the Ball approach nearer to the Center, and you will ſee and hear it ſtrike on the other Side at *m*, which ſhews its Motion to be accelerated; but if then (the Table ſtill going round with the ſame Velocity) you ſlack your Hand, the Ball in receding from the Center will ſtrike againſt M, which ſhews it to be retarded, becauſe it moves ſlower than the Parts of the Table under it, as it mov'd faſter than the Table in the other Caſe. Annotat. Lect. V. Plate 24. Fig. 11.

The Concluſion from the 15th Experiment (*viz.* that in the ſame Orbit the centrifugal Force is as the Square of the Velocity) would follow from any other Variation of the Periodical Times; as for Example, if the String of the Pulley H* be put on the Groove of 3 Inches, and that of the Pulley K on the Groove of 2 Inches, each Planet being of equal Weight, and at the ſame Diſtance from the Center; the ſwifteſt Planet P will (in its Tower) raiſe a Weight of $2\frac{1}{4}$ Pounds, whilſt the ſloweſt *p* raiſes only 1 Pound; which Weights are to one another as 9 and 4; that is, as the Squares of the periodical Times 2 and 3 reciprocally, or the Squares of the Velocities directly. * Plate 25. Fig. 1.

In a Word, let the *Central Forces* differ in any Manner whatſoever, one may compare them together by the Help of what has been already explained; for *they are always in a* Ratio *compounded of the* Ratio *of the Quantity of Matter in the revolving Bodies, of the* Ratio *of the Diſtances from the Center, and laſtly of the inverſe* Ratio *of the Squares of the periodical Times.* And the Compariſon is thus made: Multiply the Quantity of Matter in each Body by its Diſtance from the Center, and divide the Product by the Square of the periodical Time, and the Quotients of the Diviſions will be in the ſaid compound *Ratio*, that is, as the central Forces.

Experiment. *Plate* 25. *Fig.* 1.

Let the Wheel-ſtring be on the 2 Inch Groove of the Pulley K, and the 3 Inch Groove of the Pulley H, which will make the periodical Times of the Planets P and *p* be to one another as 2 to 3; by the Addition of 4 Ounces to the Weight-bearing Piece in the Tower S, the whole Weight will be of 6 Ounces, and let the Planet P be of two Ounces, and at 8 Inches Plate 25. Fig. 1.

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Plate 25. Fig. 1.

from the Center. The Planet *p* must be of 4 Ounces and 12 Inches from the Center, and the Weight that draws it in the Tower *s*, must be of 8 Ounces. Turn the Wheel, and both Weights will raise at the same Moment.

Multiply 2 (the Weight of the Planet P) by 8 in Distance, and the Product is 16, which being divided by 4 (the Square of its periodical Time) gives 4 for the Quotient, (for $2 \times 8 = 16$, and $\frac{16}{4} = 4$. Then multiply 4 (the Weight of the Planet *p*) by its Distance 12, and the Product 48 being divided by 9 (the Square of the periodical Time of *p*) you will have $5\frac{1}{3}$ (for $4 \times 12 = 48$, and $\frac{48}{9} = 5\frac{1}{3}$.) Now these Numbers 4 and $5\frac{1}{3}$ are to one another as 12 and 16, or as 6 and 8, which are the Weights in the Towers that express the Quantities of the Central Forces.

Whenever the Quantities of Matter are equal, you need only divide the Distances by the Squares of the periodical Times to determine the Proportion of the Central Forces to one another.

And in that Case, *if the Squares of the periodical Times are to one another, as the Cubes of the Distances, the Quotients of the Divisions will be in an inverse Ratio of the Squares of the Distances; and you will find the central Forces to be in the same Ratio:* which is the Case of the Planets and Comets in the Heavens.

This may be very well illustrated by the following

EXPERIMENT. *Plate* 25. *Fig.* 1.

Let the Wheel-string go over the 3 Inch Groove of the Pulley K, and the 6 Inch Groove of the Pulley H, to make the periodick Time of the Planet P be 3, while that of *p* is 6. Let P and *p* be of four Ounces each, P being 5 Inches from the Center, and *p* at the Distance of 8 Inches from the Center. Then if P is fasten'd to 10 Ounces in its Tower S, and *p* to 4 Ounces in its Tower *s*, both Weights will rise at once when the Wheel is turn'd about.

The central Forces are here as 4 to 10, and the inverse *Ratio* of the Squares of the Distances is the Ratio of 25 to 64 (which last Number should strictly be $62\frac{1}{2}$, and may be so if the Distance from the Center be made the square Root of $62\frac{1}{2}$, but we choose 8 Inches to avoid Fractions) near enough for an Experiment. The Squares of the periodical Times are 9 and 36, which also agree near enough with the Cubes of the Distances, *viz.* 125 and 512; for 512 would become only 500, the exact *Ratio* wanted, or much nearer, if the Distance from the Center had been the square Rout of $62\frac{1}{2}$, as we said before.

4. [16. Page 338. *Others*, &c. *alledge, that the* Momentum *is not as the Product of the Mass by the Velocity of the moving Body, but as the Mass multiplied into the Square of the Velocity.*

[24. Page

[24. Page 346. *The Weight*, &c. *multiply it by the Velocity, it will give us the* Momentum, &c. Annotat. Lect. V.

Till within these few Years, all Mathematicians and Philosophers agreed in the Opinion, that the *Momentum*, or moving Forces of Bodies, is made up of the Mass, or Quantity of Matter, multiplied into the Velocity, as we have explain'd in the 2d Lecture *. Monſ. *Leibnitz* (if I am rightly inform'd) was the first that rejected the old Opinion, affirming, that the Force of Bodies in Motion, was made up of the Mass multiplied by the Square of the Velocity, applying it to the Fall of Bodies, and saying, that the Stroke made by falling Bodies was always proportionable to the Heights from which they fell, which Heights are † as the Squares of the Velocities. But his Error lay, in not considering the Time (as we have shewn;) for the Velocities alone are not the Cause of the Spaces describ'd, but the Times and the Velocities together: for otherwise the Effect would be greater than the Cause, which is absurd. Several ingenious Men have endeavour'd to defend Monſ. *Leibnitz*'s Opinion by subtle, tho' fallacious, Arguments, having first deceiv'd themselves by misapplied Observations and Experiments.

* L. 2. No. 1, 2, 3, &c. Page 45.

† L. 5. No. 16. Cor. 1.

Some have distinguish'd the Actions exerted on Bodies (as for Example, the Force of Gravity) into a *living* and a *dead* Force (*vis viva*, & *vis mortua*) calling that a *living Force*, whereby it produces a visible Effect on a Body, and a *dead Force*, that which is destroy'd by a contrary Cause; as when a Body is kept from falling by an Obstacle, or being in the Scale of a Balance is kept from descending by a Counterpoise in the opposite Scale. Were we to allow of this Distinction, yet the common Experiment of the most simple mechanical Organ, I mean the Balance, shews that the *vis viva* and the *vis mortua*, are both in the simple Proportion of the Velocity multiplied into the Mass. For Example, 4 Pounds being placed at the Distance of 6 Inches from the Center of Motion, and 2 Pounds at the Distance of 12, will have a *vis viva*, if the Balance be put into a swinging Motion. Now these Forces appear to be equal, because, with contrary Directions, they soon destroy one annother; but they are to one another in the simple *Ratio* of the Velocity multiplied in the Mass, *viz.* $4 \times 6 = 24$, and $2 \times 12 = 24$. Whereas if the Forces in this Case had been as the Mass into the Square of the Velocity, the Weight 2 placed at near 8 Inches and an half, would have destroy'd the Motion of the Weight 4 at 6 Inches distance, and have reduc'd it to an *Æquilibrium*. That is, $6 \times 6 \times 4 = 144$, and 8,426, &c. $\times$ 8,426, &c. $\times 2 = 144$. The Case is the same in the *vis mortua*; for then 2 at the Distance 12, keeps 4 *in Æquilibrio* at the Distance 6, and the least Alteration either of Weights or Distances, will destroy the *Æquilibrium*. The same Thing is true in the *Leaver*, *Pulley*, *Axis in Peritrochio*, *Inclin'd Plane*, *Wedge*, and *Screw*.

The *Momentum* of Bodies appearing so plainly by this common Experiment to be the Product of the Mass by the Velocity, others have said, that the particular Make of the mechanical Organs was the Cause of this

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Phænomenon, and that this Action of Bodies upon one another, by means of Instruments, must be call'd *Pressure*, distinguishing between *Force* and *Pressure*, allowing *Pressures* of Powers to be to one another reciprocally as the Masses multiplied into the Velocities; but denying that Forces are in that *Ratio*. Now to shew that Forces, which we also call the *Momenta* of Bodies, are in the *Ratio* above-mentioned, let us impartially consider the following Experiment communicated first to me by Mr. *George Graham*.

EXPERIMENT. *Plate* 24. *Fig.* 1.

Plate 24. Fig. 1.

The Machine represented by the Figure has been already describ'd (*Page* 300,) only we must suppose the Arc F E to be divided into 24 Degrees on each Side, counting from the Edges A D and C B of the flat square Pendulum, instead of 18 mark'd in the Figure; and that the Pendulum weighs 2 Pounds, and the Weight W also two Pounds.

Now if the Pendulum without the Weight W, be drawn up to the 24th Degree on the Side of E, when you let it go, it will rise to 24 on the other Side; but if a Person, who holds the String L in his Hand (whilst the Weight W hangs half an Inch above A B C D, when at the lowest Place) lets go the String just at the Moment that the flat Pendulum comes to the lowest Place, the Pendulum receiving thereby an Addition of Matter equal to its own, goes only to 12 on the Side of F. If we let the Pendulum fall from 20 Degrees, and W be let down upon it at the lowest Part of the Vibration, then it will (thus loaded) go but to 10 Degrees; and when you let it go from 12, if it receives the Weight W at Bottom, it will go but to 6.

Now to compare together the two Opinions, we must examine, which of them gives the *Momentum* the same before and after the Reception of W.

OLD OPINION.

In the first Half of the Vibration.				*In the last Half of the Vibration.*			
Velocities,	Mass,	*Momenta.*		Velocities,	Mass,	*Momenta.*	
24	2	48		12	4	48	
20		40		10		40	
12		24		6		24	

NEW OPINION.

In the first Half of the Vibration.				*In the last Half of the Vibration.*				
Vel. Sq.	Vel. Mass,	*Momenta.*		Vel. Sq.	Vel. Mass,	*Momenta.*		
24—576	2	1152		12—144	4	576	instead of	1152
20—400		800		10—100		400		800
12—144		288		6— 36		144		288

But

But to have the *Momenta* as they ought to be according to the new Opinion, the Velocities, after the Reception of W on the Pendulum, ought to have been 17 —, 14, 1 +, an 8,48 +; for 17 — × 17 — = 288, and 14, 1 + × 14, 1 + = 200, and 8,48 + × 8, 48 = 72; which Numbers respectively multiplied by the Mass 4 of the Pendulum and Weight, will give for Products 1152, 800, and 288, the same *Momenta* that the Pendulum had in the first Half of the Vibration. Annotat. Lect. V.

N. B. *As it requires a little Practice and Dexterity to let* W *fall just upon the Middle of* A B C D, *one must make several Trials, and have a Circle drawn upon the Plate* A B C D, *to shew the exact Middle of it; but the Experiment is conclusive, tho'* W *does not fall precisely into the Circle: for when it falls beyond it (for Example, when you let the Pendulum go from* 12) *then will the compound Pendulum go as far beyond* 6 *towards* F, *and come as far short of* 6 *towards* E. *The same happens, whatever Height the Weight falls from. The Experiment also succeeds equally, whether the Bottom of the Weight and upper Surface of the Plate be rough or smooth, soft or hard. Nay, there is another Way of making the Experiment, easier than letting fall the Weight; and that is done by fixing the String that holds the Weight, so that the Plate can but just slip under it in its Vibration, and then screwing the curv'd upright Piece* w x w *to the Plate, represented now by* a b c d, *in such manner, that the curv'd Piece, or Semi-cylinder, which is something bigger than the Circle* w w (*Base of the Weight when on*) *shall, at the Bottom of the Vibration, embrace the Weight* W, *and carry it along, without its falling upon the Plate. In this last Case the Experiment will also succeed, whether the Weight has its cylindric Surface of Lead, Iron, Brass, or even soft Leather. I made the Experiment these different Ways, in order to remove several Objections; tho' they were of so little Force, that they may be easily answer'd without; and they are as follows.* Pl. 24. Fig. 1.

Object. 1. Ths Shock made by the Fall of the Weight W, hinders the compound Pendulum from going so far as it ought to do.

Answer. The Pendulum is in an horizontal Motion, when it receives the Weight W, which by its Fall gives a perpendicular Stroke, which (by what has been † said of two Forces acting at the same time upon a Body) can neither advance nor retard it towards the Side where it is going, but only make it go lower, if the Strings that suspend the flat Pendulum yield a little. For Example, let *e f* (**Plate* 24. *Fig.* 1) be the Line in which the Pendulum moves at the lowest Part of its Vibration, which we will consider as a streight Line, taking the Tangent instead of a small Arc; let the Weight W fall in the Line w *e*. Now if we suppose the Strings to yield, so as to become longer by the whole Depth *e g*, then will the Pendulum be acted upon by two Forces, as *e f*, and *e g*, which will make it go in the † Lect. 3. N°. 85. *Pl. 24. F. 1.

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Plate 24. Fig. 1.

the Diagonal *e l*; if the Strings ſtretch but to *h*, then will the Body go in the Diagonal *e m*; if they ſtretch only to *i*, the Body will go in the Diagonal *e n*; and laſtly, if the Strings either don't ſtretch at all, or by their Re-action reſtore themſelves wholly, then is the whole Effect of the Shock of the Fall of W deſtroy'd, and the Body will go in the Line *e f*, as if there had been no ſuch Stroke. Now in all theſe Caſes, the compound Pendulum goes from the Line *e g* to the Line *f l* in the ſame Time, and the Edge of the Plate muſt ſhew the ſame Number of Degrees upon the graduated Index below it. To theſe Objectors alſo, the Succeſs of the Experiment when ſoft Leather is glued under the Lead, muſt give Satisfaction; but making the Experiment without letting the Weight fall at all (when the little Semi-cylinder *w x w* of the Plate takes the Weight along with it) muſt be a full Conviction.

Object. 2. The Friction of the Weight W, and the Plate A B C D, one againſt another, occaſions the Loſs of Motion, and hinders the compound Pendulum from going to thoſe Diſtances, that are agreeable to the new Opinion.

Anſwer. This Objection is too looſe and general; for the Objectors ought to ſhew how much Motion Friction can deſtroy in other Caſes, and that there is as much Motion loſt here. Whereas, upon Examination, the ſuppos'd Cauſe will appear unequal to the Effect. For the Motion loſt here, according to the new Opinion, muſt be half of the whole Motion of the compound Pendulum; whereas Friction, at moſt, never deſtroys above a Third. Beſides, the Friction of one Part of the Pendulum againſt another, does no way affect the Motion of the whole Pendulum, or of the Syſtem of thoſe two Bodies, that Way in which it is carried. For Sir *Iſaac Newton* has demonſtrated in his *Principia*, Book the 1ſt, in the Corollaries to the Laws of Motion—*That if Bodies, any how mov'd among themſelves, are urged in the Direction of parallel Lines by equal accelerative Forces; they will all continue to move among themſelves, after the ſame manner as if they had been urged by no ſuch Forces.* Now, this is the Caſe of our compound Pendulum, all whoſe Parts for the ſmall Time of the Shock or Friction, may be conſidered, as carried along in right Lines; that is, in the Line *e f*, and its Parallels. The Anſwer by way of Experiment is, that when I have made the Bottom of W very ſmooth, as alſo the Surface of the Plate (as I was deſired to do by ſome of the Objectors) it has been more difficult to make the Experiment; but the Succeſs has been the ſame: For whenever the Weight W had its Center a little behind the Center of the Circle *w w*, perhaps by ſlipping backwards, the compound Pendulum came ſhort of the 12, 10, or 6 Degrees forwards towards F; but then it exceeded as much when it came back towards E; whereas the Diminution of Friction ought to make it go beyond thoſe Numbers according to the new Principle.

Tho

Tho' this is sufficient (at least as it appears to me) to convince any Person that is attentive to the Thing, of the Truth of the old Opinion; yet, in justice to the two ingenious Professors of *Leyden* and *Utrecht*, I ought to mention an Experiment, which they made in opposition to this Experiment of the flat Pendulum. Annotat. Lect. V.

EXPERIMENT. *Plate* 28. *Fig.* 6. Plate 28. Fig. 6.

A B G is a vertical Board, having an upright Piece A D fix'd to it with two Screws *y y*, at the Top of which Piece there is fix'd an horizontal Arm D E. From the said horizontal Arm are let down two fine Fiddle-strings C *m*, *c n*, which by the two little Hooks *m n*, sustain the Brass Cylinder W. This Cylinder is a Box with a Cone H, screwing into it, to shut up any Weights contain'd in it, and at the same Time to cut the Air the better, when the Cylinder moves with the conick End foremost. *t t* is a little Tail or Ruler fix'd to the other End of the Cylinder on its Axis continued, with several vertical Holes in it, to receive the Pin P, which goes through it behind *r r*, a strong Brass-Plate fix'd at right Angles to the Board A B G, by means of its Return R screw'd tight against the Board; which Plate has an Hole to transmit the Tail *t t*, whereby the Spring *s s* (fasten'd also to the End of the Cylinder) is bent. When the Pin P is suddenly pull'd out, the Cylinder is shot forward by the unbending of the Spring, in the Line H *h*; and the sliding Ruler O *n* being drawn out, so as to have its first Division begin where the String *c n* hangs, that String in the Motion of the Cylinder will successively pass over the Divisions upon the Ruler, and shew how far the Cylinder is shot forward in every Experiment. The Cylinder, with its Tail, Spring, and Head, weighs just one Pound; but by putting different leaden Cylinders into it, it may be made to weigh 2, 3, or 4 Pounds; and likewise the Spring may have different Degrees of Force given to it, according as the Tail *t t* is drawn farther through the Plate *r r*, towards R, and kept in its Place by the Pin P. N. B. *The 7th Figure represents Part of the Cylinder with its Tail and Spring, and the rectangular Plate, drawn by a larger Scale, and mark'd with the same Letters.* Plate 28. Fig. 7.

When the Spring bent to the same Degree, does, by pulling out the Pin P, successively shoot forward the Cylinder differently loaded; you will find, by observing the String *c n*, to what Distances the Cylinder is thrown. Now it always happens, that in those Cases, whatever the Weight of the Cylinder is, the Product of its Mass by the Square of the Velocity, is always the same. That is, if the Cylinder weighing 1, the Spring be adjusted so, as to throw it to the Degree 8 mark'd on the Ruler or Tangent-Line *n* O; the Cylinder loaden with 2, will by the same Bent of the Spring be thrown to 5,65, *&c.* if loaden with 3, it will go to 4,62, *&c.* and if loaden with 4, it will go to 4. These Gentlemen therefore alledge, that they have made use of Elasticity in their Experiment, as I have made use of Gravity in mine; and that as their Experiment is agreeable

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able to the new Opinion, and mine is agreeable to the old; therefore nothing is to be concluded from either of them.

Plate 28. Fig. 6.

Indeed, one would imagine so at first; and one would think that (according to the old Opinion) the Spring which shoots the Weight 1 to 8 Degrees, ought to shoot the Weight 4 only to 2 Degrees; whereas we see it shoots to 4 Degrees, since the same Force must produce the same *Momentum*. But here lies the Fallacy. If the Force of the Spring acted no longer upon the Weight 4, than it does upon the Weight 1, it would shoot it but to 2; but the Spring acts as long again upon the Weight 4, as it does upon the Weight 1; as is evident, because the Weight 4 moves but with half the Velocity of the Weight 1, the Spring quitting the Plate *r r*, as soon again, when it drives forward the Weight 1. So that an equal Cause acting twice the Time, must produce the same Effect, as a double Cause acting during the same Time as the single one.

When a Truth has been prov'd by plain Reasoning and simple Experiments; there is no Occasion to take any Notice of such subtle Reasonings, and complex Experiments, that are made use of to perplex it, for the Confirmation of the Truth. But, if the Experiments made to establish one Opinion, and overthrow another, tho' they have not answered that End, yet lead us to some new Discovery, or clear up some other Truth not generally known; they certainly merit our Attention and thorough Examination. Dr. *s'Gravesande*, and Dr. *P. Muschenbroek*, are too curious in making, and too faithful in relating their Experiments, not to have us credit the Facts: Therefore, tho' I deny the Conclusion, which they deduce from the Impressions made on Clay, or other soft Substances, by Sheres, Cones, or Cylinders, I think, there are very useful Consequences to be drawn from their Experiments in respect of Percussion, and the Resistance of soft Bodies. I likewise believe Professor *Polenus* to have made his Experiments carefully, tho' I have not the Honour to know him. Therefore I will, in the Beginning of my next Volume, consider this Dispute fully, and hope to shew the entire Fallacy of the Reasonings, which have been publish'd in favour of the new Opinion, and fully explain, why the Experiments alledged, are not conclusive in that Point. It is not proper to do it now; because, I have not yet explain'd the Laws of the Congress, or Shock of Bodies, which this Volume, already swell'd to too great a Bulk, allows me no room to do. I hope I shall then be able to settle the Dispute, by proving the three following Propositions.

1. *That the Cause, which accelerates Bodies in their Motion downwards, and retards them in their Motion upwards, does (abstracting from the Resistance of the Air) not accelerate them in their Fall with more Difficulty, or slower than it retards them in their Rise, which is a Supposition made in favour of the new Opinion.*

2. *That Impressions, or Pits of equal Capacities or Depths, made on soft Substances by hard Bodies, by striking them with unequal Velocities, are not*

not proportionable to the Momenta, *or Forces of those percutient Bodies; tho' they be proportionable to the Masses multiplied into the Squares of the Velocities.* Annotat. Lect. V.

3. *That Experiments of the Congress of soft and elastick Bodies do not prove the new Opinion, but confirm the old.*

In the mean Time, I refer the curious Reader for Satisfaction to the *Philosophical Transactions*, where he will find Dissertations upon this Subject, in the following Numbers, N°. 371, 375, 376, 396, 400, and 401.

In N°. 371. There is a Paper of Dr. *Henry Pemberton*, to shew the Experiments of *Polenus* inconclusive for the new Opinion.

In N°. 375. I endeavour to confirm the old Opinion, by an old Experiment on the Balance, and some Experiments of the Congress of Steel Balls.

In N°. 376. I endeavour to shew the Fallacy of *Polenus*'s Experiments by two new ones.

In N°. 396. Mr. *John Eames*, F. R. S. makes Remarks upon the new Opinion, relating to the Forces of moving Bodies, in the Case of Collision of *Non-elastick* Bodies: And in the same *Transaction* shews, in another Dissertation, that the Proof for the new Opinion, drawn from the Doctrine of Composition and Resolution of Forces (when a Body by its Impulse bends several Springs) equally proves both Sides of the Question, and thereby proving too much, does in reality prove nothing at all; and therefore is far from deserving the Name of a Demonstration.

In N°. 400. The said Mr. *Eames* has made Remarks upon some Experiments in *Hydraulicks*, which seem to prove, that the Forces of equal moving Bodies are as the Squares of their Velocities.

In N°. 401. There is a Letter of the late Rev. Dr. *Samuel Clarke*, concerning the Proportion of *Velocity* and *Force* in *Bodies in motion.*

Yet I cannot but blame the Doctor's uncivil Treatment of those Gentlemen, who defend the new Opinion, in Expressions very rude, and not at all relating to the Argument itself: Neither can I be of his Opinion, in his taxing Dr. *s'Gravesande* with "*intending to raise a Dust of Opposition against Sir* ISAAC NEWTON's *Philosophy*;" since I know that Professor does, and always did, very much esteem Sir *Isaac Newton*'s Works; and that both he, and Dr. *Petrus van Muschenbroek*, Professor at *Utrecht*, do teach the *Newtonian* Philosophy, tho' they differ from him in relation to the *Momentum* of Bodies.

[25. Page 347. *The Pile may enter into the Earth farther than in that Proportion*, &c.——Page 348. *Nails*, &c. *give way to the Hammer nearly according to the Square of the Velocity*, &c.——*But the Reverse must be done in the Battering Ram.*

Tho' the Experiments made by the ingenious Professors *Polenus*, *s'Gravesande*, *Muschenbroek*, and others, do not prove the *Momenta* of Bodies to be as the Mass into the Square of the Velocity, for the Reasons given in the *Philosophical Transactions*, which I have referr'd to, and for other Reasons

Annotat. Lect. V. (which I reſerve for that Part of my Work, wherein I ſhall conſider the Congreſs of Bodies, and which, I hope, will be ſatisfactory to thoſe Gentlemen themſelves) yet they have a great deal of Merit, becauſe they do certainly ſhew, that the ſame Body ſtriking ſoft Subſtances with different Velocities, does make Impreſſions upon them, in Proportion to the Squares of thoſe Velocities; and thereby give us a Principle to direct the Practice of ſome mechanical Operations, which was not very well known before. Accordingly, a one-handed Hammer, ſtriking upon a hot Iron, when it has 4 times the Velocity of a two-handed Hammer, which is 4 times heavier, will do twice the Work in forging the Iron, tho' the *Momentum* is the ſame in both; but the Introceſſion, or Yielding of the Iron, is not the ſame in both. So in driving Piles, when the Earth is equally yielding, if we raiſe the Rammer four times as high, the Pile will enter into the Earth four times eaſier; but when the Pile (tho' ſhod with Iron) ſtrikes upon a Rock, or is driven in ſtony Earth, the Caſe is not the ſame, but in a mix'd Proportion between that of the Maſs into the Velocity, and the Maſs into the Square of the Velocity.

Now in a Battering Ram, the Effect will be greater, when a great Quantity of Matter moves with a ſmall Velocity, than when the ſame *Momentum* is produc'd by a little Quantity of Matter which moves with a great Velocity. The explaining of this will give ſome light into the Diſpute concerning the new and the old Opinion. For, if we ſuppoſe ACDE* a Piece of Wall (whoſe horizontal Section is repreſented here) ſtruck by a Cannon Ball, or a Battering Ram at B, the Effect in the firſt Caſe will be this, that the Cannon Ball, (ſuppoſing it to have a ſufficient Velocity) will go thro' the Wall, making the Hole *a b* B *c* thro' the Wall, or at moſt, the Hole *d c* B *e*, in the Form of a truncated Cone, ſhaking a little more of the Wall, than the Parts of it, which are very near that Hole, becauſe the Velocity is ſo great, that the Motion has not Time to be communicated very far, whilſt the Bullet is paſſing. But if a Battering Ram, with the ſame *Momentum*, but a Velocity ſo much leſs as it has more Matter, ſtrikes the Wall at B, it will ſhake it round about to a great Diſtance, the Motion having Time to be communicated; and, if the ſucceſſive Blows be continued, ſo that the ſecond may be given, before the Shaking given by the firſt is over; after ſeveral Blows, a large Piece of Wall will be thrown down, ſuch as FGHI. This is what Sir *Chriſtopher Wren* has often practis'd in the beating down of old Buildings and Ruins; in order to erect new Edifices in the Place where old ones formerly ſtood: And he ſaid, he found no Way ſo expeditious for demoliſhing old Walls, whoſe Mortar was almoſt as hard as the Stone itſelf.

* Plate 28. Fig. 8.

L. 2. No. 9. and Annot. 3. In War indeed, Cannon is preferable to the ancient Battering Rams, for many Reaſons: ſome of which have been given in *Lect.* 2d, N°. 9. and the 3d *Annotation.* And in thoſe Caſes we are to obſerve, that the Beſiegers ſhoot obliquely, chooſing rather to ſtrike the Walls obliquely from different Batteries, tho' with leſs *Momentum*, that the ſecond Batt may beat down what the firſt has ſhak'd: This being more effectual, than

than to make many small Holes in a Wall by shooting perpendicularly. To illustrate this yet further, I will here repeat what I have already said in the *Philosophical Transactions* (N°. 376.) *viz.* "That when a Door is "half open, and moving very freely on its Hinges; if a Pistol be fir'd "against it, the Ball will go through the Door without moving out of its "Place; but if we take a large Weight of Lead, and throw it against "the Door, with the same Force as the Pistol Bullet mov'd, the Door "will be remov'd from its Position, and carried out of the Place on its "Hinges by the Stroke; because in the first Case, the Motion of the Ball "is communicated but to a few Parts of the Door, and in the last it is "diffus'd all over it. Nay, the Door will be mov'd by the Stroke, even "though there should be a prominent Part in the Lead no bigger than a "Pistol Bullet, in order to strike the Door upon no more of its Surface, "than the Bullet has done." Annotat. Lect. V.

5. [32. Page 352. *A Fluid will resist sometimes as much as a Solid*, &c.] Let us suppose a Piece of Wood, as a Plank one Inch thick, to be of the same specifick Gravity as Water; and that Water of the same Thickness covers a Surface of Clay; and let us examine what will be the Effect of a Bullet striking the Solid or the Fluid. Let us again suppose, that the Bullet striking the Water with the Velocity 1, passes through it, overcoming by its *Momentum* the Resistance as 1, which arises from the Density of the Water, and also another Resistance as 1, which arises from its Tenacity. If the Bullet, with the same Velocity, strikes the Wood, whose Resistance from its Density is as 1, but whose Resistance from the Tenacity is as 9; that is, whose whole Resistance is 10; it is certain, that the Bullet cannot penetrate through the Wood with the Velocity 1, but it must have the Velocity 10, to overcome the Resistance above-mentioned.

Now, if the Bullet, with the same Velocity 10, comes against the Water, it will meet with a Resistance, as 100, on account of the Density †, and also the Resistance 10, on account of the Water's Tenacity *, that is, 110. Then if the Bullet with the same Velocity strikes the Wood, and we suppose the Resistance of the Wood, from its Quantity of Matter, to increase as the Square of the Velocity, as it does in the Water (though it does not increase in so great a Proportion) the whole Resistance of the Wood will be 190, that is, 100 from its Density, and 90 from its Tenacity. Here the whole Resistance of the Water, is to the whole Resistance of the Wood, as 110 to 190, or as 11 to 19. If the Velocity of the Bullet be doubled, that is, becomes 20, the whole Resistance of the Water will be $400+20=420$; and the Resistance of the Wood will be $400+180=580$: Then will the Resistance of the Water be to that of the Wood, as 42 to 58, or 21 to 29, the Difference being now less than before. If the Velocity of the Bullet be 40, the Resistance of the Water will be $1600+40=1640$, and that of the Wood $1600+360=1960$; and then the Resistance of the Water will be to that of the Wood, as 164 to 196, or 41 to 49, the Difference being still less. Therefore,

† L. 5. 35.
* Page 350, 351.

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fore, as the Velocity of the Bullet increaſes, the Reſiſtance of the Water continually comes nearer to that of the Wood. Now, if the Wood be leſs denſe than the Water, the Reſiſtance of the Water (on increaſing the Velocity of the Bullet) will ſoon be greater than that of the Wood. Likewiſe, tho' the Wood be as denſe as the Water, if the Reſiſtance, on account of the Quantity of Matter, does not increaſe ſo faſt as it does in the Water (as I ſaid above, and Experience ſhews) by increaſing the Bullet's Velocity, the Reſiſtance of the Water will ſoon overtake that of the Wood. The following Experiment has been made to confirm this Aſſertion.

A Sail was ſpread horizontally in a Pond about 2½ Feet under the Surface of the Water, and a Muſket with a ſmall Charge being fir'd obliquely to the Surface of the Water, but perpendicularly to an half-inch Deal Board fix'd under the Water over the Sail, the Bullet went through the Board under the Water, and after that through the Sail-Cloth. The Experiment was made again with a greater Charge, and the Bullet ſtruck againſt the Board without going through it, making but a ſmall Impreſſion in the Board; yet its Roundneſs was alter'd more than the Reſiſtance of the Board ſeem'd likely to have caus'd, and therefore the Bruiſe in the Lead was thought owing to the Reſiſtance of the Water. Upon increaſing the Charge a third Time, the Ball fell upon the Sail without reaching the Board, and was much beaten out of ſhape. At laſt putting in a Charge pretty nearly equal to the Proof (that is, the Weight of Powder equal to the Weight of the Bullet) the Bullet was beaten to pieces upon the Surface of the Water. N. B. *I cannot exactly tell in what Proportion the Charges were increas'd, becauſe I did not ſee the Experiment myſelf; but I had the Account from Perſons of Credit that ſaw them.* That the Reſiſtance of the Water was greater than that of the Wood appear'd by this, *viz.* that when the Shot was made againſt a Plank of 1½ Inch thick in the Air, the Bullet went through more and more eaſily, the more the Charge was increas'd.

This great Reſiſtance of Water is farther confirm'd, by an Accident that happen'd to myſelf.

Some Years ago, on a rejoicing Day, the Honourable Colonel *Samuel Horſey*, with ſeveral other Perſons, and myſelf, went out upon the *Thames* in a great Barge belonging to the *Vintners* Company, where we play'd off ſome Fire-works. It happen'd, that a Water-rocket (whoſe Property is to go under Water ſeveral times and riſe again, and at laſt burſt on the Top of the Water) came up, when it was ready to burſt, under the Stern of the Barge, being thereby prevented from coming up to the Surface of the Water, and burſting where it was, gave the Barge a great Shock, ſo as to give it a ſenſible Lift, which I plainly felt, being directly over the Place, and wonder'd at the Force of ſo ſmall a Quantity of Powder, there being much leſs than an Ounce of Powder to make the Report. A little after, ſome Gentlemen and Ladies, being in another Barge, of leſs than half the Bigneſs of ours, came near us, and deſired me to throw ſome of the Water-rockets between the two Barges, that

that they might the better obſerve their Motion. I threw ſeveral; but at laſt one of them in its laſt Riſe, ſtopping under the Middle of their Barge, broke there, and made ſo great a Hole in the Barge's Bottom, that there was only Time for the Company to get out of their Barge into ours, and the Watermen to row to Shore from the Middle of the *Thames* (at *Mortlake*, where it is but narrow) before the Barge was half full of Water. Becauſe in the Exploſion of the Powder, whoſe Velocity, in its Expanſion, is exceedingly great, the Water reſiſted, like a Solid, and conſequently the Powder made its way through the Bottom of the Barge, whoſe Planks were leſs denſe than the Water, and not very thick. Annotat. Lect. V.

After this, to try the Effect of the Exploſion of Gun-powder under Water; I loaded a Water-rocket, ſo that it ſhould break under the Water, and having ſet Fire to it, threw it into a Pond, that cover'd an Acre of Ground: And ſo great was the Shock, that ſeveral Perſons, who ſtood round the Pond, felt it like a momentaneous Earthquake.

6. [39. Page 359. *Sir* Iſaac Newton *has demonſtrated it*, &c.]

Take Sir *Iſaac Newton*'s two Demonſtrations in his own Words.

BOOK I. SECT. 2. PROPOSITION I. THEOREM I.

" *The* Areas, *which revolving Bodies deſcribe by* Radii, *drawn to an immoveable Center of Force, do lie in the ſame immoveable Planes, and are proportional to the Times in which they are deſcrib'd.* [Plate 28. Fig. 9.] Pl. 28. F. 9.

" For, ſuppoſe the Time to be divided into equal Parts; and in the firſt " Part of that Time, let the Body by its innate Force deſcribe the right " Line A B. In the ſecond Part of that Time, the ſame would (by Law " 1.) if not hinder'd, proceed directly to *c*, along the Line B *c* equal to " A B: ſo that by the *Radii* A S, B S, *c* S drawn to the Center, the " equal Areas A S B, B S *c*, would be deſcribed. But when the Body is " arriv'd at B; ſuppoſe, that a centripetal Force acts at once with a " great Impulſe, and turning aſide the Body from the right Line B *c*, " compells it afterwards to continue its Motion along the right Line B C. " Draw *c* C parallel to B S, meeting B C in C; and at the End of the ſe- " cond Part of the Time, the Body (by *Cor.* 1. of the Laws) will be " found in C, in the ſame Plane with the Triangle A S B. Join S C, " and, becauſe S B and C *c* are parallel, the Triangle S B C will be " equal to the Triangle S B *c*, and therefore alſo to the Triangle S A B. " By the like Argument, if the centripetal Force acts ſucceſſively in " C, D, E, *&c.* and makes the Body in each ſingle Particle of Time, to " deſcribe the right Lines C D, D E, E F, *&c.* they will all lie in the " ſame Plane; and the Triangle S C D will be equal to the Triangle " S B C, and S D E to S C D, and S E F to S D E. And therefore, in " equal Times, equal Areas are deſcrib'd, in one immoveable Plane: And, " by Compoſition; any Sums S A D S, S A F S, of thoſe Areas, are

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Plate 28. Fig. 9.

" one to the other, as the Times in which they are describ'd. Now, let " the Number of those Triangles be augmented, and their Breadth be di- " minish'd *in infinitum*; and (by Cor. 4. *Lem.* 3.) their ultimate Perime- " ter A D F will be a curve Line; and therefore the centripetal Force by " which the Body is perpetually drawn back from the Tangent of this " Curve, will act continually; and any describ'd Areas S A D S, S A F S, " which are always proportional to the Times of Description, will, in this " Case, also be proportional to those Times. *Q. E. D.*

" *Cor.* 1. The Velocity of a Body attracted towards an immoveable " Center, in Spaces void of Resistance, is reciprocally as the Perpendicu- " lar let fall from that Center on the right Line that touches the Orbit. For " the Velocities in those Places A, B, C, D, E are as the Bases A B, B C, " C D, D E, E F, of equal Triangles; and these Bases are reciprocally as " the Perpendiculars let fall upon them.

" *Cor.* 2. If the Chords A B, B C of two Arcs, successively describ'd " in equal Times, by the same Body, in Spaces void of Resistance, are " compleated into a Parallelogram A B C V, and the Diagonal B V of this " Parallelogram, in the Position which it ultimately acquires, when those " Arcs are diminish'd *in infinitum*, is produced both ways, it will pass " through the Center of Force.

" *Cor.* 3. If the Chords A B, B C, and D E, E F of Arcs describ'd " in equal Times, in Spaces void of Resistance, are compleated into the " Parallelograms A B C V, D E F Z; the Forces in B and E, are one " to the other in the ultimate Ratio of the Diagonals B V, E Z, when " those Arcs are diminished *in infinitum*. For the Motions B C and E F " of the Body (by *Cor.* 1. of the *Laws*) are compounded of the Motions " B *c*, B V, and E *f*, E Z; but B V and E Z, which are equal to *c* C and " F *f*, in the Demonstration of this Proposition, were generated by the " Impulses of the centripetal Force in B and E, and are therefore propor- " tional to those Impulses.

" *Cor.* 4. The Forces by which Bodies in Spaces void of Resistance, " are drawn back from rectilinear Motions, and turned into curvilinear " Orbits, are one to another as the versed Sines of Arcs describ'd in equal " Times; which versed Sines tend to the Centre of Force, and bisect the " Chords, when those Arcs are diminished to Infinity: For such versed " Sines are the Halves of the Diagonals mention'd in *Cor.* 3.

" *Cor.* 5. And therefore those Forces are to the Force of Gravity, as the " said versed Sines to the versed Sines perpendicular to the Horizon of those " parabolic Arcs, which Projectiles describe in the same Time.

" *Cor.*

" *Cor.* 6. And the same Things do all hold good (by *Cor.* 5. of the *Laws*) when the Planes in which the Bodies are mov'd, together with the Centers of Force which are placed in those Planes, are not at rest, but move uniformly forward in right Lines. Annotat. Lect. V.

" PROPOSITION II. THEOREM II.

" *Every Body, that moves in any curve Line describ'd in a Plane, and by a* Radius, *drawn to a Point, either immoveable, or moving forward with an uniform rectilinear Motion, describes about that Point* Areas *proportional to the Times, is urged by a centripetal Force directed to that Point.*

" *Case* 1. For every Body, that moves in a Curve Line, is (by *Law* 1.) turned aside from its rectilinear Course, by the Action of some Force, that impels it. And that Force, by which the Body is turned off from its rectilinear Course, and is made to describe, in equal Times, the equal least Triangles SAB, SBC, SCD*, &c. about the immoveable Point S (by *Prop.* 40. *Book* 1. *El.* and *Law* 2.) acts in the Place B, according to the Direction of a Line parallel to *c*C, that is, in the Direction of the Line BS; and in the Place C, according to the Direction of a Line parallel to *d*D, that is, in the Direction of the Line CS, &c. And therefore acts always in the Direction of Lines tending to the immoveable Point S. Q. E. D. * Plate 28. Fig. 9.

" *Case* 2. And (by *Cor* 5. of the *Laws*) it is indifferent, whether the Superficies in which a Body describes a curvilinear Figure be quiescent, or moves together with the Body, the Figure describ'd, and its Point S, uniformly forwards in right Lines.

" *Cor.* 1. In non-resisting Spaces or Mediums, if the *Areas* are not proportional to the Times, the Forces are not directed to the Point in which the *Radii* meet; but deviate therefrom *in consequentia*, or towards the Parts to which the Motion is directed, if the Description of the *Areas* is accelerated; but in *Antecedentia*, if retarded.

" *Cor.* 2. And even in resisting Mediums, if the Description of the *Areas* is accelerated, the Directions of the Forces deviate from the Point in which the *Radii* meet, towards the Parts to which the Motion tends.

SCHOLIUM.

Body may be urged by a centripetal Force compounded of several [illegible]s. In which Case the Meaning of the Proposition is, that the Force " which results out of all, tends to the Point S, &c."

I have

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I have contriv'd a Machine to represent this mechanically, wherein little Brass Ball representing a Planet, moves in an Ellipse about a Body representing the Sun in one of the *Foci* of that Ellipse, describing, by a *Radius vector*, Areas proportional to the Time. N. B. *I shall give a Figure and Description of it at the End of this Volume.*

——*That the centripetal Force is the same as Gravity.*

Sir *Isaac Newton* demonstrates this in the 4th Proposition of the third Book of his *Principia.*

"PROP. 4. THEOR. 4. *That the Moon gravitates towards the Earth;* "*and by the Force of Gravity is continually drawn off from a rectilinear Mo-* "*tion, and retain'd in its Orbit.*

Sir *Isaac*, having given an Account of the Moon's mean distance, according to several Astronomers, in the Beginning of this Proposition, goes on in these Words: "Let us assume the mean Distance of 60 Semi-dia- "meters in the *Syzygies*; and suppose one Revolution of the Moon, in "respect of the fix'd Stars, to be compleated in 27 d. 7 h. 43 m. as "Astronomers have determined; and the Circumference of the Earth to "amount to 123249600 *Paris* Feet, as the *French* have found by Mensu- "ration. And now, if we imagine the Moon, deprived of all Motion, "to be let go, so as to descend towards the Earth, with the Impulse of "all that Force, by which (by *Cor. Prop.* 3.) it is retained in its Orb; "it will, in the Space of one Minute of Time, describe in its Fall "15 $\frac{1}{12}$ *Paris* Feet. This we gather by a *Calculus*, founded either upon "*Prop.* 36. Book 1. or (which comes to the same Thing) upon *Cor.* 9. "*Prop.* 4. of the same Book. For the versed Sine of that Arc, which "the Moon, in the Space of one Minute of Time, would by its mean "Motion describe at the Distance of 60 Semi-diameters of the Earth, is "nearly 15 $\frac{1}{12}$ *Paris* Feet, or more accurately 15 Feet, 1 Inch, and 1 "Line $\frac{4}{9}$. Wherefore, since that Force, in approaching to the Earth, in- "creases in the reciprocal duplicate Proportion of the Distance, and, "upon that account, at the Surface of the Earth, is 60 × 60 times great- "er than at the Moon; a Body in our Regions, falling with that "Force, ought, in the Space of one Minute of Time, to describe "60 × 60 × 15 $\frac{1}{12}$ *Paris* Feet; and, in the Space of one Second of Time, "to describe 15 $\frac{1}{12}$ of those Feet; or more accurately, 15 Feet, 1 Inch, "and 1 Line $\frac{4}{9}$. And with this very Force, we actually find, that Bo- "dies here upon Earth do really descend. For a Pendulum oscillating "Seconds in the Latitude of *Paris*, will be three *Paris* Feet, and 8 Lines $\frac{1}{2}$ "in Length, as Mr. *Huygens* has observ'd. And the Space, which a "heavy Body describes by falling in one Second of Time, is to the "Length of this Pendulum, in the duplicate *Ratio* of the Circumference "of a Circle to its Diameter, (as Mr. *Huygens* has also shewn;) and is there-

"fore

" fore 15 *Paris* Feet, 1 Inch, 1 Line $\frac{7}{9}$. And therefore the Force, by which the Moon is retain'd in its Orbit, becomes, at the very Surface of the Earth, equal to the Force of Gravity, which we observe in heavy Bodies there. And therefore (by Rule 1. and 2.) the Force by which the Moon is retained in its Orbit, is that very same Force, which we commonly call Gravity. For, were Gravity another Force different from that; then Bodies descending to the Earth, with the joint Impulse of both Forces, would fall with a double Velocity, and in the Space of one Second of Time, would describe $30\frac{1}{6}$ *Paris* Feet; altogether against Experience, *&c.*

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SCHOLIUM.

" The Demonstration of this Proposition may be more diffusely explained after the following manner: Suppose several Moons to revolve about the Earth, as in the System of *Jupiter* or *Saturn*; the periodic Times of these Moons (by the Argument of Induction) would observe the same Law which *Kepler* found to obtain among the Planets; and therefore their centripetal Forces would be reciprocally as the Squares of the Distances from the Center of the Earth, by *Prop.* 1. of this *Book*. Now, if the lowest of these was very small, and was so near the Earth, as almost to touch the Tops of the highest Mountains; the centripetal Force thereof, retaining in its Orb, would be very nearly equal to the Weights of any terrestrial Bodies, that should be found upon the Tops of those Mountains, as may be known by the foregoing Computation. Therefore, if the same little Moon should be deserted by its centrifugal Force, that carries it through its Orbit, and so be disabled from going onwards therein, it would descend to the Earth; and that with the same Velocity as heavy Bodies do actually fall with, upon the Tops of those very Mountains; because of the Equality of the Forces that oblige them both to descend. And, if the Force by which that lowest Moon would descend, were different from Gravity; and if that Moon were to gravitate towards the Earth, as we find terrestrial Bodies do upon the Tops of Mountains, it would then descend with twice the Velocity, as being impelled by both these Forces conspiring together. Therefore, since both those Forces, that is, the Gravity of heavy Bodies, and the centripetal Forces of the Moons, respect the Center of the Earth, and are similar and equal between themselves, they will (by Rule 1. and 2.) † have one and the same Cause. And therefore the Force, which retains the Moon in its Orbit, is that very Force which we commonly call Gravity; because, otherwise, this little Moon at the top of a Mountain, must either be without Gravity, or fall twice as swiftly as heavy Bodies used to do.

† *See the Beginning of Sir* Isaac's *third Book.*

[40. Page 361, 362. *Since the Angles*, &c.——*only decrease half way from* A *to* P, &c. *And the Angles*, &c.——*only increase half way from*

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P *to* A, &c.] This will be made evident by proving, that the Angle made by a Line drawn from either of the *Foci*, to any Point in the Periphery of an Ellipse, and the Tangent to the Curve in that Point, will be the least possible, or at its *Minimum*, when that Point is at either of the Ends of the lesser Axis; this is, S H T * is less than S I *t*. And this we shall prove by the Help of the two following *Lemmata*.

*Pl. 20. F. 19

LEMMA I.

In every quadrilateral Figure, inscribed in a Circle, the Sum of the two Rectangles form'd by the opposite Sides, are equal to the Rectangle form'd by the two Diagonals of that quadrilateral Figure.

This is a known Property of the Circle, and has been demonstrated by *Ptolemy*, and others; that is, $\overline{AC \times DB} + \overline{BC \times AD} = \overline{AB \times DC}$ †.

†Pl. 29. F. 11.

LEMMA II.

If an Arc of a Circle be divided into two equal, and into two unequal Parts; I say, that the Sum of the Chords of the two equal Arcs, will always be greater than the Sum of the Chords of the two unequal Arcs. That is, if the Arc A C K B *is divided into two equal Parts at* C, *and into unequal Parts at* K; A C + C B *is greater than* A K + K B.

In order to prove this; through C and the Center *k*, draw the Diameter C D, and the Diagonals and Chords A B, D K, A D, B D. It is evident (by the 2d and 29th of the 3d of *Eucl.*) A D = B D, and they may be substituted for one another. It is evident also, that D K will always be less than C D (by *Eucl.* 15th of the 3d.) Now by the last *Lemma* $\overline{AK \times DB} + \overline{KB \times AD} = \overline{AB \times DK}$; now since D K has been prov'd less then D C, it must follow, that $AB \times DC$ is greater than $AB \times DK$; therefore $\overline{AC \times DB} + \overline{BC \times AD}$ is greater than $AK \times DB + KB \times AD$; and since the Factors D B and A D are equal, it must follow, that A C + C B is greater than A K + K B.

Pl. 28. F. 10.

Let a Circle in *Fig.* 10. be describ'd passing through the Points S, H, *s*, 'tis evident (by *Eucl.* 21st of the 3d) that the Angle S H *s* and S M *s*, are equal; and 'tis evident by the 2d *Lemma*, that S H + H *s*, is greater than S M + M *s*. Now by the Property of the Ellipse S I + I *s* is always equal to S H + H *s*; being always equal to A P; it must therefore follow, that S I + I *s* is greater than S M + M *s*: whence it appears, that the Point I is out of the Circle S H M *s*, and therefore the Angle S I *s* is less than S M *s*, or its equal S H *s*. Now by the Property of the Ellipse demonstrated in all the Conick Writers (*vide Elementa Conicorum Lud. Trevigar. Lib.* 2. *Prop.* 10.) the Angles *s* I *g* and S I *t* are equal, and *s* H G is equal to S H T; therefore since *s* I S is less than *s* H S, it must follow, that S I *t*, is greater than S H T; and what has been prov'd of I, may be prov'd of the F, or any other Point in the Semi-ellipse A H P; therefore the Truth of the Proposition advanced is demonstrated.

8. [42. Page 364. *Comets — are Planets moving in very excentrick Orbits*, &c.] One may have a full Account of Comets, and their Motions, in Dr. *Halley*'s Treatise of Comets, which was first printed by itself, and then reprinted at the End of the *English* Translation of Dr. *Gregory*'s Astronomy; as also in Sir *Isaac Newton*'s 3d Book of his *Principia*, from the Beginning of the 39th *Prop.* to the End of the 42d, where his general *Scholium* begins. The Reader may be fully satisfied about Comets by consulting those Authors. However, it may not be improper to give a short Account of them here, by way of Abstract. Annotat. Lect. V.

COMETS are a Sort of excentrick Planets, which move in very long Ellipses about the Sun in one of the *Foci* of those Ellipses, whose periodical Revolutions take up such a long Space of Time, that the same Man has never yet seen the same Comet twice. So that the Astronomy of Comets is but lately brought to any tolerable Perfection, for want of a sufficient Number of Observations: And the Accounts we have of them in former Ages, are from Historians, and not Astronomers: which last, while they took pains to observe the Planets and fix'd Stars, neglected such Comets as appeared in their Times, being carried away with the Opinion, that they were only Meteors near the Earth, and not to be reckoned among the celestial Bodies. But in the last Age, Comets were first found to be higher than the Moon, and consequently to belong to the heavenly Bodies; and for about 70 Years last past, Astronomers have been very diligent in observing such as have appear'd. Sir *Isaac Newton*, from his own Observations, and those of others, has given a Method how, from only three Observations of a Comet, made at different Times, to find out all that Part of its Orbit, in which it will continue visible to us; and Dr. *Halley* has first shewn, that the same Comet returns again after a certain Number of Years, and that the whole Orbit is elliptical. From Astronomers and Historians, we are inform'd of 25 Appearances of Comets within these last 400 Years, tho' the Comets belonging to our System may be fewer in Number; because the same Comets have been seen several times, but were not known to be the same, till Dr. *Halley* shew'd it, and settled the Orbits of three of them; making out, that the Comet seen in 1661 had been seen before, and observ'd to go the same Way, in the Year 1532; which makes its Period to be of 129 Years. The Comet seen in the Year 1680 and 1681, appear'd before in the Year 1106, so that its Period is about 575 Years. Lastly, The Comet of 1682 appeared in the Year 1607 and 1531, which shews its Period to consist of 75 Years. So that the Number of the Comets which have been seen will be reduced to 22. And if, according to Mr. *Whiston*'s Conjectures, the Comet of 1682, appear'd also in 1456; and that of 1681 in 532, as also 44 Years before [illegible] Christian *Æra*, as Sir *Isaac Newton* and Dr. *Halley* have since discover'd, the Number will not be increas'd, since those Periods are before the Time of the first of the 25. But yet we cannot say, there are but 22, because several of them may have been at such a distance from the

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Earth, as to have escap'd Observation; as the last might have done, if the Right Honourable the Lord *Paizley* had not discovered the last by chance in the Year 1723, and given notice of it to other Astronomers, that they might also observe it.

The Comets are reckon'd not to be less than the Moon, nor much bigger than *Venus*; and those are the least, which come nearest to the Sun, lest they should agitate the Sun too much, as they come by it at their *Perihelion*, and likewise disturb those Planets which are near the Sun. That they are not very large appears, because they are not visible till they come nearer to us than *Jupiter*'s Orbit; for as they are opake Bodies, that reflect Light from the Sun, they must have been seen as far as *Saturn*, if they had been as big.

Whereas all the Planets have the Planes of their Orbits in, or near, the Plane of the Ecliptick, the most inclin'd making an Angle but of few Degrees; the Comets have their Orbits inclin'd to the Ecliptick in very great Angles, moving towards all Parts of the Heavens; and by that means are less liable to disturb the Planets, because they come but twice in one Revolution into the Plane of the Ecliptick, and when they are at their *Aphelia*, very distant from the Sun, they are also very distant from each other, and thereby not so subject to attract each other with sufficient Force to disturb one another's Orbits sensibly; for when they move very slowly at their *Aphelia*, they are easily drawn out of their usual Course.

Comets are considerably rarer than the Planets; for if they were as solid, they would not have those Tails, which are observable, as they come near enough to be warmed by the Sun, with a Degree of Heat, something less than that of the Planet *Mars*: Those Tails being Vapours rais'd from their *Nucleus*, or Globe, so as to make a very large Atmosphere, whose Particles being thrown off on that Side, which is opposite to the Sun, give that Appearance, which is call'd a *Tail*, when we see it at right Angles, or at any great Angle; a *Beard*, when we see it obliquely; and a *Coma* or *Hairiness*, when we see it with so great an Obliquity, as to be almost in the Line of its Axis, so that the Vapour appears to encompass the Comet like a *Periwig*, or *Head of Hair*. And these three Appearances are owing to the different Positions of the Earth in respect to the Comet; for a Tail of the same Bigness varies optically according to the Position of the Eye.

But then the Tail itself is continually changing; for often the Comet begins to appear without any sensible Tail, and by degrees the Tail is generated, and increases continually, as the Comet comes nearer to the Sun, and is the biggest of all when the Comet is just past the *Perihelion*, having then receiv'd its greatest Degree of Heat.

If the celestial Spaces, in which the Comets move, were not void of Matter, we could not see their Tails, because they are abundantly rarer than our Air, since one may plainly see the fix'd Stars through them; we see the Light of the Sun reflected from those Particles that float in the Air, when a Sun-beam of 1 or 2 Inches wide is let into a dark Room; so is the Light of the Sun beam reflected to us from the Particles

cles of the Tails of Comets, because there is no other sensible Matter near them: For if there was a celestial Matter about them, tho' 50000 times rarer than our Air, it would reflect Light enough to confound that reflected from the Tail of the Comet in one undistinguish'd Glare: So that Comets would not appear to have any Tails at all; or, at most, but very small ones. Annotat. Lect. V.

The Particles or Vapours, that form the Tail, seem to be at first darted every way from the Comet; but on that Side, which is towards the Sun, they turn back again in parabolick Lines, so as to go on the contrary way, and to help to increase the Tail on the Side opposite to the Sun, as if the Sun's Rays gave them an Impulse, and drove them that way. The 12th *Figure* of *Plate* 28, represents a Comet, and the lower Part of its Tail, as Dr. *Hook* carefully observ'd it through a Telescope near its *Perihelion*, with the purer Part of its Atmosphere, winding itself into the Tail, and the cloudy Part of it plac'd round about the central Solid. But, as the Comet of 1680 and 1681 was the most remarkable, and most nicely observed by Sir *Isaac Newton*, and many other Astronomers; it will be sufficient to give an account of that, and the Figure of that Part of its Orbit, wherein it was visible to us, such as Sir *Isaac Newton* has given it in the last Edition of his *Principia*. And, as I can't give his Sense better than in his own Words, I shall take the liberty to make use of them. Pl. 28, F. 12.

" I have given (*Pl.* 28. *Fig.* 13.) a true Representation of the Orbit, " which this Comet describ'd, and of the Tail which it emitted in seve- " ral Places, in the annexed Figure; protracted in the Plane of the Tra- " jectory. In this Scheme ABC represents the Trajectory of the Co- " met, D the Sun, DE the Axis of the Trajectory, DE the Line of " the Nodes, GH the Intersection of the Sphere of the Orbit of the " Earth, with the Plane of the Trajectory, I the Place of the Comet, " *Nov.* 4. *Anno* 1680. K the Place of the same, *Nov.* 11. L the Place " of the same. *Nov.* 19. M its Place, *Dec.* 12. N its Place, *Dec.* 21. " O its Place, *Dec.* 29. P its Place *Jan.* 5, following. Q its Place, " *Jan.* 25. R its Place *Feb.* 5. S its Place, *Feb.* 25. T its Place, *March* " 5. And V its Place, *March* 9. In determining the Length of the " Tail I made the following Observation. Pl. 28. F. 13.

" *Nov.* 4. and 6. The Tail did not appear; *Nov.* 11. the Tail just be- " gan to show itself, but did not appear above half a deg. long, through " a 10 Foot Telescope; *Nov.* 17. the Tail was seen by *Ponthæus* more " than 15 deg. long, *&c.*

Then Sir *Isaac* proceeds, in giving an Account of several Persons, as well as himself, who observ'd the Tail, and all its Changes, in Increase and Decrease, as it appear'd to them in several Countries; and at last he goes on thus.

" *Feb.* 25.

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" *Feb.* 25. the Comet was without a Tail, and so it continued till it " disappear'd.

" Now, if one reflects upon the Orbit describ'd and duly considers " the other Appearances of this Comet, he will be easily satisfied, that " the Bodies of Comets are solid, compact, fix'd, and durable, like the " Bodies of the Planets. For, if they were nothing else but the Vapours " of Exhalations of the Earth, of the Sun, and other Planets, this " Comet in its Passage by the Neighbourhood of the Sun, would have " been immediately dissipated. For the Heat of the Sun is as the Den- " sity of its Rays: that is, reciprocally as the Square of the Distances of " the Places from the Sun. Therefore, since on *Dec.* 8. when the " Comet was in its *Perihelion*, the Distance thereof from the Center of " the Sun, was to the Distance of the Earth from the same, as about 6 " to 1000; the Sun's Heat on the Comet was at that time to the Heat " of the Summer's Sun with us, as 1000000 to 36, or as 28000 to " 1. But the Heat of boiling Water is about 3 times greater than the " Heat, which dry Earth acquires from the Summer-Sun, as I have " try'd; and the Heat of red-hot Iron (if my Conjecture is right) is about " three or 4 times greater than the Heat of boiling Water. And therefore " the Heat which dry Earth on the Comet, while in its *Perihelion*, " might have conceived from the Rays of the Sun, was about 2000 times " greater than the Heat of red-hot Iron. But by so fierce a Heat, " Vapours and Exhalations, and every volatile Matter, must have been " immediately consum'd and dissipated.

" This Comet, therefore, must have conceiv'd an immense Heat from " the Sun, and retain that Heat for an exceeding long Time. For a " Globe of Iron of an Inch diameter, expos'd red-hot to the open Air, " will scarcely lose its Heat in an Hour's time; but a greater Globe " would retain its Heat longer in Proportion of its Diameter, because the " Surface (in Proportion to which it is cool'd by the Contract of the am- " bient Air) is in this Proportion less, in respect of the Quantity of " the included hot Matter. And therefore a Globe of red-hot Iron, " equal to our Earth, this is about 40000000 Feet in Diameter, would " scarcely cool in an equal Number of Days, or in above 50000 Years. " But I suspect, that the Duration of Heat may, on account of some " latent Causes, increase in a yet less Proportion, than that of the Dia- " meter; and I should be glad, that the true Proportion was investigated " by Experiments.

" It is further to be observ'd, that the Comet in the Month of *De-* " *cember*, just after it had been heated by the Sun, did emit a much " longer Tail, and much more splendid, than in the Month of *November* " before, when it had not yet arriv'd at its *Perihelion*. And universally, " the greatest and most fulgent Tails always arise from Comets, imme- " diately after their passing by the Neighbourhood of the Sun. There- " fore the Heat receiv'd by the Comet, conduces to the Greatness of the " Tail. From whence, I think, I may infer, that the Tail is nothing

" else but a very fine Vapour, which the Head, or *Nucleus* of the Comet Annotat.
" emits by its Heat, *&c.* Lect. V.

After this, our incomparable Author gives an Account of three several Opinions concerning the Tails of Comets; and having refuted two of them, goes on to prove the third (which is also his own) in these Words——

" That the Tails of Comets do arise from their Heads, and tend to-
" wards the Parts opposite to the Sun, is further confirm'd from the
" Laws, which the Tails observe. As, that lying in the Planes of the
" Comets Orbits, which pass through the Sun, they constantly deviate
" from the Opposition of the Sun towards the Parts, which the Comets
" Heads in their Progress along these Orbits have left. That to a Spec-
" tator, placed in those Planes, they appear in the Parts directly opposite
" to the Sun; but as the Spectator recedes from these Planes, their De-
" viation begins to appear, and daily becomes greater. That the Devi-
" ation, *cæteris paribus*, appears less, when the Tail is more oblique to
" the Orbit of the Comet, as well as when the Head of the Comet ap-
" proaches nearer to the Sun; especially, if the Angle of Deviation is
" estimated nearer the Head of the Comet. That the Tails, which have
" no Deviation, appear strait; but the Tails which deviate, are likewise
" bent into a certain Curvature. That the Curvature is greater, when
" the Deviation is greater; and is more sensible, when the Tail, *cæteris*
" *paribus*, is longer: For in the shorter Tails, the Curvature is hardly to
" be perceiv'd. That the Angle of Deviation is less near the Comet's
" Head, but greater towards the other End of the Tail; and that, be-
" cause the convex Side of the Tail regards the Parts, from which the De-
" viation is made, and which lie in a right Line, drawn out infinitely
" from the Sun, through the Comet's Head. And that the Tails, that
" are long and broad, and shine with a stronger Light, appear more re-
" splendent, and more exactly defin'd on the convex than on the concave
" Side. Upon which accounts, it is plain, that the *Phænomena* of the
" Tails of Comets depend upon the Motions of their Heads, and by no
" means upon the Places in the Heavens in which their Heads are seen;
" and, that therefore the Tails of Comets do not proceed from the Re-
" fraction of the Heavens, but from their own Heads, which furnish the
" Matter that forms the Tail. For as, in our Air, the Smoke of a heated
" Body ascends, either perpendicularly, if the Body is at Rest, or oblique-
" ly, if the Body is moved obliquely; so in the Heavens, where all Bodies
" gravitate towards the Sun, Smoke and Vapours must (as we have already
" said) ascend from the Sun, and either rise perpendicularly, if the smoak-
" ing Body is at rest; or obliquely, if the Body in all the Progress of
" its Motion, is always leaving those Places, from which the upper or
" higher Parts of the Vapour had risen before. And that Obliquity will
" be least, where the Vapour ascends with most Velocity, to wit, near the

" smoaking

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" smoaking Body, when that is near the Sun. But because the Obliquity " varies, the Column of Vapour will be incurvated; and, because the Va- " pour in the preceding Side, is something more recent, that is, has as- " cended something more late from the Body, it will therefore be some- " thing more dense on that Side, and must on that account reflect more " Light, as well as be better defin'd.

Then Sir *Isaac*, shewing how very rare our Air must be at the Distance of one Semi-diameter of the Earth from its Surface; makes use of it, as an Argument, to shew, that a very small Quantity of Air and Vapour is abundantly sufficient to produce all the Appearances of the Tails of Comets. Then goes on shewing—" That we may pretty nearly determine " the Time spent during the Ascent of the Vapour from the Comet's " Head, to the Extremity of the Tail, by drawing a right Line from " the Extremity of the Tail to the Sun, and marking the Place where " that right Line intersects the Comet's Orbit. For the Vapour, that is " now in the Extremity of the Tail, if it has ascended in a right Line " from the Sun, must have begun to rise from the Head, at the Time " the Head was in the Point of Intersection. It is true, the Vapour " does not rise in a right Line from the Sun, but retaining the Motion, " which it had from the Comet before its Ascent, and compounding that " Motion with its Motion of Ascent, arises obliquely. And, therefore, " the Solution of the Problem will be more exact, if we draw the Line, " which intersects the Orbit, parallel to the Length of the Tail; or rather " (because of the curvilinear Motion of the Comet) diverging a little from " the Line or Length of the Tail. And by means of this Principle I fonnd " that the Vapour, which *Jan.* 25. was in the Extremity of the Tail, " had begun to rise from the Head before *Dec.* 11. and therefore had " spent in its whole Ascent 45 Days; but that the whole Tail, which " appear'd on *Dec.* 10. had finish'd its Ascent in the space of the two " Days then elapsed from the Time of the Comet's being in its *Perihelion*. " The Vapour, therefore, about the Beginning, and the Neighbour- " hood of the Sun, rose with the greatest Velocity, and afterwards con- " tinu'd to ascend with a Motion constantly retarded by its own Gravity; " and the higher it ascended, the more it added to the Length of the " Tail. And while the Tail continu'd to be seen, it was made up of " almost all that Vapour, which had risen since the Time of the Comet's " being in its *Perihelion*; nor did that Part of the Vapour, which had " risen first, and which form'd that Extremity of the Tail, cease to ap- " pear, till its too great Distance, as well from the Sun, from which it re- " ceiv'd its Light, as from our Eyes, render'd it invisible. Whence also " it is, that the Tails of other Comets, which are short, do not rise " from their Heads, with a swift and continual Motion, and soon after " disappear; but are permanent and lasting Columns of Vapours and Ex- " halations; which ascending from the Heads with a slow Motion of ma- " ny Days, and partaking of the Motion of the Heads, which they had

" from the Beginning, continue to go along together with them through Annotat.
" the Heavens. From whence again, we have another Argument proving Lect. V.
" the celeſtial Spaces to be free and without Reſiſtance; ſince in them,
" not only the ſolid Bodies of the Planets and Comets, but alſo the ex-
" tremely rare Vapours of Comets Tails, maintain their rapid Motions
" with great Freedom, and for an exceeding long Time."

Then after having ſhewn, why the Tails of Comets, when they are very near the Sun, riſe to ſuch prodigious Heights from their Heads, and yet do not quit the Heads in the Motion of the Comets; he goes on.

" The Tails, therefore, that riſe in the *Perihelion* Poſitions of the Co-
" mets, will go along with their Heads into far remote Parts, and toge-
" ther with the Heads will either return again from thence to us, after
" a long Courſe of Years; or rather, will be there rarified, and by de-
" grees quite vaniſh away. For afterwards, in the Deſcent of the Heads
" towards the Sun, new ſhort Tails will be emitted from the Heads with
" a ſlow Motion; and thoſe Tails by degrees will be augmented im-
" menſely; eſpecially in ſuch Comets as in their *Perihelion* Diſtances de-
" ſcend as low as the Sun's Atmoſphere. For all Vapour in thoſe free
" Spaces, is in a perpetual State of Rarifaction and Dilatation. And
" from hence it is, that the Tails of all Comets are broader at their up-
" per Extremity, than near their Heads. And it is not unlikely, but
" that the Vapour, thus perpetually rarify'd and dilated, may be at laſt
" diſſipated, and ſcatter'd through the whole Heavens, and by little and
" little be attracted towards the Planets, by its Gravity, and mix'd with
" their Atmoſphere. For, as the Seas are abſolutely neceſſary to the Con-
" ſervation of our Earth, that from them the Sun, by its Heat, may ex-
" hale a ſufficient Quantity of Vapours, which being gathered together
" into Clouds, may drop down in Rain, for watering of the Earth, and
" for the Production and Nouriſhment of Vegetables; or being conden-
" ſed with Cold on the Tops of Mountains (as ſome Philoſophers with
" Reaſon judge) may run down in Springs and Rivers; ſo for the Conſer-
" vation of the Seas, and Fluids of the Planets, Comets ſeem to be re-
" quir'd, that from their Exhalations and Vapours condens'd, the Waſtes
" of the Planetary Fluids, ſpent upon Vegetation and Putrefaction, and
" converted into dry Earth, may be continually ſupplied and made up.
" For all Vegetables entirely derive their Growth from Fluids, and after-
" wards in great meaſure are turn'd into dry Earth by Putrifaction, and
" a Sort of Slime is always found to ſettle at the Bottom of putrified
" Fluids. And hence it is, that the Bulk of the ſolid Earth is conti-
" nually increaſed, and the Fluids, if they are not ſupply'd from without,
" muſt be in a continual Decreaſe, and quite fail at laſt. I ſuſpect more-
" over that 'tis chiefly from the Comets that Spirit comes, which is in-

Annotat. Lect. V.

" deed the smallest, but the most subtil and useful Part of our Air, and " so much requir'd to sustain the Life of all things with us.

" The Atmospheres of Comets, in their Descent towards the Sun, by " running out into the Tails, are spent and diminish'd, and become nar- " rower, at least on that Side, which regards the Sun; and in receding " from the Sun, when they less run out into the Tails, they are again " enlarged, if *Hevelius* has justly mark'd their Appearances. But they " are seen least of all, just after they have been most heated by the Sun, and " on that account then emit the longest and most resplendent Tails, and " perhaps at the same time the *Nuclei* are environ'd with a denser and " blacker Smoak, in the lowermost Parts of their Atmosphere. For " Smoke, that is raised by a great and intense Heat, is commonly the " denser and blacker; thus the Head of that Comet, which we have " been describing, at equal Distances both from the Sun and from the " Earth, appear'd darker after it had pass'd by its *Perihelion*, than it did " before. For in the Month of *December* it was commonly compar'd with " the Stars of the third Magnitude; but in *November*, with those of the " first or second. And such as saw both Appearances, have described the " first, as of another and greater Comet than the Second."

Sir *Isaac* confirms this, by the Observations of several Astronomers; then gives a Problem concerning a Comet's Trajectory (to which, and all that we have omitted of his on that Subject, I refer the mathematical Reader) and several Tables of Observations of Comets Places; and at last he speaks in general of Comets in the following Words:

" Because of the great Number of Comets, of the great Distance of " their *Aphelia* from the Sun, and of the Slowness of Motions in the " *Aphelia*, they will by their mutual Gravitations, disturb each other; So " that their Excentricities, and the Times of their Revolutions will be " sometimes a little increased, and sometimes diminished. Therefore we " are not to expect, that the same Comet will return exactly in the same " Orbit, and in the same periodick Times. It will be sufficient, if we " find the Changes no greater, than may arise from the causes just spoken " of.

" And hence a Reason may be assign'd, why Comets are not compre- " hended within the Limits of a Zodiack as the Planets are; but being " confin'd to no Bounds, are with various Motions dispersed all over the " Heavens; namely to this Purpose, that in their *Aphelia*, where their " Motions are exceeding slow, receding to greater Distances one from " another, they may suffer less Disturbance from their mutual Gravita- " tions. And hence it is, that the Comets, which descend the lowest, " and therefore move the slowest in their *Aphelia*, ought also to ascend the " highest.

" The

" The Comet, which appeared in the Year 1680, was in its *Perihelion*, Annotat.
" less distant from the Sun, than by a sixth Part of the Sun's Diameter; Lect. V.
" and because of its extreme Velocity in that Proximity to the Sun, and
" some Density of the Sun's Atmosphere, it must have suffer'd some Re-
" sistance and Retardation; and therefore, being attracted something near-
" er to the Sun in every Revolution, will at last fall down upon the Body
" of the Sun. Nay, in its *Aphelion*, where it moves the slowest, it may
" sometimes happen to be yet farther retarded by the Attractions of other
" Comets, and in consequence of this Retardation descend to the Sun. So
" fixed Stars, that have been gradually wasted by the Light and Vapours
" emitted from them for a long time, may be recruited by Comets that
" fall upon them, and from this fresh Supply of new Fuel, those old
" Stars acquiring new Splendor, may pass for new Stars. Of this kind
" are such fix'd Stars as appear on a sudden and shine with a wonderful
" Brightness at first, and afterwards vanish by little and little."

9. [46. Page 367.—*Action and Re-action are equal and contrary, as well in the Repulsions as Attractions.*] So the Gravitation betwixt the Earth and its Parts, is equal; for if we suppose the Earth FEHIK * to be divided *Pl. 28. F. 14.
into two equal parts, it is evident, that those Parts would come together with equal Forces. But if they be divided into two unequal Parts, *viz.* EFG and EIG by a Plane as EG; they will then also gravitate, or press towards one another with equal Forces (see *Lect.* I. N° 11.) for if L. I. N° 11.
we cut off from the Part EIG, the smaller Part HIK equal to EFG; then will the greater Part EGHK, become the Obstacle, upon which the two equal Quantities of Matter EFG and HIK do press, whilst itself remains *in Æquilibrio* between them. If we were to suppose EFG to be separated from EIG, in their Tendency towards each other by their Gravity, the little one would move so much faster towards the great one, as it has less Matter than the great one; so that the *Momentum* being equal in both, they will at meeting come to Rest, destroying each other's Motion. For if the great one had more Force, it would go *in infinitum*, carrying EFG before it in the Direction IF.

The better to illustrate the Law of Action *and* Re-action, *I think it may not be improper, to give my Reader the following Problem, communicated to me by a Friend.*

PROBLEM.

To find what is the Force with which the Breech of a Cannon is acted upon, as it shoots out a 24 lib. *Ball.*

According to *Mersenus*, a 24 Pounder, weighing 6400 lib. it gives its Ball an uniform Velocity of 600 *Paris* Feet in a Second, which makes 640 *English* Feet. Now, let the Weight of the Cannon be called $= c =$

Annotat. Lect. V.

6400 lib. the Weight of the Ball $=b=24$ lib. and the uniform Velocity, which the Powder gives it $=V=640$ Feet in a Second, the Velocity of the Cannon $=u$. The *Momenta* of the Cannon, and the Ball given by the ſame Force of Powder, will be equal; therefore $cu=bV$; conſequently, $6400 : 24 :: 640 : 2{,}4=u=$ the Velocity of the Cannon, if it recoil'd on an horizontal Plane perfectly ſmooth. But if the Cannon cannot recoil, the Force of the Powder acting but by way of Preſſure, will give the Cannon a Shock, with a Force as $15630=6400\times 2{,}4$. Therefore, when the Cannon recoils, Part of the Force of the Powder is employ'd in giving it a Velocity of 2,4 Feet in a Second, and the remaining Part acts upon the Cannon by way of Preſſure. If then we would find how much the whole Preſſure 15630 is diminiſh'd, when the Recoil is of 2,4 Feet in a Second; ſince the whole Preſſure is to the partial Preſſure, as the whole Velocity is to the partial Velocity, the two Parts will be, as $640-2{,}4$: is to 2,4; or as 637,6: is to 2,4; or as 796 : 3. Therefore we muſt divide 15630 in the ſame Proportion, making $797 : 3 :: 15630-x : x$; conſequently $797+3$ (800) $3 :: 15630 : x=57{,}6$; which Quantity being taken from 15630, leaves a Preſſure as 15572,4 when the Cannon recoils freely. But the Platform of the Battery not being perfectly ſmooth, and commonly riſing backwards, the Recoil of the Cannon, which in ſuch a Caſe is oblig'd to riſe, will be ſo little, that one may conſider very near the whole Force of 15630 as acting on the Cannon, and whatever ſtops it from recoiling.

Now, as it is not ſufficient, only to compare the Weights and Velocities of the Ball and Cannon (becauſe the Force, which is expreſs'd by the Number 15630, might have been expreſs'd by any other Number, as it would have been by 184320, if we had taken 7680 Inches, inſtead of 640 Feet in a Second for the Velocity of the Ball) we muſt ſhew, what Weight expreſs'd in Number of Pounds preſſing upon the Cannon, as a Weight laid on, is equal to the Force with which the Powder preſſes the Gun from within, as it throws out the Ball; which will be thus found. Let us ſuppoſe the Length of the Cannon within to be of 12 Feet, and that the Powder, as it expands in firing, drives out the Ball with a Velocity uniformly accelerated, ſo that the Ball, when out of the Cannon, ſhall by an uniform Velocity move 24 Feet in the ſame time, that it mov'd 12 Feet in the Cannon by the accelerated Velocity. Therefore as the Ball, by that uniform Velocity, moves 640 Feet in a Second, it ſpends but $\frac{1}{26\frac{2}{3}}$ and $\frac{2}{3}$ of a Second in moving 24 Feet, and conſequently ſpends but $\frac{1}{26\frac{2}{3}}$ in running the Length of the Inſide of the Cannon. Now, ſince by accelerated Velocities the Spaces gone through, are as the Squares of the Times then will $\overline{\frac{1}{26\frac{2}{3}}}\Big|^2$: be to $\overline{1\text{ Second}}\Big|^2$; or, 1 : to $\overline{26\frac{2}{3}}\Big|^2$ $(=711\frac{1}{9})$ :: as 12 Feet : to $8533\frac{1}{3}$ Feet, which the Ball would go in a Second with the accelerated Velocity, which it has in the Cannon. If therefore Gravity, with an ac-

accelerated Force of 1,6 Feet in a Second, gives the Ball a Force of 24 lib. Weight; the Action of the Powder, which is capable of giving it in the same time a Velocity of 8533⅓ Feet in a Second, must give it a *Momentum*, or Force against the Cannon, equal to a Weight of 12800 lib. because 16 Feet: are to 24 lib. :: as 8533 ⅓ Feet : to 12800 lib. since *Action* and *Re-action* are equal. Annotat. Lect. V.

If we allow the Cannon any sensible Recoil, the Effect of the Pressure of the Powder must be diminish'd, by the same Analogy as we diminish'd the *Momentum* in the former Consideration.

——*A Pendulum, whose Length is* 39 *Inches and* 2 *Tenths* English *Measure, will perform one Vibration in one Second of Time.*] That Length (according to a Measure, that has been taken at a Mean from two Standard Measures of an *English* Yard, one of which is reposited at *Guildhall*, and the other at the *Exchequer*) is too much, 39,13 Inches, being the Length of a Second-Pendulum, which has been found, by comparing together many Experiments, none of which gave the Length above 39,133, nor less than 39,125. But I made use of 39,2 Inches, because it has been commonly reckon'd the true Length, till more accurate Experiments have been made by Mr. *George Graham*, and others, within these 15 or 20 Years.

§ 10. [68. Page 387. *The whole Curve so describ'd is call'd a Cycloid.*] Let the Line B C † represent Part of the Earth's Surface, on which a Wheel or generating Circle rolling, does with a Point of its Circumference describe a Cycloid, as we have shewn; it is plain, that so small a Part of the Earth's Surface, which a Wheel runs over in one Revolution, may be consider'd as a Plane; and also, that if the generating Circle should run along the under Side of the Line C B, it would by a Point of its Circumference, first applied to one End as C, describe the same Sort of Curve, or Cycloid C G B, as if it had ran above the Line; which Curve would terminate at B, the Base of the Cycloid C B being equal to the Circumference of the generating Circle, and the Curve C G B equal to 4 times the Diameter. But if the generating Circle should run along the inner Surface of a Sphere, as from the Point *c* to *b*, instead of running from *c* to K along a Tangent of equal Length with the Curve *c* A *b*, the Cycloid *c* G *b* will be shorter and less curve, than if its Base *c* A *b* had been strait, as one may see by comparing it with the Cycloid C G B, produced by the same generating Circle. This is call'd an Epicycloid. Now, if we suppose a large Wheel, or generating Circle, to be so big, that its Diameter shall bear a sensible Proportion to the Diameter of the Earth, its Revolution under the Earth's Surface would generate such a Cycloid: And if the generating Circle should be half as big as a Diameter of the Earth, as the Circle *x* γ, the Point *x* by the rolling of the generating Circle along the Hemisphere, or Semi-circle *x* A β, would describe *x* β a Diameter of the Earth; so Pl. 29. Fig. 1.

† We have taken 16 Feet instead of 16,1 to avoid Fractions.

that

Annotat. Lect. V. Pl. 29. F. 1.

that the Cycloid would degenerate into a strait Line. Now, in all these Epicycloids, the Vibrations of a Pendulum, or of an heavy Body, oscillating on either Side of a middle Point G or γ, *&c.* will be *isochronal.* For tho' we have shewn, that the Reason of a Body's falling faster in a Cycloid, than in an Arc of a Circle, or any other oblique Line, was because the Body set out at first in the Direction in which Gravity acts; and here the Epicycloids being less curve than the common Cycloid, one would imagine, that they would lose that Advantage of Steepness at first: Yet we can shew, that in all the Epicycloids suppos'd within the Earth, as *c* G *b*, or even in that which becomes a strait Line, as $\varkappa\beta$, Bodies begin their Motion in the same Direction as Gravity, whose Action tends towards the Center γ; for wherever the generating Circle begins its Revolution, a Line as γc, drawn from the Center of the Earth (in which Gravity acts) passes through the Center of the generating Circle, and the describing Point: Whereas in the common Cycloid we build our Demonstrations upon the Supposition, that the Lines, in which Gravity acts, are parallel to one another, which Supposition is very proper, considering the great Distance of the Center of the Earth, whereto those Lines converge.

L. 1. An. 11.

We have shewn in the 11th Annotation of *Lect.* 1. *Page* 34 and 35, as a Consequence of the Earth attracting according to its Quantity of Matter; that the Force of the Action of Gravity is greatest at the Earth's Surface, decreasing as the Squares of the Distances from its Center increase, when Bodies are remov'd outwards farther and farther from the Earth's Surface; and that if Bodies are suppos'd to come nearer to the Center, within the Earth, the Force of Gravity acting upon them, will decrease directly as the Distance from the Center. If therefore (supposing the Earth penetrable) several Bodies, as for Example 4, were placed, one at the Surface at β (about 4000 Miles from the Center γ) another at G (2500 Miles from it) another at H (2000 Miles from the Center) and the fourth at L (but 1000 Miles from it) the Force of Gravity acting upon those several Bodies to drive them to the Center, would be respectively as 4000, 2500, 2000, and 1000; that is, as their Distances from γ: and consequently, if they should set out at the same Moment of Time they would also arrive at the Center at the same Instant. So, that if there was an Hole through the Earth along the Diameter of it as $\beta\varkappa$, a Body falling from β, which, according to Mr. *Whiston*'s Calculation, would come from the Surface to the Center in 21 Minutes and 9 Seconds, would go from β to $\varkappa$ in double the time, and so vibrate backwards and forwards in the Diameter $\beta\varkappa$, like a Pendulum: And if it began its Motion at H, it would take up as much time in going to I, vibrating backwards and forwards (or rather up and down) in the Line HI. *Now therefore, if the* isochronal *Vibration in a Diameter of the Earth, which follows from a Supposition of the Attraction of the Earth, being proportional to its Quantity of Matter, be also a Consequence of the* isochronal *Vibrations of Pendulums in a Cycloid (which has been mathematically and experimentally demonstrated*

Fig. 1.

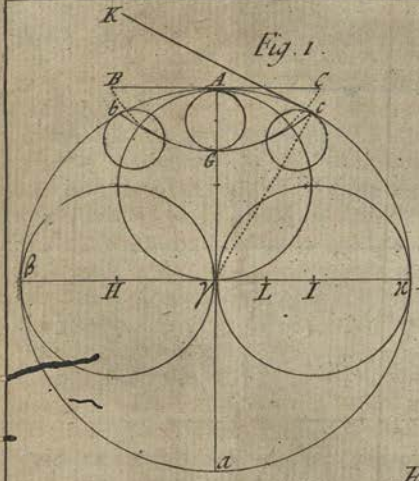


Fig. 4.

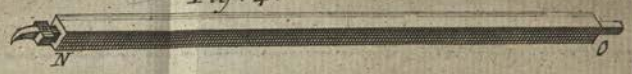


Fig. 2.

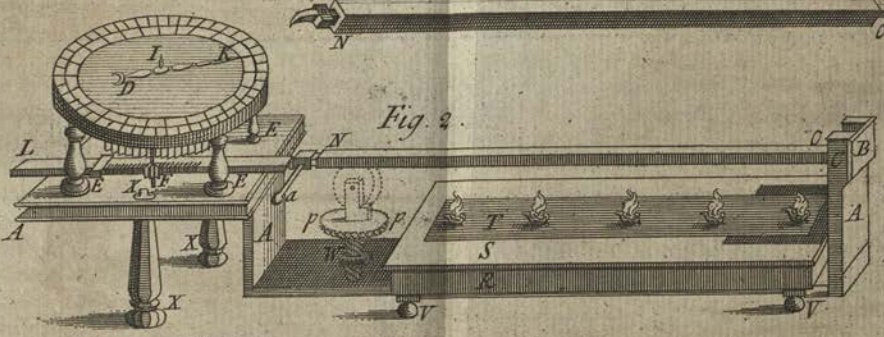
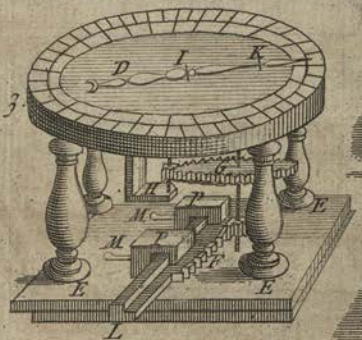


Fig. 3.



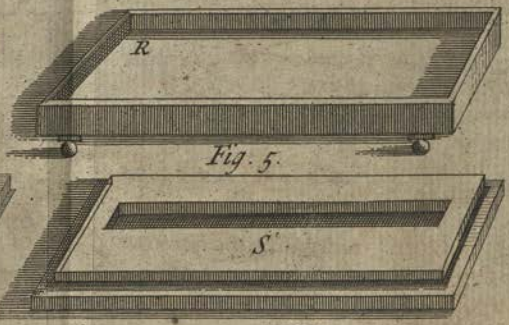


Fig. 5.

Fig. 6

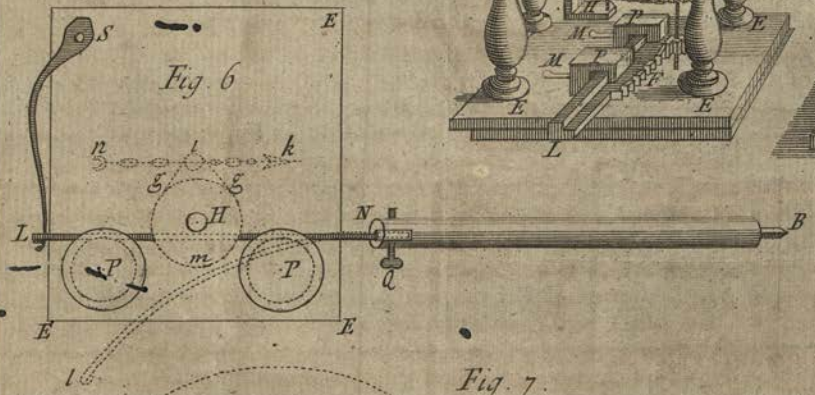


Fig. 7.

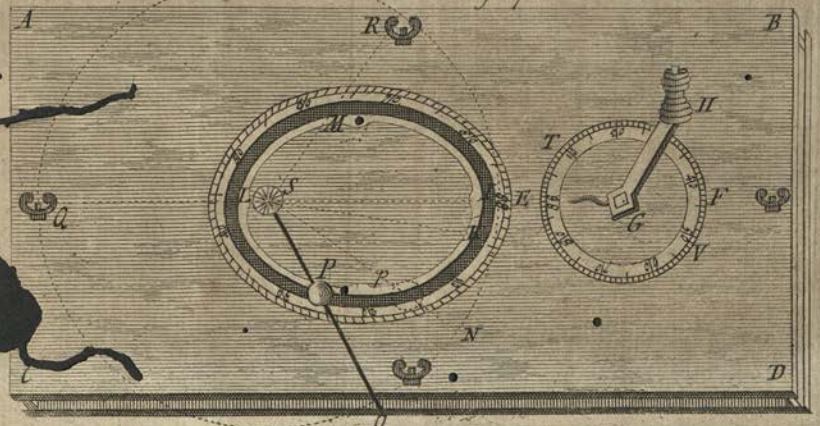
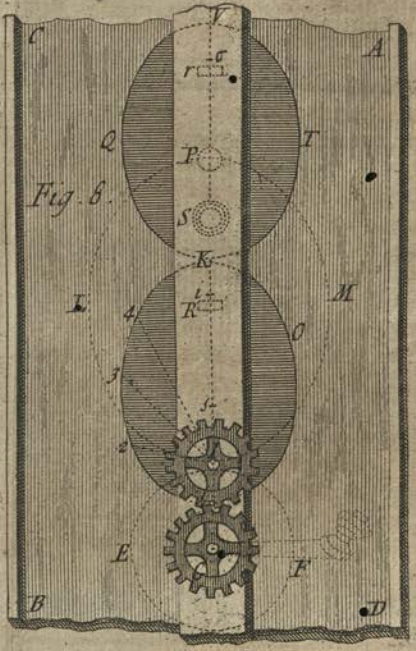


Fig. 8.



must follow that that Supposition is true——tho' there are many other ways of proving it. Annotat. Lect. V.

11. [70. Page 383. *The Way to measure the least Alterations of Dimensions in Metals, so as to make it sensible*, &c.] Dr. *Petrus van Muschenbroek*, the ingenious Professor of Astronomy, *&c.* at *Utrecht*, contriv'd a Machine, which he calls a *Pyrometer*, for this Purpose, whose Description in his own Words, is as follows.

The 2d *Figure* of *Plate* 29, represents the whole Machine, with all its Parts together as it is used. Pl. 29. F. 2.

" AAA is an Iron turn'd up perpendicularly at one End, which Re-
" turn is 1 $\frac{8}{10}$ Inches high. The other End, which is distant from it $\frac{3}{4}$
" Inches, is also turn'd up, but then it is turn'd back again, so as to
" make a broad horizontal square Plate, the Side of which Square is 2
" Inches. The Iron itself is 1 Inch wide and $\frac{3}{10}$ of an Inch thick, which
" Thickness was requir'd, lest it should easily or soon grow hot, which
" would hinder the Accuracy of the Experiments.

" Upon the Iron Plate stands a Brass Machine, which is drawn by it-
" self in *Fig.* 3. where it is represented larger, and seen from another Side,
" the better to discover its Parts, which for that purpose are mark'd with
" the same Letters as in *Fig.* 2. This is fix'd to the Iron by two Screws
" X, X, which are its Legs. D is a circular Plate of 2 $\frac{1}{10}$ Inches Diame-
" ter, divided into 300 equal Parts, which we have not number'd in the
" *Figure*, by reason of their Smallness. This divided Plate stands upon
" four Pillars E, E, E, E, which join it to the lower Brass Plate: Between
" these two Plates there is a perpendicular Steel Arbor or Axis F, which
" has on its lower Part a Pinion of 6 Leaves or Teeth, and on its upper
" a Wheel of 60 Teeth mark'd G: There is also another Axis I H, sup-
" ported by a Cock that comes down from the upper Plate, and which
" Axis with its upper End passes through the said upper Plate, so as to
" receive the Hand or Index I K; having at its lower End a Pinion of 6
" Leaves to take the Teeth of the Wheel G. The Hand by one Turn of
" the Pinion H is carried round to all the Divisions. There is besides, a
" little Rack or streight Piece L with Teeth, which takes the Leaves of
" the Pinion F, while the Rack slides along under two small Cocks P, P,
" where it is pressed towards the Pinion F by help of two small Screws M,
" M, or drawn from it as there is Occasion, that the Teeth may take pro-
" perly without sticking or being too loose. The Teeth of the Rack are
" 25 in an Inch, and as it moves forward and backward, the Pinion F is
" carried round, and consequently the Wheel G, which carries round the
" Pinion H, together with the Hand I K. Let us suppose the Rack to
" have run the Length of 1 Inch; then F and G will have turn'd round 4
" times and $\frac{1}{6}$; and consequently the Pinion H, will have gone round
" 10 × $4\frac{1}{6}$, that is, $41\frac{2}{3}$ times, because H turns round 10 times for G once:

" So

Annotat. Lect. V. " So that the Index IK, will have mov'd round $41\frac{2}{3}$ times; and because the upper-Plate is divided into 300 Parts, the Hand must have run over 300 Divisions $41\frac{2}{3}$ times; that is, 12500 Parts. Therefore, when the Hand goes but from one Division to another, the Rack moves but the 12500th Part of an Inch. Now, since the Divisions are still large enough to observe the Motion of the Hand to the Half of one of them, one may perceive when the Rack has mov'd $\frac{1}{25000}$ Part of an Inch. Care has been taken, that the Teeth and Pivots should have but just Play enough to go free, without which, one can make no accurate Experiment. There is a Female Screw goes through the Head of the Rack.

Pl. 28. F. 4. " The 4th *Figure* represents a square Bar, or Paralselopiped of Metal, upon which the Experiment is made, $5\frac{8}{10}$ Inches long, and $\frac{3}{10}$ of an Inch thick. One End of it O has a small Tail, that it may communicate no sensible Degree of Heat to the Iron A A; it is received in a Notch near B, and fix'd by the Screw C. Its other End N has an Hole in it, through which Hole goes the Screw Q, which makes it fast to the Rack L; but that End is also made small (as you see in the *Figure*) lest the Heat should be communicated to the Rack.

" The Bar being thus fix'd, cannot became longer, without shooting forward the Rack L, and thereby moving round the Hand IK, by means of the Wheel and Pinions F, G, H; so likewise, when it becomes shorter, they must move the contrary Way. Now, lest the Weight of the Bar should hinder the Motion of the Rack, I have thrust in a Piece of a Watch-Spring between the square Iron and Brass Plates E A, exactly of such a Strength as to support a Bar as much as its Gravity would depress it; so that the Rack would then move with all possible Freedom. However insignificant this Particular may seem, yet it is very necessary to be nice in it; for in making Experiments with Bars of different Metals, I was oblig'd to draw out, or thrust in my Spring more or less, according to their different Weights. The Hand, or Index IK moves so freely, that you will perceive it to move when the Bar of Tin is made fast to the Machine, if you but apply your warm Hand to the Bar; for Tin requires but a very small Degree of Heat to alter its Dimensions, as this Experiment proves. But to apply more Heat conveniently, especially the Flame of burning Spirits, there is a

Pl. 29. F. 5. " Box made of Brass (see *Fig.* 5.) $3\frac{1}{2}$ Inches long, $1\frac{2}{10}$ Inch wide, and $\frac{4}{10}$ Inch deep, which is cover'd at top with a Piece of blue Stone (such as will bear the fire, call'd in *Dutch*, *Lye*) which is represented in the *Figure* the underside upwards. It was necessary to make this Cover of Stone, lest it should grow too hot, and set fire to the Spirit within the Lamp, as will happen when the Cover is made of Metal. It has a long Hole cut through the Middle, into which is let in a Brass Plate T, which has 5 small equidistant Holes $\frac{6}{10}\frac{5}{0}$ Parts of an Inch asunder, and $\frac{7}{100}$ Inch in Diameter, to transmit the Wicks of the Lamp. The Lamp has 4 Feet, which closely take in the Iron A between t

" that in every Experiment the Flame may exactly come against the Mid- Annotat.
" dle of the Bar; but the Bottom of the Lamp must not touch the Lect. V.
" Iron, which in such a Case would be heated, and by its lengthening
" would disturb the Experiment: And thus, in no Experiments was it
" at all warm'd. The Distance between the Bottom of the Bar and
" the upper Surface of the Lamp, must be half an Inch, that the
" Cotton Wicks that stand up $\frac{3}{10}$ of an Inch, may give an exact
" and equal Flame. The Cotton Threads must be very fine and even,
" and 5 of them twisted together must make a Wick of about $\frac{5}{100}$ of an
" Inch in Diameter. I took particular Care of every Circumstance in ma-
" king Experiments with this Machine; because the least Omission might
" lead one into great Errors: For if the Wick is drawn up too high thro'
" the Cover of the Lamp, the Flame will be too large, which shews the
" Necessity of keeping it to the same Height: Likewise in Experiments,
" that are made with highly rectified Spirit of Wine, if there was not
" always an equal Quantity put into the Lamp, the Flame would be
" supply'd too fast or too slow, and so the Heat would vary; therefore I
" took care always to make my Experiments with the Lamp half full of
" Spirits, whereby I had the best Flames, which were of a cylindrick
" Form, from the Lamp to the Metal which they heated; only they
" widen'd a little at Top—the Diameter of their Base was $\frac{25}{100}$ Parts of
" an Inch. And to prevent the Motion of the Air, or the Breath of one's
" Mouth from moving the Flames, I cover'd the whole Instrument with
" a Glass; except just the Dial-Plate, which came up above the Glass,
" the better to observe the Index.

" This being all ready, I proceeded to examine how much Iron, Steel,
" Copper, Brass, Tin, and Lead were expanded by one Flame: then
" how much they were expanded by two; then by three; then by four;
" and lastly, by all the five Flames: Then also, whether there was any Dif-
" ference when two Flames burned next to each other, or two more
" distant Flames.

" One Day when it began to freeze, and *Farenheyt*'s Thermometer was
" at 32 Degrees, the Wind in the West, the Sky clouded, and the Mer-
" cury in the Barometer stood at 29 $\frac{4}{12}$, I laid the Metals above-men-
" tioned close to one another, upon a Stone, that they might be cool'd
" equally: Then I applied them successively to the *Pyrometer*, and having
" first lighted one Flame, I observ'd their Expansion: Then having taken
" them off from the *Pyrometer*, I set them a cooling till they became as cold
" as at first, and try'd them on the *Pyrometer* with two Flames; and so
" on till I tried the Effect of all the five Flames upon them. For Brevi-
" ty's sake, I have digested into the following Table what those Effects
" were. The Degrees of Expansion are mark'd in Parts, of which each
" is the $\frac{1}{12500}$ Part of an Inch. It is to be observ'd of Tin, that it
" will easily melt, when heated by two of the Flames burning next to
" one another, therefore one cannot try it with more Flames than two.

Annotat. Lect. V. " Lead commonly melts with three Flames, which are next to one another, provided they burn long.

" A TABLE;

" *Shewing how Metals are expanded by the Heat of Flames* " *of Spirit of Wine, different in Number, but equal* " *in Bigness.*

Expansion by one Flame in the Middle.	Iron 80	Steel 85	Copper 89	Brass 110	Tin 153	Lead 155
By two Flames in the Middle next to one another.	117	123	155	220		274
By two Flames 2½ distant from one another.	109	94	92	141	219	263
By three Flames in the Middle next to one another.	142	168	193	275		
By four Flames in the Middle next to one another.	211	270	270	361		
By all the five Flames.	230	310	310	377		

" These Experiments were made several times over; and tho' they did " not always agree, yet they never differ'd so much as five Degrees, " which is so little as not to be worth Notice; and this may be owing " to the Play of the Teeth, which cannot be avoided in these Experi- " ments. But I always took the Mean of a great Number of Experi- " ments.

" From

Annotat. Lect. V.

" From these Experiments it appears at first View, that Iron is the " least rarified of any of these Metals, whether it be heated with one or " more Flames: And therefore is the most proper for making such Ma- " chines or Instruments, which we would have to be least altered by Heat " and Cold.

" Therefore it is very proper to make the Rods of Pendulums for " Clocks, of Iron: They are not so good of Steel; and much worse of " Brass; yet sometimes they are made of Copper, because it is not so " liable to rust; but yet, that is wrong. So likewise the Measures of " Yards or Feet should generally be made of Iron, that their Length may " be as near as possible the same, Summer and Winter.

" 2dly, The Expansion of Lead and Tin by only one Flame, is pretty " near the same.

" 3dly, The same Flame expands Tin and Lead almost the double " of Iron; for their Expansions are as 155 to 80; that is, nearly as 2 " to 1.

" 4thly, The Flames which are next to one another, and act upon the " metalline Rods, cause a greater Rarifaction, than when they have a " sensible Interval between them: For two neighbouring Flames expanded " Iron 117 Degrees, and two farther asunder, but 109; and the same " was observed in all the other Metals.

" Now, this happens, because all the Parts of the Fire do not rise di- " rectly up and apply themselves to the Rods over them; but some fly " off sidewise from every Part of the Flame. Now, when two Flames " are at a distance from one another, the lateral fiery Parts fly off from " the Flames, and do not act upon the Metals; whereas, when they are " near together, those Parts which fly off from the Side of one Flame " towards the other, are reflected back from that Flame, as from a *Spe-* " *culum*, and coming back into their own Flame again, are driven up- " wards, and enter into the Metal, which thereby receiving a greater " Quantity of Fire, must consequently swell the more.

" 5thly, Now let us compare together the Expansions of the same " Metal, produced by one, two, three, or more Flames: Two Flames " did not give double the Expansion of one; nor three Flames three " times the Expansion, but less; and the Expansions differ so much the " more from the *Ratio* of the Number of the Flames, as there are more " Flames acting at the same time. I shall hereafter give the Demonstra- " tion of that *Phænomenon*; now I shall only give the Proportions of the " Expansions observ'd in the following short Table.

Annotat. Lect. V.

In Iron.	In Steel.	In Copper.	In Brass.	In Lead.
80:117 :: 1:1 37/80	85:123 :: 1:1 38/85	89:155 :: 1:1 66/89	110:200 :: 1:1 90/110	153:263 :: 1:1 110/153
80:142 :: 1:1 52/80	85:168 :: 1:1 83/85	89:193 :: 1:2 15/89	110:275 :: 1:2 55/110	
80:211 :: 1:2 51/80	85:270 :: 1:3 15/85	89:270 :: 1:3 33/89	110:361 :: 1:3 31/110	
80:230 :: 1:2 70/80	85:310 :: 1:3 55/85	89:310 :: 1:3 43/89	110:377 :: 1:3 47/110	

" 6thly, Before Metals are, from the same Degree of Cold, brought " to melt, they are not equally expanded; but some are more expanded, " and some less. For Tin began to run when it was rarefied 219 De- " grees: But Brass, which was expanded 377 Degrees, was far from be- " ing red-hot, and consequently very far from melting; and Copper was " expanded 310 Degrees, whereas perhaps it must have been expanded " twice as much before it could melt.

So much translated from Dr. *Muschenbroek*, * serves to shew some of the many Uses of the *Pyrometer*; which I have improved by the following Changes.

1st, Instead of square Rods of Metal, to make the Experiments upon, I use cylindrick ones; because I can more certainly have them of the same Size, by drawing them through the same Hole of a strong Steel Plate, such as Wire-drawers use: And by that means, I am sure, that the Bulks of the Metals compar'd are equal.

2dly, Instead of the Pinion F, I have a small Roller of Steel truly round, but not polish'd, and filed on the Surface in the Direction of its Axis, so that it may be consider'd as a little Wheel with an infinite Number of Teeth: Then the Wheel G upon the same Axis has no Teeth, but only a Groove to receive a fine Watch-chain (or even an Horse-hair) whereby it carries round a Roller at H, with a little Groove, upon the Axis of which Roller the Index is fix'd at the upper End I.

3dly, That the Chain whereby G carries H, may be neither too slack, nor too tight, the whole Dial Plate (which in my *Pyrometer* is square) and the Cock and Pinion H are driven towards, or from the Wheel G, by a Screw fastened to the upper Frame-Plate, which Plate receives the Top of the Pillars and Arbor of G, whilst the Dial-Plate slides upon it.

N. B. *There is no such Plate under the Dial-Plate in Dr.* Muschenbroek's Pyrometer.

Pl. 29. F. 2. 4thly, Instead of the Rack NL (*Fig.* 2.) I have a long thin Plate of Steel about $\frac{1}{100}$ Parts of an Inch broad, rough filed, so as to move the first Roller F, by rubbing against it. It is tempered of a Spring Temper, and a little curv'd, so as to have its convex Side bear against F; but

* See the Notes of his Latin Translation of the Experiments of the *Academia del Cimento*, Part the 2d, from *Page* 12, to *Page* 18, &c.

which

when it is fasten'd to the Rod at N, there is a Spring fix'd to the lower Brass Plate, which draws it streight and tight by its End L, in the Direction N L, and instead of the Cocks P, P (*Fig.* 3.) there are two Pullies placed horizontally whose broad vertical Grooves receive and direct the Steel Plate or Ruler, that supplies the Place of the Rack. Annotat. Lect. V. Pl. 29. F. 3.

5thly, Instead of the Watch-Spring to support the Bars, I have a small Brass Roller of $\frac{4}{10}$ of an Inch Diameter, whose Axis is horizontal; and this Roller is raised up from its Pedestal, that is fix'd to the Iron at W, by a Screw, so as to come under every Rod of Metal, and support it at its End N.

The 6th *Figure* represents, the lower Frame Plate, with my Alterations. Pl. 29. F. 6.

B N is the round Rod of Metal to be tried, with the Steel Plate made fast at N, by the Pin Q. This Steel Plate, whose natural Situation is represented by the pointed Line N *l*, is here kept in the strait Line N L, by means of the Spring S L, which hooks in the End of it at L, drawing it in the Direction N L; and it is directed by the Grooves of the Pullies P, P, so that its Side, that is inclin'd to be convex when left at liberty, now presses on the Roller H, which it turns by its Friction, as it accedes towards, or recedes from L. The pointed Circle G *g*, represents the Wheel above on the same Axis as H: *g i* G, the Watch-Chain, carrying round the last Roller *i*, an Index *k i k*, as in Dr. *Muschenbroek*'s *Pyrometer*. The rest is easily comprehended from what has been said, and a Sight of the 2d *Figure*, which represents the supporting Brass Roller, rais'd and depressed, by turning round the screw'd Plate *p p*. Pl. 29. F. 2.

N. B. *There is no Inconvenience in using the Horse-hair if you take care to place its Knot at* G; *for as the Motion is made in the Direction* G, m g, *the Knot will never go so far as* g, *and therefore in all its Motion bear equally in the Groove of the Wheel* G.

By using Rollers, the Shake of Teeth is wholly avoided in my *Pyrometer*; so that the Index begins to move the very Moment, that Heat is apply'd to the metalline Rods; and if you blow out the Flame, that very Instant the Index begins to return, which cannot happen where there are Teeth.

I mean the quick Return when the Heat is remov'd. I must own, that if we consider the Motion of the Index only one way, when once the Teeth take, and it begins to move, it will go on regularly; therefore Dr. *Muschenbroek*'s Experiments may be depended upon. However I shall try them again with my *Pyrometer* the first Leisure. I must also do the Doctor the Justice to own, that he has signified to me in one of his Letters

Annotat. Lect. V. Letters, that if he had not loſt his Workman, he would have made a new *Pyrometer* without Teeth.

12. [73. *Who aſſert, that the Earth is an oblong Spheroid (like an Egg) higher at the Poles than the Æquator*, &c.] Monſieur *Caſſini* ſays, that the Earth is an oblong Spheroid, higher at the Poles than the Æquator, making the Axis longer than a Diameter of the Æquator about 13 *French* Leagues, which he deduces from comparing his Father's Meaſures of the Meridian from *Paris* to the *Pyrenean* Mountains with thoſe of Monſ. *Picard*, of which an Account may be ſeen in the *Memoirs* of the Royal Academy for 1713; then having afterwards continued the Meridian, which is drawn through *France*, from *Paris* to *Dunkirk*, he ſtill draws Conſequences to prove the Earth an oblong Spheroid; but then makes the Axis exceed the æquatorial Diameter 34 Leagues.

Now, in order to prove this, M. *Caſſini*, taking the Meaſures above-mentioned to be exact enough, not only to determine the Magnitude of a Degree of the Earth, correſponding with a Degree of a great Circle of the Heavens; but alſo to ſhew, even the Difference in the Degrees of the Earth (reckoning thoſe, that were meaſur'd in the South of *France*, to exceed thoſe towards the North, by a certain Number of Fathoms and Feet) demonſtrates, that if the Degrees of the Earth are longer towards the Æquator than the Poles, the Plane of the Meridian muſt be an Ellipſe, whoſe long Axis is that of the Earth.

Monſieur *Caſſini*'s Demonſtration is certainly right; but the Meaſures taken could not, in the Nature of Things, determine any thing about the different Length of Degrees ſo near to one another; becauſe as that Difference in one of the Suppoſitions is but of 11 or 12 Fathoms, and in another of about 31 at moſt, the Latitude muſt be taken to an Exactneſs, exceeding to the Nature of any Aſtronomical Inſtruments yet made, and much more than that of the Inſtruments made uſe of by the Gentlemen, who carried on the Meridian.

For in the firſt place, the Inſtrument, with which they took the Obſervations for the Latitude at the two Ends of the Meridian, was a 10 Foot Sector, where the 200th Part of an Inch anſwers to 8 Seconds of a Degree. Now the 200th Part of an Inch, being one of the leaſt viſible Parts that we can ſee in a divided Line, they could not take an Angle nearer than that; nay, their Inſtrument, according to their own Deſcription of it, was divided but to every 20 Seconds. They allow, that 16 Toiſes (or Fathoms) upon the Surface of the Earth, anſwer to 1 Second in the Heavens; and they don't pretend to have taken an Obſervation nearer than to about 3 Seconds, which therefore cannot determine a Difference leſs than 48 Toiſes; whereas the Degrees are only ſuppoſed to decreaſe (at moſt) 31 Toiſes each from *Collioure* (the Southern Part of their Meridian) to *Dunkirk*. But an Error of 8 Seconds would make a Difference of 128 Toiſes on the Surface of the Earth, above 10 times greater than the Difference of the Degrees in the firſt Suppoſition, and

4 times

4 times greater than that Difference in the last. Besides, the Latitude was not observ'd in the intermediate Places between *Paris* and *Collioure*, with the above-mention'd Instrument of 10 Foot *Radius:* but they made use of a Quadrant, whose *Radius* was only 39 Inches, and sometimes an Octant of three Foot *Radius*. Nay, they say themselves in their Account, that it is not the Observations made at the Ends of the Meridian, that we are to deduce the Difference of the Length of a Degree from, but the Altitudes taken at several Places between the Extremes; and if we grant that they can take an Angle very well to 4 or 5 Seconds with the greater Instrument, they cannot come nearer than 12 or 15 Seconds with the Quadrant or Octant, which we must depend upon for the Difference of the Measure of Degrees. So that upon the whole, we are to determine a Length of 31 Toises, by an Instrument, which is liable to err above 200. Annotat. Lect. V.

And indeed, upon a strict-Examination of the Account of the Measures of Degrees, I found many Errors and Inconsistencies; particularly in the measuring of the Height of Mountains, where 9 Observations of the Barometer for measuring the Height of them in the South of *France*, which are said to confirm their trigonometrical Measures, so entirely clash with one another, as to determine nothing at all. Nay, the Conclusion, that Degrees on the Earth increase in going towards the Æquator, is only drawn from some pick'd Observations; for if they are all compar'd together, the Degrees will appear from thence to increase in going Northwards.

Now, abstracting from Observations, the ingenious Mons. *Mairan* has endeavour'd to prove, that in an oblong Spheroid, Pendulums must be made shorter to swing Seconds at the Æquator, than in greater Latitudes: but the Principles he has assumed, are not to be allow'd him; and if they were, it would follow from them, that a Pendulum swinging Seconds at *Paris*, must be shorten'd an Inch to make it swing Seconds at the Æquator: whereas the Fact is, that such Pendulums carried from *Paris* to the Æquator, are only shorten'd $\frac{1}{10}$ of an Inch. Therefore, M. *Mairan*, by proving a great deal too much, proves nothing in this Case. Those that are curious to know all that relates to this Dispute, may consult the *Philosophical Transactions*, N°. 386, 387, 388 †, and 389; in which, I think, I have fully prov'd the oblate spheroidical Figure of the Earth, according to Sir *Isaac Newton*, and shewn the oblong Spheroid to be impossible.

† *N. B.* There are some *Errata* in my second Dissertation, which are corrected in the last Abridgment of the *Transactions*, by Messieurs *Reid* and *Gray*.

F I N I S.

At the Desire of several Friends, I have here subjoin'd a short Description of my Planetarium; *an Instrument, which I made lately, to shew the Motion of the heavenly Bodies.*

Description of the PLANETARIUM.

MACHINES and Movements for representing the Motions and Appearances of heavenly Bodies, have been justly esteem'd in all Ages; especially since the *Copernican* System has been generally receiv'd; not only as the most probable Hypothesis, but as Sir *Isaac Newton* has, from the Laws of Gravity, prov'd it the true System of the World *—at least, a System of so much of the Universe as relates to us Inhabitants of the Earth; that is, so far as we can discover, not only by our Eyes, but by means of Telescopes.

By Help of such Machines, a great many Persons, who have not time to apply themselves to the Study of Astronomy, and yet are desirous to be acquainted with the celestial Appearances, in a few Days may get a competent Knowledge of several Phænomena, and especially, be cured of the common Prejudices against the Motion of the Earth.

When such Astronomers, as are skill'd in Mechanicks, have made or contriv'd these kind of Movements, they have answered the Ends propos'd, by shewing so much as the Machine was intended to represent; but, when Instrument-makers, without a competent Knowledge of Astronomy, or proper Instructions from Persons of Skill, have made improper Additions to such Machines as had been contriv'd by Astronomers (under pretence of great Improvements) merely to make them pompous and costly; the true Intention of the first Inventors has been destroy'd, and the Buyers have paid dear for false Notions of Astronomy.

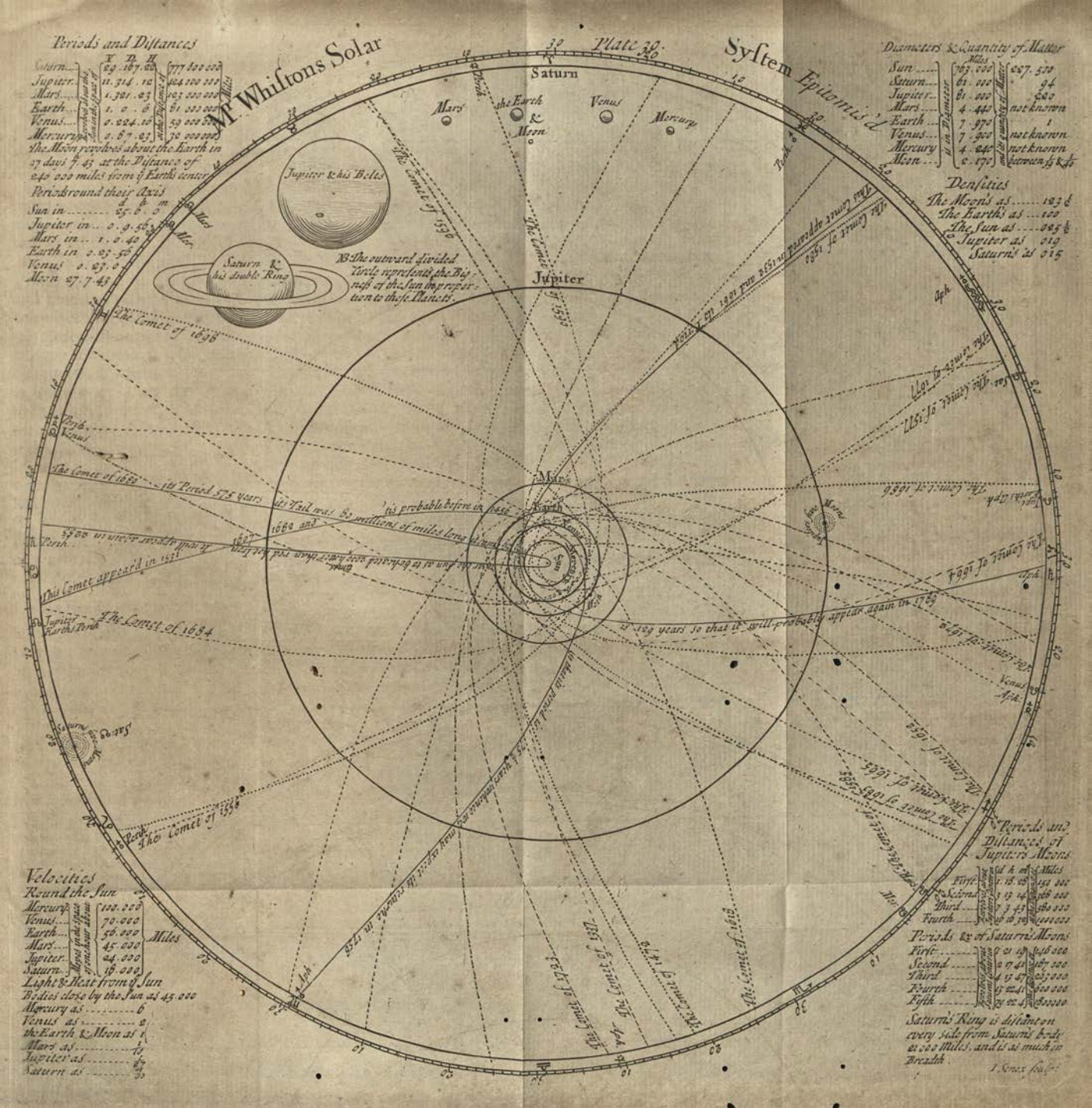
Mr. *George Graham* (if I am rightly informed) was the first Person in *England*, who made a Movement to shew the Motion of the Moon round

* We must not forget, that the Rev. and Ingenious Mr. *James Bradley*, *Savilian* Professor of Astronomy at *Oxford*, has demonstrated it also of late from Astronomical Observations.

the

Mr Whistons Solar System Epitomis'd

Plate 30.



the Earth, and of the Earth and Moon round the Sun, about 25 or 30 Years ago. In this Machine every thing, that was ſhewn, was well and properly executed——as the Phænomena of Day and Night, and their gradual Increaſe and Decreaſe, according to the Seaſons, the Places of the Earth where the Sun is ſucceſſively vertical, and ſeems to deſcribe its Parallels, the real annual Motion of the Earth, which gives the Sun an apparent annual Motion, the Rotation of the Sun about its Axis, the periodical and ſynodical Month, the ſolar and ſidereal Days, the ſucceſſive Illumination of all the Parts of the Moon, *&c.* This Machine being in the Hands of an Inſtrument-maker, to be ſent with ſome of his own Inſtruments to Prince *Eugene*, he copied it, and made the firſt for the late Earl of *Orrery*, and then ſeveral others, with Additions of his own. Sir *Richard Steele*, who knew nothing of Mr. *Graham*'s Machine, in one of his Lucubrations, thinking to do juſtice to the firſt Encourager, as well as the Inventor, of ſuch a curious Inſtrument, call'd it an *Orrery*, and gave Mr. *J. Rowley* the Praiſe due to Mr. *Graham*.

Since that Time, the *Orreries* have been much in vogue, and executed in the moſt ornamental Manner, firſt by Mr. *Rowley*, then by other Inſtrument-makers; but the Addition of the reſt of the Planets and the Satellites, reckon'd by the Ignorant to be an Improvement, does but give confus'd Ideas, and falſe, as to Diſtances and Proportions of Bigneſs; which will always be ſo, whilſt the Orbit of the Moon about the Earth is fix'd to the ſame Machine, that has the primary and ſecondary Planets. The general Syſtem (in which our Earth has but a ſmall Part) ought to be by itſelf—The Sun, Moon, and Earth, ought alſo to be ſhewn by themſelves;——and a Syſtem of a primary Planet (as Jupiter, for Example) with its Satellites in their true Proportion of Magnitudes and Diſtances, ſhould alſo be repreſented by itſelf.

Theſe Conſiderations (and the Deſire of giving a true Notion of the celeſtial *Phænomena*, in the plaineſt and moſt expeditious manner, to ſuch Perſons as do me the Honour to frequent my Courſes of Experimental Philoſophy) induc'd me to contrive and make a Movement with all the Requiſites above-mentioned, that might ſhew in true Proportion what can be ſo repreſented in a celeſtial Machine.

I have made it to conſiſt of ſeveral Parts to be put on and taken off ſucceſſively; and have call'd it a PLANETARIUM, whoſe Deſcription I am now going to give:. But before I enter upon it,

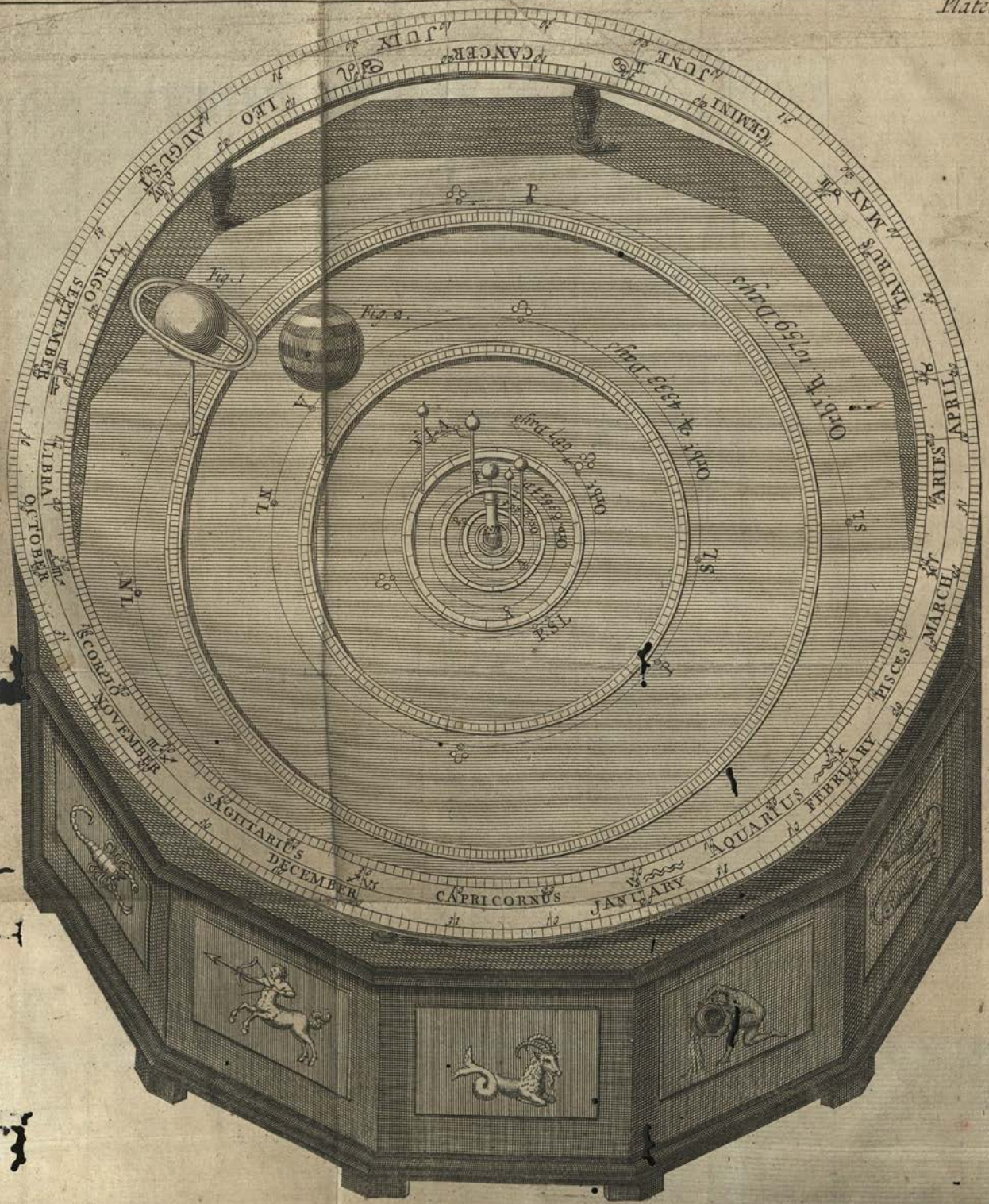
I muſt deſire the Reader to caſt his Eye upon the 30th *Plate*, which is Mr. *Whiſton*'s Scheme of the *Solar Syſtem* epitomiz'd, whereby he will make himſelf acquainted with the mean Diſtances, Magnitudes, Periods, Quantities of Matter, Revolutions about the Axes, Denſities, and mean Velocities in their Orbits, of all the primary and ſecondary Planets; and the Orbits of ſeveral Comets, the Periods of three of them. Only I muſt obſerve ſome Things relating to this Scheme, which (if not conſider'd) might lead into Error. Pl. 30.

1. Tho' we see the Distance of the Planets from the Sun set down in Miles, we are not to imagine, that we can measure their Distances to a few Miles, as we really can the Moon's Distance; because the Semi-diameter of the Earth (whose Quantity is known in Miles) being about $\frac{1}{60}$th Part of the Moon's Distance, is a sufficient Measure for the Moon's Distance; whereas it is scarce a twenty thousandth Part of the Sun's Distance, therefore very insufficient for a Measure of that Distance. There are indeed other Methods of coming at the Sun's Distance, as by the Parallax of Mars, but none of them can ascertain it to a Million of Miles. We must stay till the next Transit of Venus across the Sun's Disk in the Year 1761, *May* the 26th, when we may, by observing that Transit, find out the Sun's Distance within a 500th Part of the whole Distance, as Dr. *Halley* has shewn in the *Philosophical Transactions* N°. 348. p. 454. and even that Advantage will bring us but within 15 or 16000 Miles of the Truth. However, we know the proportionable Bigness and Distances of the Bodies revolving round the Sun; that is, if the Sun's Distance from the Earth be, as here set down, 81 Millions of Miles, all the other Distances and Diameters are right: We know those Distances in Diameters of the Sun, which is the Way that most Astronomers express them; for whenever that comes to be known, all the rest will be known too, and therefore their respective Proportions may be represented by Schemes and Machines: But not exactly what Proportion they bear to the Moon and Earth.

2. Venus is here set down, as turning upon her Axis in 23 Hours; but since Mr. *Whiston* publish'd his Scheme, *Signior Bianchini* has observ'd, that she is 24 Days and 8 Hours revolving round her Axis, which Axis is inclin'd to the Plane of her Orbit in an Angle of 15 Degrees.

3. The Moon's Velocity in her Orbit, which is the only Planet, whose real Velocity we know, has been omitted here. She moves 2300 Miles in an Hour. The other Planets only move in an Hour the Miles set down, supposing the Sun at the Distance of 81 Millions of Miles from the Earth.

To give a more easy Idea of the Distances and Diameters of the Bodies, we will take them at a Mean in round Numbers; and supposing the Distance from the Earth to the Sun, to be divided into 10 Parts, Mercury will be distant from the Sun 4 of those Parts, Venus 7, Mars 15, Jupiter 52, and Saturn 95. If we call the Sun's Diameter 100; that of ♄ will be $7\frac{5}{10}$; of ♃ 10; of ♂ $\frac{55}{100}$; of ☉ 1; of ♀ 1; of ☿ $\frac{54}{100}$, and of the Moon $\frac{285}{1000}$; and the Satellites of ♃ and ♄ are conjectur'd to be about as big as our Earth.


JULY
CANCER
JUNE
GEMINI
MAY
TAURUS
APRIL
ARIES
MARCH
PISCES
FEBRUARY
AQUARIUS
JANUARY
CAPRICORNUS
DECEMBER
SAGITTARIUS
NOVEMBER
SCORPIO
OCTOBER
LIBRA
SEPTEMBER
VIRGO
AUGUST
LEO
Fig. 1
Fig. 2
Orb. ♃ 4333 Days
Orb. ♄ 10759 Days
Orb. ♂ 687 Days
Orb. ♀ 224 Days
P
A
N.L.
S.L.
P.S.L.

4. Tho' the different Curvature of the Parts of the Ellipſes, in which the Planets move, is not ſenſible enough to be expreſs'd even in a larger Scheme; yet the Excentricities might have been expreſs'd, but that it was omitted to avoid Confuſion among the Comets Orbits. The Excentricities of the Planets may be expreſs'd by ſuppoſing the Line of the mean Diſtance of each, divided into 1000 equal Parts, and then the Excentricities will be the following Numbers of thoſe Parts.

		Equal Parts			
Imagine the mean Diſtance of	Saturn	divided into 1000	their Excentricity is 55	of thoſe Parts	The Moon's Excentricity is $\frac{55}{1000}$ Parts of its Diſtance from the Earth.
	Jupiter	1000	48		
	Mars	1000	93		
	the Earth	1000	17		
	Venus	1000	7		
	Mercury	1000	210		

The PLANETARIUM, (*Plate* 31.)

Is fix'd in a Frame of Ebony about 6 Inches high, and 3 Foot in Diameter, contain'd by 12 vertical Planes, on which are repreſented the 12 Signs of the Zodiac. The upper Surface is flat of poliſh'd Braſs, on whoſe outward Circumference are ſcrew'd in ſix Braſs Pillars, which ſupport a large flat ſilver'd Ring repreſenting the Ecliptick, with ſeveral Circles drawn upon it. The three innermoſt are divided into 12 Parts for the Signs of the Zodiack, each of which is divided into 30 Degrees; and among thoſe Degrees are grav'd in their proper Places, the *Nodes*, *Aphelia*, and greateſt North and South Latitudes of the Planets. Between the next two Circles are mark'd the Cardinal Points. The next three Circles, have the Months and Days of the Months, according to the *Julian* Account; and the three laſt have them likewiſe engrav'd, according to the *Gregorian* Account. But in the 31ſt *Plate* I have omitted the *Julian* Account, as I have omitted the *Gregorian* in the 32d *Plate*. Upon the Braſs Surface of the Machine, are graduated Silver Circles, which carry the Planets (repreſented by ſilver'd Balls) upon Arbors or Stems, that raiſe them up to the Height of the Plane of the Ecliptick; and by turning about the Handle or Winch of the *Planetarium*, all the Planets move at their proportional Diſtances from a little gilt Ball in the middle, which repreſents the Sun, and perform their Revolutions according to their periodical Times. There are fix'd Indices of ſteel'd Steel, which ſhew the Longitudes of the Planets, by pointing to the Diviſions of the ſilver'd Rings or Circles, as they move round. But as thoſe Circles, being concentrick, give only the mean Diſtances; the true Orbits, according to their Excentricities, are graved on the Out-ſide of each Circle, with the periodical Times taken from the Tables, to ſhew what the Revolutions are nearer than can be

perform'd by any Machine. The *Nodes* and *Aphelia*, with the Places of greatest North and South Latitudes, are also mark'd on those Orbits. As the Distances are in their true Proportions to each other; so likewise are the Bodies of the Planets in their just Proportions to one another. But it cannot be expected, that the Diameters of the Planets should be in Proportion to the Diameters of the Orbits; because taking Jupiter under 3 Inches Diameter, and the Earth a little more than a quarter of an Inch, as in my Machine, it would require the System to be of the Bigness of a Mile and $\frac{3}{4}$, the Orbit of Saturn about 9000 Feet in Diameter, and so of the rest; which would make the Machine 3000 times bigger than it is. And if we would suit the Bodies to the Machine as it is, the Bodies must be 3000 times less, which would render them all invisible, but the Sun; and that would be less than $\frac{1}{100}$th Part of an Inch. For this Reason, I could not put in a Ball big enough to represent the Sun in respect of my Planets; but we are to suppose the Sun (in respect of them) as big as the inner Circle of the Silver Ring, which represents the Ecliptick.

Now in the ORRERIES *(as they call them) there is shewn no Proportion of the Orbits to one another; nor of the Bodies to one another.*

As the Orbit of the Moon, and the Orbits of Jupiter and Saturn, are quite lost in this Proportion of the Orbits of the primary Planets, much more are the Satellites themselves; therefore the Satellites are not put on in this Position of the Machine. But Saturn's Ring is join'd to Saturn's Body, according to its Proportion, and the Inclination of its Plane to the Plane of Saturn's Orbit: And as the Planet is carried round, the Ring always moves parallel to itself, as it does in the Heavens. Thereby we see why the Inhabitants of the Earth, in one Revolution of Saturn, see
Pl. 31. F. 1. the Ring twice in the most open Situation of the *Ansæ*, as in *Figure* 1 *, and twice, as if it had no Ring, that is, when the Edge of the Ring is towards the Earth (the Plane of the Ring going through the Observer's Eye) and the successive increasing and decreasing of the visible Bigness of the *Ansæ*:

The 2d Figure represents Jupiter with his Belts, and the Spots whereby his Revolution has been observ'd.

When the Spectators have look'd long enough upon the Machine, to have a compleat Idea of the proportionable Bigness of the Planets; Jupiter and Saturn are taken off, and then there are put on another Jupiter and another Saturn 3 times less than the former, in order to put Satellites about them (and at the same time the Moon is joined to the Earth) and shew how the Satellites accompany their primary Planet in its Course round the Sun. These Satellites, which are Pearls upon crooked Stems, do not turn by Clock-work round their Primaries, (as has been done in some of the large *Orreries)* but are only set by the Hand; because, to do it, would be only a needless Expence, to give a false Notion of their Bigness, Distances, and Inclination of their Orbits, in respect of their Primaries.

ries. But to give a right Notion of Jupiter and his Satellites, and Saturn and his Satellites; there is ſhewn for each of theſe Planets a Syſtem a-part, where the Diſtances from the Primary, and the Bigneſs of the Satellites, are expreſs'd: And in this Syſtem, tho' Jupiter is but of about an Inch Diameter, the outermoſt Satellite is as far diſtant from Jupiter's Center, as Saturn is from the Sun in the Machine, which ſhews the Inconſiſtency and Diſproportion of making the Satellites to move round Jupiter in an *Orrery*. Saturn's Satellites are ſtill more improperly put in; becauſe four of them move in Orbits very much inclin'd to Saturn's Ecliptick, (*viz.* in an Angle of above 30 Degrees) and the 5th has its Orbit almoſt in the ſame Plane as Saturn's Ecliptick, with a Diameter greater than the Diameter of the whole *Planetarium*, even when Saturn is 3 times leſs than the Saturn of the *Planetarium*.

The next Thing put on, is a Contrivance to ſhew, that all the Confuſion of the Planets Motions in the *Ptolemaic Hypotheſis* (call'd their Stations and Retrogradations) is not really, but apparently ſo, in the *Copernican*, or true Syſtem of the World. And this is done by two Steel Indices, one of which being always applied to the Sun, and ſucceſſively to the Top of the Stem of the Planet to be examin'd, whilſt the other is apply'd to the Earth (as a Center) and the ſaid Planet; by turning the Handle of the Machine, the *Heliocentrick* * and *Geocentrick* Places of the Planet are ſeen on the Ecliptick at the ſame Time; ſhewing, why the Planets ſeem to go backwards and forwards when view'd from the Earth; tho' they go all the while regularly from Weſt to Eaſt, as they would be ſeen from the Sun.

Then to ſhew the true Inclination of the Orbits of the Planets; having taken off all the ſilver'd Balls, which repreſented the Planets before, and their Stems, ſix Orbits of gilt Braſs Wire are put on, by means of Pillars ſet upon the Orbits, that are grav'd upon the great Plane of the Machine, ſo high, as to have one Part above, and the other below, the Plane of the above deſcrib'd Ecliptick; whilſt the Nodes, in two oppoſite Parts of the Wire Circles are exactly in the Plane of the Ecliptick, and the *Aphelia* and *Perihelia*, according to the right Diſtances from the Sun, directly over the Letter A and P in each Orbit grav'd below. For Example, in Jupiter's Orbit, two Pillars of equal Height (and the Height of the Plane of the Ecliptick above the great Plate) ſtand upon the Holes at ☊ and ☋, to ſuſtain that Part of the Orbit, which is in the Node; a Pillar a little higher ſtands at N L, and one a little lower at S L. The Heads of theſe Pillars being round, repreſent Jupiter at each Node, and at the greateſt North, and the greateſt South Latitudes. Beſides theſe four Balls, there are two others moveable upon the Wire, to be plac'd, one at the *Aphelion*, and other at the *Perihelion*, or in any other Places to ſhew the Longitude or Latitude of the Planet at any Time; for one of the Steel Indices above-mentioned, laid over the

* Planets ſeen from the Sun, or from the Earth.

Center

Center of the Sun and the Planet, ſhews its *Heliocentrick* Longitude, at the ſame time, that a moveable Arc of a Circle vertically ſtanding upon the Degree of the Ecliptick, which the Index is over or under, ſhews the *Heliocentrick* Latitude: The *Geocentrick* Longitude and Latitude, being ſhewn in the ſame manner, when the Index comes from the Earth. *The other five Wire Orbits are fix'd the ſame Way.* There is alſo a double Index opening like a Pair of Compaſſes, the Hole of whoſe Center being plac'd on the Earth's Stem, the Legs taking in the Orbits of Mercury or Venus, the greateſt Elongation of thoſe Planets is ſhewn by a graduated Arc, that meaſures the angular Diſtance of thoſe Legs.

N. B. *The greateſt Elongation of* ☿ *is about* 28 *Degrees, and of* ♀ *about* 48 *Degrees.*

There is alſo a Wire bent into a parabolick Figure, and ſupported by three Pillars, to ſhew the lower Part of a Comet's Orbit; that is, ſo far as we can obſerve of it. The Head of each Pillar repreſents the Comet at that Place; and the fine Silver Wires extended from the Comet in its *Perihelion*, ſhew the Tail of the Comet when it is biggeſt.

As the Obſervations of the Eclipſes of Jupiter's Satellites are of great Uſe in Aſtronomy; I have made a particular Syſtem of Jupiter and his Satellites applicable to the Machine, whereby the Diſtances, Bigneſſes, and true Periods of the Satellites, in reſpect to Jupiter, are truly ſhewn; that when we have taken off the Sun and the Wheel-work, which carries ☿, ♀, ☉, and ♂, and put in the Wheel-work belonging to the Syſtem of Jupiter (each of theſe being put on or off in one Piece) ſome of the Wheels, that were before in the *Planetarium*, help to move Jupiter and his Satellites in ſuch manner; that if they be once ſet by the Tables, you will by turning the Winch, ſee what Eclipſes of any of the Satellites will appear at any Time. There is a Dial-plate with three Hands or Indices in one Corner of the *Planetarium*, where the Handle for this Purpoſe is applied, one of which Hands ſhews the Hours of Jupiter's Rotation about its Axis in our Hours, and the Hours of Jupiter (that is, dividing his Rotation once about its Axis into 24 Parts), the next Index ſhews the Hours of the Motions of any of the Satellites, and the third Hand ſhews the Days.

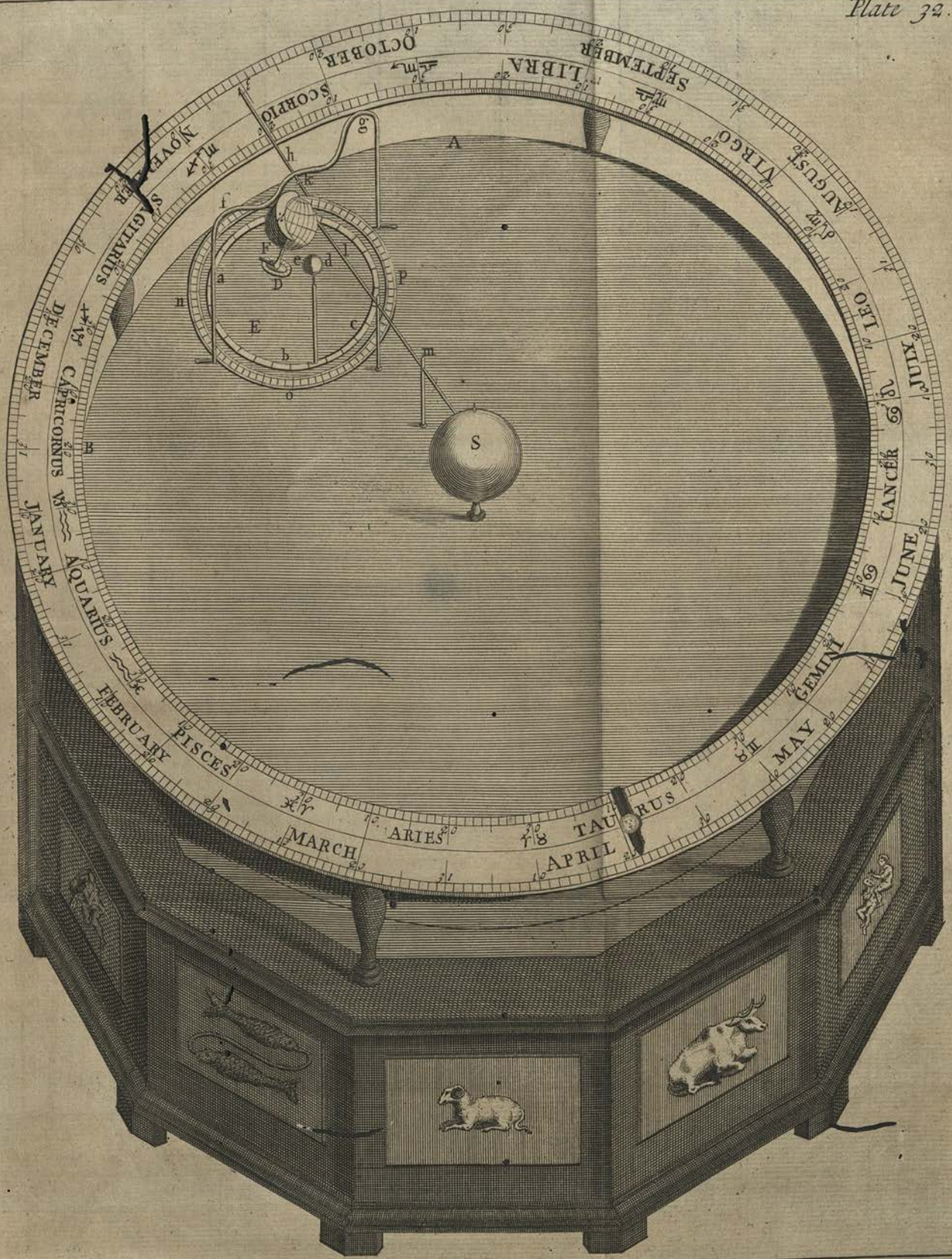
By looking at Jupiter and the Satellites, as they are mov'd by turning the Handle; together with the Uſe of the Lantern, to repreſent the Sun; we viſibly ſee the following Things, as they really happen.

1. The Immerſion; that is, when a Satellite enters into Jupiter's Shadow.

2. The Emerſion of a Satellite out of the Shadow.

3. When the Immerſions, or when the Emerſions are viſible.

4. When


OCTOBER
LIBRA
SEPTEMBER
SCORPIO
VIRGO
AUGUST
NOVEMBER
SAGITTARIUS
LEO
JULY
DECEMBER
CAPRICORNUS
CANCER
JUNE
JANUARY
AQUARIUS
GEMINI
MAY
FEBRUARY
PISCES
TAURUS
MARCH
ARIES
APRIL
A
B
S
P
D
E
F
J
a
b
c
d
e
f
h
k
m
n
o

4. When a Satellite is hidden by the Body of ♃, before it comes into, or after it is gone out of the Shadow.

5. When a Satellite may be ſeen to croſs the Body of ♃.

6. When a Satellite makes a Solar Eclipſe in ♃, by throwing its Shadow upon ſome Part of its Surface.

7. When Satellites eclipſe one another.

So much is ſhewn by the PLANETARIUM *in reſpect of the general Syſtem: As for the Repreſentation of the Phænomena of the Sun, Moon and Earth in reſpect to each other, there ought to be another Machine; but for Conveniency of Carriage, I have contriv'd to change the above-mentioned Machine into one for this Purpoſe, tho' there be ſome Trouble in making the neceſſary Changes.*

Plate 32. repreſents the *Planetarium* with only the Sun, Moon, and Earth.

1. The Ecliptick being ſet upon other Pillars, is raiſed about 2 Inches higher than it was before.

2. The Surface of the Machine within thoſe Pillars, is a Plate painted blue, inſtead of the poliſh'd Braſs before; becauſe the blue Colour of the Sky depends wholly upon the Earth's Atmoſphere, and not a blue Arch or Canopy at an immenſe Diſtance.

The Clock-work under this Plate, applying itſelf to the Clock-work within the Machine, and the Winch being fix'd to another Part of the Machine, the following Motions will be perform'd.

3. The Sun (which is here a gilt Braſs Ball of 2 Inches Diameter) turns round his Axis from Weſt to Eaſt once in 25 Days 6 Hours; every Turn of the Handle being now a Day, which was a Month in the general Syſtem, and the Hours being mark'd by an Index, and divided Plate, at the Bottom of the Axis of the Earth.

4. The annual Motion being ſtopp'd, the Earth (whoſe Center, as well as that of the Sun, is in the Plane of the Ecliptick) turns in 24 Solar Hours round an Axis, inclin'd to the Plane of the Ecliptick in an Angle of $66\frac{1}{2}$ Degrees from Weſt to Eaſt, not moving out of its Place. But when the annual Motion is not lock'd, then it turns about its Axis once in 23 Hours and 56 Minutes, and at the ſame Time is carried round in its annual Orbit, from Weſt to Eaſt alſo; its Axis is all the while remaining parallel to itſelf.

An

An Account of the Phænomena explain'd only by the Rotation of the Earth about its Axis.

The Earth is here repreſented by a ſilver'd Ball of an Inch Diameter, on which are drawn ſeveral Circles, as is done upon the common Globes, *viz.* the two Polar Circles, the Tropicks, the Æquator, the Ecliptick, and the Parallel of *London*; likewiſe 24 Meridians, one of which grav'd in thicker than the reſt, repreſents the Meridian of *London*, which Place is expreſs'd by a ſtrong Spot or Point in 51 ½ Degrees of North Latitude. From a rais'd and bent Wire-Arch *f k g* hangs a Plate like a Creſcent, wanting 47 Degrees of a whole Circle at Bottom, that it may not fall foul upon the Axis of the Earth. The Plane of this Plate does always paſs through the Center of the Earth, and facing the Sun, is always perpendicular to a Ray coming from the Sun to the Earth, and conſequently divides the enlighten'd from the darken'd Part of the Earth: Therefore I call it a *Solar Horizon*; and W, E mark'd upon it, ſhew the Weſt and the Eaſt.

In a Line ſuppos'd to be drawn from the Center of the Sun to the Center of the Earth, is fix'd upon two Props, the Solar Ray or Wire *m l*, to ſhew on what Part of the Earth the Sun ſhines perpendicularly at any Day, Hour, or Minute of the whole Year.

There is an Index *h*, which ſhews the Place of the Earth on the Ecliptick, and likewiſe oppoſite to it, there is another Index (with a little Sun grav'd upon it) to ſhew the Sun's Place in the oppoſite Part of the Ecliptick.

Taking hold of the blue Plate, the Earth muſt be brought to the firſt Degree of Capricorn, which will be ſhewn by its Index; then the Axis of the Earth (always making an Angle of 66 ½ Degrees, with the Plane of the Ecliptick) muſt be ſo ſet, that the North Pole may be towards the Sun, and a Plane paſſing through the Axis of the Earth, and Number XII, in the Dial-Plate D, may alſo paſs through the Sun's Center. This gives the Situation of the Earth for the longeſt Day in our Northern Hemiſphere; the Solar Index, which points to the firſt Degree of Cancer, pointing alſo to the 10th Day of *June*: Set the Meridian of *London* to the Solar Ray, and the Index of the Dial-Plate D at Bottom to XII. and you will have the Situation of *London* at Noon for the longeſt Day of the Year, where an Inhabitant being ſuppos'd to ſtand with his Face towards the South, will ſee the Sun before him at its greateſt Altitude.

Now, by gently turning the Handle of the Machine, the Point repreſenting *London*, or an Inhabitant of it, will (by the Rotation of the Earth) be carried away towards the Eaſt, while the Sun ſeems to move Weſtward, or towards the right Hand; and when *London* is brought to the Eaſtern Part of the Solar Horizon, you will ſee the Hour of Sun-ſet

ſhewn

ſhewn by the Index on the Earth's Axis and Dial-Plate D. Turn on, and *London* will move in the ſhaded Part of the Earth on the other Side of the Solar Horizon in reſpect of the Sun. When the Index is come to the other Number XII on the Dial-Plate, *London* is come to Midnight: As you turn on, *London* will come out from under the Solar Horizon at W, the Index below then pointing to the Hour of Sun-riſe, an Inhabitant perceiving the Sun as coming out of his Horizon in the Eaſt, or on his left Hand. You may then alſo obſerve, that in the whole Turn of the Earth about its Axis, the Solar Ray only points to the Tropick of Cancer; no other Countries, but thoſe that are under that Tropick, having the Sun vertical. And this would happen all the Year round, if the Earth had no annual Motion; neither would there be any Change of Seaſons, or Alteration of the Length of Days. Here by obſerving, how the Solar Horizon cuts the ſeveral Parallels, one may perceive, that all of them (except the Æquator) between the North and South Polar Circles, are unequally divided into diurnal and nocturnal Arcs, the former being greateſt on the North-ſide of the Æquator, and the latter (that is, ſuch as are behind the Solar Horizon) on the South-ſide of it. The Northern Polar Circle in this Situation, is always enlightened, being on the Sunny Side of the Solar Horizon, whilſt all the Regions contained within the South Polar Circle continue in the dark, notwithſtanding the Earth's diurnal Revolution; the annual only being capable to change their uncomfortable Situation of perpetual Darkneſs and Winter, to Day, Spring, Summer, *&c.*

Now, let the Movement for the annual Motion be unlock'd, and join'd to the diurnal; then by turning the Handle, the Ball repreſenting the Earth, will move truly as the Earth does, and repreſent all the *Phænomena* depending upon the two Motions.

Firſt then, whilſt it continues to turn upon its own Axis, once in a Day, it goes forward in the *Magnus Orbis* from Weſt to Eaſt, according to the Order of the Signs (or *in conſequentia*, as Aſtronomers call it) as is ſhewn by its Index, pointing ſucceſſively to all the Signs and Degrees of the Ecliptick, whilſt the Sun, round which it moves, ſeems to deſcribe the ſame Orbit, alſo *in conſequentia*, or from Weſt to Eaſt, at the Diſtance of ſix Signs; that is, ſeeming to ſet out from the firſt Degree of Cancer, juſt as the Earth does really ſet out from the firſt of Capricorn, as the Solar Index *i* (over *Taurus* in *Plate* 32.) plainly ſhews.

Now, as in the annual Motion of the Earth, the Axis always remains parallel to itſelf, the Situation of it, in reſpect of the Sun, muſt be continually chang'd; and a Plane paſſing through the ſaid Axis and the Number XII upon the Dial-Plate D, will not paſs through the Sun's Center again, when once it has left it, till at the Diſtance of half a Year, and then the North-Pole of the Earth (which was neareſt to the Sun before) will be fartheſt from the Sun. N. B. *The Situation of the Earth, in reſpect of the Pole Star, to which the Axis points, never ſenſibly changes;*

because the whole Diameter of the Earth's Orbit bears no Proportion to the Distance of the fix'd Stars, which are 400000 *times farther from us than the Sun, and therefore their Parallax can't amount to one Second Minute of a Degree.*

Secondly, Observe, as the Earth moves on in the Ecliptick, that the North Polar Circle does gradually get under the Solar Horizon, which cuts it into two equal Parts, as the Earth enters into *Aries* (and the Sun seemingly into *Libra*) as it does also every parallel Circle all over the Earth, making the Diurnal and Nocturnal Arcs equal; and this is call'd the Æquinox, the Days and Nights being then of 12 Hours all over the Earth, as is also shewn by the horary Index at D: And both Poles being equally enlightened, have the Sun in their Horizon; likewise the Solar Ray, after having successively pointed to all the Parallels supposed between the Tropick of Cancer and the Æquator, now points to the Æquator, whose every Part (for this Day, or one Turn of the Handle) successively receives the perpendicular Light and Heat of the Sun; which is what the *Ptolemaicks* call the Sun being in (or describing) the Æquator.

By continuing to turn the Handle, the Earth goes on gradually *in consequentia*, and the Solar Horizon shews how the Days continue to decrease as the diurnal Arcs shorten; and by degrees the whole Space contain'd by the North Polar Circle hides itself from the Sun, till it is quite behind the Solar Horizon, when the Earth comes to the first Degree of Cancer; which shews, that all the Countries within that Space have no Day at all, whilst the Countries within the South Polar Circle, being now on the Sunny-side of the Solar Horizon, have no Night at all. Now, the Solar Ray points to the Tropick of Capricorn, which by the Rotation of the Earth, the Sun seems (or is said) to describe. The Sun's Index also being in Capricorn, and likewise pointing to the 11th Day of *December*, shews, that we have the shortest Day at *London*, whose Measure is shewn by the horary Index. The Earth being still carried on, the Solar Ray will describe the Parallels over again, being at the Æquator, when the Earth enters Libra (and the Sun seemingly Aries), and the Solar Horizon divides all the Parallels into equal diurnal and nocturnal Arcs, the Poles being now in the Plane of the Solar Horizon. Then as the Earth goes on, more of the North Polar Circle comes out towards the Sun, the diurnal Arcs, and consequently the Days, in our Northern Hemisphere increase, whilst those beyond the Æquator decrease, till at last the Earth comes to the first Degree of Capricorn, from whence it mov'd at first: Then also is the Axis of the Earth return'd to the same Situation in respect to the Sun, that it had at first.

Thirdly, Take off the Earth and its Solar Horizon, and put on another Earth of 3 Inches Diameter, whose Center will still be in the Plane of the Ecliptick. This Earth having a graduated Meridian, and Pins (representing Inhabitants) perpendicularly stuck into the Tropicks, the Æquator,

tor, and ſome Parallels between; the Solar Ray (ſhortened a little to ſerve for this Earth) will more plainly ſhew, when the Sun is perpendicular to any of theſe Inhabitants, and how much it is more or leſs oblique to thoſe without the Tropick in different Seaſons, Pins being alſo fixed in thoſe Places. The Solar Ray will likewiſe exactly ſhew the Degrees of the Sun's Declination upon the Degrees of the Meridian of this Globe.

Fourthly, Take off this Globe, and put on another, which is only of 2 Inches Diameter, of Ivory, having a Map of the World grav'd upon it, and a Silver Horizon to ſet eaſily for any Place. By this the Globe with its Horizon differently ſet, and the Solar Ray, are ſhewn the Phænomena of the *right* Sphere, of the *oblique* Sphere, and of the *parallel* Sphere. Having ſet the Horizon for any Place, you ſee eaſily by the Solar Ray, when the Sun is above or below the Horizon of that Place, and at what Point of the Compaſs it riſes in that Place all the Year round: Whether it has Amplitude or not, whether Northern or Southern, and how much more or leſs in one Latitude than another. That is, how much and when, the Sun riſes and ſets North or South of the Points of due Eaſt and Weſt.

Fifthly, Put the little Earth on again, and having brought it to the firſt Degree of Capricorn, ſet the Hour-Index at XII towards the Sun, as alſo the Meridian of *London*. Then we are to conſider, that a Plane paſſing along the Axis of the Earth, and the Pole Star (at an immenſe Diſtance) does alſo paſs through the Meridian of *London*, and the Center of the Sun. Now, having unlocked the annual Motion, that the Earth may turn about its Axis without moving out of its Place, every Turn of the Handle will carry round the Meridian of *London* till it comes back into its Place again, and *London* faces the Pole Star as before; that is, returns into the Plane, out of which it was carried; and this is called a *Sidereal* Day. Now, as the Sun is in the ſame Plane as the Axis of the Earth and the Pole Star, the Sidereal Day and Solar Day will be the ſame, becauſe the Meridian of *London* comes to the Sun's Center at the ſame Time that it comes to the Pole Star. But as this depends upon the Immobility of the Earth's Center, it is not true in Fact; therefore to know what does really happen, we muſt put on the annual Motion. Then one Turn of the Handle not only gives the Earth one Turn about its Axis; but makes it go forward almoſt one Degree in the Ecliptick. * Now, if you obſerve the Meridian of *London*, you will find, that it is not yet come to the Solar Ray; becauſe tho' it is come into the Plane, that paſſes through the Center of the Earth and the Pole Star, this Plane no longer paſſes through the Center of the Sun, becauſe the Center of the Earth has quitted its Place, and moved *in conſequentia*; ſo that the Handle muſt be moved a little farther, before the Meridian of *London* (which has gone quite round to make a Sidereal Day) comes to the Solar Ray to make a Solar Day, which is longer than the Sidereal Day, the Space of four Minutes in Time. Therefore the

Earth turns about its Axis in 23 Hours and 56 Minutes; but any Country which has Noon, or the Sun in the South, muſt make more than one Revolution about the Axis to come to Noon again. This Difference between the Solar and Sidereal Day is ſo little, that it cannot eaſily be perceiv'd in one ſingle Revolution; but if we turn the Handle 15 times, thoſe 15 Differences making an Hour (taken all together) you will ſee, that after juſt 15 Turns, the Plane paſſing through the Earth, and the Pole Star has ſo far left the Sun, that the Solar Ray inſtead of pointing to the Meridian of *London*, only points to that, which is immediately before it, which is an Hour more eaſterly, and the Hour-Hand at Bottom, which points to XII, only ſhews the Sidereal Day; but muſt by means of the Handle be brought to the Number I, before it points to the Sun's Center. If the Earth be carried on one Month in its annual Courſe, the Hour-Index, when at XII, will want 2 Sidereal Hours of the Solar Noon: After 3 Months it will want 6 Hours: After 6 Months it will want 12 Hours, or be at the Solar Midnight, when it is at the Sidereal Noon; and this is the Reaſon, why the Stars near the Sun (which any Day at Noon would be viſible about the Sun, if it was totally eclipſed; but are inviſible, becauſe their Light is ſwallow'd up in his) after 6 Months will be viſible at Midnight. After 9 Months, the Hour-Index will want 18 Hours of pointing to the Sun; and after 12 Months, or one annual Revolution of the Earth, it wants 24 Hours, or nothing; that is, one Day is loſt, which the Sidereal Year wants of the Solar Year; this Day being made up of all the Differences, or 360 times 4 Minutes; but more exactly, $365\frac{1}{4}$ times 3 Minutes and 56 Seconds.

Sixthly, Bring back the Earth to the firſt Degree of Capricorn, take off the Solar Ray, and ſtick a little Index into the Sun in the Hole, which it has to repreſent one of the Spots, which have ſhewn us the Time of its Revolution round its Axis. Unlock the annual Motion, and ſet this new Solar Index towards the Meridian of *London*, or the Hour-Index at XII. After 25 Turns and $\frac{1}{4}$, the Solar Index will be come about and point to the Center of the Earth; but the Meridian of *London* and Hour-Index, which is in the Plane of it, will be got beyond XII quite to VI. This ſhews the abſolute Time of the Sun's Revolution round its Axis. Place every thing as before, then put on the annual Motion, obſerving, that as the Solar Index points to the Center of the Earth, it points alſo to the firſt Degree of Capricorn. Turn the Handle $25\frac{1}{4}$ times, and you will find that the Solar Index does not yet point to the Center of the Earth, (tho' by pointing to the firſt Degree of Capricorn, which the Earth has now left, the Sun appears to have made one entire Revolution about its Axis) but it requires near two Turns of the Handle to make it do ſo: This is the Reaſon, why Aſtronomers, who did not believe the Motion of the Earth, in obſerving a Solar Spot, reckoned one Revolution of the Sun round its Axis, when the Spot was ſeen from the Earth in the ſame Part of the Sun's Diſk, as it had been obſerv'd at firſt; and ſo affirm'd that

the

the Sun was about 27 Days turning on its Axis; when that Revolution is but apparent, the real Revolution being ended two Days before.

Seventhly, Take out the Pin or Index from the Sun's Surface, bring back the Earth to the firſt Degree of Capricorn, and unlock the annual Motion. Then on the Moon's Orbit *a b c* fix the Moon's Stem, and put on the Moon *e d*, which is repreſented by a little Silver Ball, whoſe Diameter is $\frac{2}{7}$ths of that of the Ball repreſenting the Earth, (according to the true Proportion of thoſe Bodies in Nature) having one Side plain, to repreſent that Side of the Moon, which we always ſee, and the other Side graved to repreſent that Side of the Moon, which is never ſeen from the Earth. Set the plain Side of the Moon towards the Earth, the Body of it being placed exactly between the Sun and the Earth, as in the Time of the New-moon; then turn the Handle. You will obſerve; firſt, that 27 Turns $\frac{1}{3}$, will carry the Moon quite round its Orbit, and bring it back to the ſame Place again, which ſhews the Moon's Period is 27 Days and near 8 Hours; ſecondly, that the ſame Side of the Moon is always turn'd towards the Earth, the graved Hemiſphere of the Ball being always averſe from it; thirdly, that the Moon turns only once about its Axis, whilſt it makes one Revolution round the Earth; and fourthly, that the *Periodical* Month would be the ſame as the *Synodical*, if the Earth had no annual Motion; that is, the Revolution of the Moon, counting its Beginning from the New or Full-moon, would not end till the next New or Full-moon: Whereas in Nature, the Period of the Moon (or periodical Month) is perform'd in ſomething leſs than 27 Days and $\frac{1}{2}$; but the ſynodical Month, or Time from New-moon to the next New-moon, or from Full-moon to the next Full-moon, takes up the Time of about 29 $\frac{1}{2}$ Days: Becauſe, in the Time of one Revolution, the Earth is carried on ſo far in the Ecliptick, that when the Moon has compleated its Period round the Earth, it is no longer in a Line with the Center of the Earth and Sun; but muſt move on ſtill in its Orbit above 2 Days, to be in a Line with the Sun and Earth. This is eaſily ſhewn by the Machine. For if you bring the Earth to the firſt Degree of Capricorn, and place the Moon between the Earth and Sun; and, having put on the annual Motion, turn the Handle 27 $\frac{1}{3}$ times, the Earth will be got on ſo far in the Ecliptick, that the Moon (tho' it has compleated its Period) is not yet in a Line with the Sun; but if you hold a String over the Center of the Moon and Earth, and another over the Center of the Sun and the firſt Degree of Capricorn (which Line the Earth and Moon did ſet out from) you would find thoſe two Strings to be parallel, which ſhews that the Moon has really finiſhed its Period, tho' it does not ſeem to have done ſo in reſpect of the Sun. But if you give the Handle a little more than two Turns, you will finiſh the ſynodical Month, by bringing the Moon again between the Sun and Earth: And the Earth's Place on the Ecliptick, will ſhew that it has advanced 29 Days and an half.

Plate 32.

Eighthly, Bring back the Earth to the first Degree of Cancer, and having put on the black Cap, which belongs to the Moon, with its convex Side opposite to the Sun as at *e*, bring the Moon between the Earth and Sun; that is, to the Place of New-moon: Then turning the Handle all the Phases of the Moon will be given successively, as they happen, whilst the Moon going round the Earth in its Orbit, is also carried round the Sun, together with the Earth. For, first, the black Cap representing the shadowed Part of the Earth, does wholly cover all the white Hemisphere of the Moon, which is turned towards the Earth, thereby shewing how the New-moon comes to be invisible to us; then after a few Turns of the Handle, you may perceive, that a small Part of the white Hemisphere is come from under the Cap (that is, out of the Shadow) and makes a Crescent, and as you go on, you see the first Quarter; the Half-moon after 7 Turns, and still more and more comes from under the Cap, till 14 Turns and $\frac{3}{4}$ makes the *Phænomenon* of Full-moon, when the white Hemisphere is come quite out of the Cap, and the Earth is between the Sun and Moon. As you turn on, the Phases of the Moon gradually decrease till you come to have New-moon again. And now all the above-mention'd *Phænomena*, may be seen at the same Time, as it happens in the Heavens.

Ninthly, Take off the Sun, and in the Place of it, put the Lantern belonging to the *Planetarium*, which has two convex Glasses (without which we could not have the Rays parallel, converging, or diverging, with sufficient Splendor) to shew how the Earth and Moon are enlightened by the Sun, together with the *Phænomena* of Eclipses. Let the Room be made dark, and then you will see the *Phænomena* of Day and Night, and the Seasons and Phases of the Moon above-mentioned, very plainly, without either the solar Horizon, or Moon's black Cap, which were only made use of, when we could not compare Light and Darkness.

By making use of the bigger Earth, with Pins properly fixed for Inhabitants, you may plainly see what Geographers say concerning the Inhabitants of the Earth, in respect of their Shadows, as they call them *Ascii*, *Amphiscii*, and *Heteroscii*; you will also the better compare together the Days and Seasons of the *Antæci*, *Periæci*, and *Antipodes*: as likewise (when you use the Ivory Earth, with its Silver Horizon) you see plainly how the Sun appears to rise above our Horizon and set below it, with or without Amplitude, real Rays of Light now performing what the Solar Rays of Brass Wire did before but imitate. There is also a Contrivance of a Silver Plate, to put upon any Place of the Surface of the Earth, parallel to the moveable Horizon above-mention'd, and which turns along with it, to shew the Distinction between the sensible and rational Horizon of a Place; but at the same time to make it appear, that the Sun's immense Distance makes them so nearly co-incide, that the rising and setting is the same at the same Moment in respect to both.

The

The Moon's ſecondary Light (that is, the Light, which being reflected from the Earth, gives the dark Part of the Moon's Hemiſphere, which is turn'd towards the Earth, ſuch a faint Illumination as we receive from the Moon) whereby in ſerene Weather, at the Time of the Moon's Creſcent or Wane, we ſee the remaining Part of the Moon's Diſk of a dun Colour (nay and ſometimes ſee the whole Moon itſelf at the very Time of the New-moon) is very well ſhewn here by the Light of the Lantern; as alſo how it is, that the Earth in its Turn, becomes a Moon to the Moon.

In order to ſhew the *Phænomena* of Eclipſes; firſt, put on the little Earth, the Moon being on its Stem, and the Lantern in its Place, and you will obſerve, how the Moon in paſſing between the Sun and the Earth, throws its Shadow upon ſome Part of the Earth, whoſe Inhabitants have a total Eclipſe of the Sun in thoſe Places, which the Shadow paſſes over, and a partial one in the *Penumbra*, or near the Shadow. When the Moon being Full, paſſes on the other Side of the Earth, ſo as to loſe her Light by the Interpoſition of the Earth's Body; that is, as ſhe goes through the Shadow of the Earth, there is a total Eclipſe of the Moon, which all the Time of its Duration is ſeen by all the Inhabitants of that Hemiſphere of the Earth, which is towards the Moon at that Time. Now, tho' this Poſition of the *Planetarium* does ſhew what a Solar and a Lunar Eclipſe is, it does not repreſent what really happens in regard to the Time of Eclipſes; for it ſhews an Eclipſe every New and Full-moon, whereas we ſeldom have more than four Eclipſes in a whole Year, and ſometimes not ſo many. The Reaſon is, that tho' the artificial Earth and Moon on the Machine, do bear a juſt Proportion to one another, the Orbit of the Moon is 10 times leſs than it ought to be in Proportion to the Bodies: Beſides, the Orbit of the Moon is not in the ſame Plane with the Ecliptick, but makes an Angle of about 5 Degrees with it; ſo that unleſs the Moon be in, or near, the Nodes (or the Places where the Moon's Orbit cuts the Plane of the Ecliptick) at the Time of Full or New-moon, there will be no Eclipſe; the Shadow of the Moon paſſing northerly over, or ſoutherly under the Globe of the Earth at New-moon; and the Moon paſſing northerly over, or ſoutherly under the Shadow of the Earth at Full-moon. Some of the *Orrery*-makers indeed have made the Moon riſe and fall in its Courſe, in an Orbit properly inclined; and beſides, have made the Nodes of that Orbit move backwards, or *in antecedentia* (that is, from Eaſt to Weſt) one Revolution in 19 Years, as it happens in Nature; but neither increaſing the Moon's Orbit, nor putting on ſmaller Balls for the Moon and Earth, this Inconveniency of too frequent Eclipſes has not been avoided. Others to avoid it, have given the Moon's Orbit 3 or 4 times a greater Inclination than it really has, deſtroying one Truth to explain another.

Now, in my *Planetarium*, I ſhew the true Inclination of the Moon's Orbit, the Motion of the Nodes, and true Magnitude of the Orbit, in reſpect

respect of the Bodies, and consequently the true *Phænomena*, relating to the Time and Quantity of the Eclipses, by making the following Changes.

I take off the Moon and Earth, and upon the silver'd Circle *n p* (which is divided into 19 Parts, corresponding with 19 Years to shew the Motion of the Nodes) I fix upon Pillars an Orbit of fine gilt Wire, properly inclined with Nodes upon it, and two small Moons, which, as well as a small Earth, now to be put on, bear a just Proportion to the Bigness of the Orbit: And now by means of a little Piece of white Paper, held behind the Earth at the Time of New-moon, the Shadows of the Earth and Moon shew whether there will be an Eclipse of the Sun or not; if an Eclipse, whether partial or total, of long or short Duration, whether Northerly or Southerly; likewise by holding a Paper behind the Moon at the Time of the Full-moon, you may see whether there will be an Eclipse of the Moon or not; and if an Eclipse, of what kind, *&c.*

Tenthly, There are Parts to fix to the *Planetarium*, to shew what *Parallaxes* are, and how they are to be observ'd; as the annual, menstrual, and diurnal; making it very evident, how the Distances of the Sun, and that of the fix'd Stars, have been attempted, but not sufficiently settled by those Methods, but that of the Moon exactly discover'd. There is also a Contrivance to shew what is meant by the *Longitude* at Sea, which shews how it must be at last found by the Appulses of the Moon to the fix'd Stars, when the Theory of the Moon comes to be perfectly settled.

I have also contrived an additional Part to shew a *Celidography* upon the *Planetarium*; that is, the *Phænomena* of the Spots of *Venus*, of Day and Night, and the Seasons in that Planet, which (according to *Signior Bianchini*'s Discoveries) does in 24 Days and 8 Hours, turn round an Axis inclin'd in an Angle of 15 Degrees, to the Plane of the Orbit of the Planet, that Axis always continuing parallel to itself. There is also a Wire to represent a perpendicular Solar Ray, and a Solar Horizon, to shew how sensible a Spiral the Sun must daily seem to move in to an Inhabitant of *Venus*. The Lantern also may be put on instead of the Sun, *&c.* But I forbear fixing this additional Machine to my *Planetarium*, till other Astronomers Observations have confirm'd *Signior Bianchini*'s new and great Discoveries.

The Machine to shew mechanically, how Planets and Comets, by a Ray drawn from the Sun, describe Areas *proportionable to the Times, which I promis'd in the sixth Annotation to* Lect. 5. (Page 424,) *properly belongs to the* Planetarium; *therefore I choose to describe it here.*

See Plate 29. Fig. 7, *and* 8.

The 7th *Figure* repreſents the upper Surface of a Wooden-Frame, or Caſe of Clock-Work, held together by four Skrews. E T F V is a ſilvered circular Plate, divided into 88 Parts (88 being the Number of Days of Mercury's periodical Time) with an Handle or Winch H G, and Index G E. I K *p* P L M is a ſilvered oval Plate, divided into the ſame Number of Parts as the Circular Plate, with a Channel or Groove in it, for the little Braſs Ball or Planet P to move in. S repreſents the Sun in one of its *Foci*, L the *Perihelion*, and I the *Aphelion*. S O is an Index of Steel, which going thro' the Planet P, carries it in the elliptick Channel, with a Velocity reciprocally proportional to its Diſtance from S. The Part S P of this Index, which is intercepted between the Sun and Planet, or S and P, repreſents the *Radius Vector* (already deſcribed) which continually becomes ſhorter, as the Planet goes from the *Aphelion* to the *Perihelion*, and lengthens gradually as the Planet goes from the *Perihelion* to the *Aphelion*. The Part P O of the Index, which goes beyond the Planet, and whoſe End O does here deſcribe the Circle O Q R I N, is of no Conſequence in our Aſtronomical Conſideration. If a Planet (as for Example, Mercury) moved in a circular Orbit, concentrick with the Sun, it would not only deſcribe equal *Areas* in equal Times, but alſo equal Arcs; then by uniformly turning the Handle H, the End of the Index at E would truly repreſent the Motion of ſuch a Planet. But if the Planet moves in an Ellipſe in one of whoſe *Foci* the Sun is, and the *Perihelion* be 6 times nearer than the *Aphelion*; the Planet ſetting out from the *Aphelion* I, will move in an accelerated Motion in the Direction I K, P L, and from L will come by M to I with a retarded Motion, and ſo in all its Revolutions continually change its Velocity, deſcribing equal *Areas* in equal Times, whoſe Equality is ſhewn, and proved in the following Manner. The Planet P with its Index through it, being brought to I, ſet the Index G E at E; then by means of the Handle H, carry on the Index 10 Degrees, or from E to T, and you will perceive at the ſame time, that the Planet in the Ellipſe will only go from I to K $3\frac{1}{3}$ Diviſions, the *Radius Vector* having moved from S I to S K, whereby the *Area* S I K will be deſcribed. Then if the Planet be ſet at the *Perihelion* L (to which Place it will be brought by turning the Handle H till the Index comes to F) the *Radius Vector* on the Ellipſe will then be S L: And if the Index of the circular Plate which repreſents the Time of the Motion, be carried on from F (44 Degrees) to V (54 Degrees) the Planet will go from L the whole Arc L M, or 20 Degrees, ſetting out 6 times faſter than it did at firſt from I, the *Radius Vector* deſcribing the *Area* S L M, ſo much broader than S I K, as S L is ſhorter than S I, which makes this laſt *Area* S L M equal to the former *Area* S I K, and the Velocity of the Planet reciprocally as the Diſtance. If any intermediate Place be taken in the Curve of the Ellipſe; as

for Example, the Point *p*, whilſt the Index on the circular Plate moves 10 Degrees, the Planet will go through the elliptical Arc *p* P, the *Radius Vector* deſcribing the *Area p* S P equal to the former *Areas*.

Now, to ſhew the Contrivance of the Machine, let us take off the upper Board of the Frame, as in *Figure* 8, where the circular and oval Plates, the Planet-Handle and the Indices, are only repreſented in pricked Lines to ſhew their Places. Under the Bar or Bridge V G, are two Ovals Q T and O N, moving each other about by means of a Cat-gut in a Groove on their Edge, which Gut croſſes at K. Theſe Ovals have their Centers of Motion on their alternate *Foci* S and I. The Sun and Center of Motion of the *Radius Vector* is fixed over S. Now if the Handle be placed at I the Center of the other Ellipſe N O, that Part of the Circumference of the Ellipſe Q T, which touches the Circumference of the Ellipſe N O, is carried round with Velocities proportionable to the Diſtances from I, or to the Length of the Lines I K, I 4, I 3, I 2, I 1, *&c.* which repreſent unequal Leavers. Now, in the preſent Situation repreſented by the Draught, the Velocity of the Planet P in *Perihelio* is greateſt of all; for P S being equal to S K, P has the Velocity of the Point K, which is led by the longeſt Leaver I K. But when the Ellipſe N O has made half a Turn, the Ellipſe Q T changes its Situation, its Point V coming to *s*, where it is led by the Leaver I 1, ſix times ſhorter than before, and the Planet having ſlipped along its *Radius Vector*, quite to its End, is over the Point I at its *Aphelion*, where it has 3 times leſs Velocity. The Reſt in relation to the different Situation of the Ovals may be eaſily conceived by carefully obſerving the Figure. As for the two Wheels whoſe Numbers of Teeth are equal (no matter what the Numbers are) they are only made uſe of to keep the Circle and Oval on the Surface of the Machine (*Fig.* 7.) at a proper Diſtance, and to give the Planet and the Index of the circular Plate the ſame Direction.

N. B. *The Ovals of Wood, and the Oval ſilvered Plate on the Machine, muſt be ſimilar; but they may be any kind of Ellipſes. I have taken one here, more excentrick than any Planet's Orbit, only to make the* Phænomena *the more ſenſible; tho' not ſo excentrick as a Comet's Orbit, which would have exceeded the Bounds of the Machine.*

The END of the DESCRIPTION of the PLANETARIUM.

AN ALPHABETICAL

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